

The Mathematical Association of Victoria

Trial Exam 2018

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 11 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale .
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (5 marks)

a. Find $\frac{dy}{dx}$ when $y = 2x \log_e(2x)$.

2 marks

b. Find $f'(0)$ when $f(x) = \frac{e^{x^2}}{e^x + 1}$.

3 marks

TURN OVER

Question 2 (4 marks)

The graph of the function $f(x) = 18x^2 - ax^4$ where $a \in R^+$ has a stationary point at $x = -\frac{3}{\sqrt{5}}$.

- a.** Show that the value of a is 5.

2 marks

- b.** Hence state the coordinates of the stationary points for the graph of f .

2 marks

Question 3 (4 marks)

Let $g : \left[\frac{1}{2}, \infty\right) \rightarrow \mathbb{R}$, where $g(x) = \sqrt{(2x-1)}$.

- a. Show that the equation of the tangent to the curve at $x = 1$ is $y = x$.

2 marks

- b. Find the area bounded by the graph of the function g , the line $y = x$ and the x -axis.

2 marks

TURN OVER

Question 4 (3 marks)

A company is investigating the efficiency of one of its machines which packages muesli. A random sample of 400 packets was taken and the 95% confidence interval for the proportion of underweight muesli packets was found to be (0.024, 0.132).

- a.** Find the sample proportion from which this interval was obtained. 1 mark

- b.** Using the nearest integer value for z , determine the standard deviation of the sample. 2 marks

Question 5 (6 marks)

- a. Find the general solution to $\sqrt{3}\sin(2x) = \cos(2x)$.

2 marks

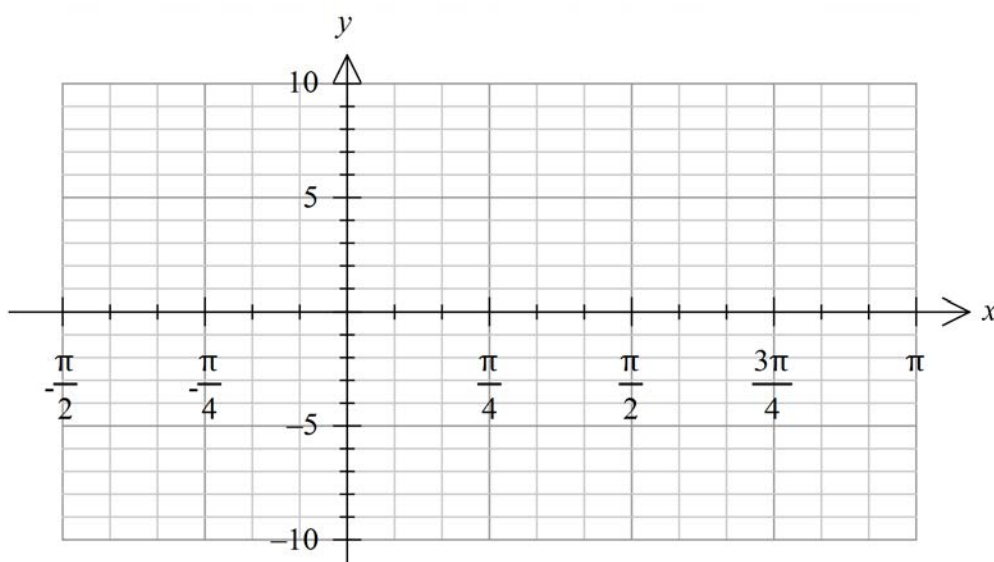
- b. Show that there are no stationary points in the graph of $y = \sqrt{3}\tan(2x)$.

1 mark

- c. **Hence** sketch the graph of $y = \sqrt{3}\tan(2x)$ for $x \in \left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$. On your graph label the

points where $\sqrt{3}\tan(2x) = 1$ and the x -intercepts with their coordinates. Label any asymptotes with their equations.

3 marks

**TURN OVER**

Question 6 (4 marks)

Solve $2\log_2(x-2) + \log_2(x) = 0$ for x .

Question 7 (4 marks)

The height of water in a dam is modelled by the function h with rule $h(t) = -20 \sin\left(\frac{\pi t}{12} + 10\right) + 20$, where h is the height in metres at time t hours after 9 am.

Find the average value of the height of water in a dam, from 9 am to 9 pm on a certain day, giving your answer in the form $a \cos(b) + c$, where a , b and c are real constants.

TURN OVER

Question 8 (5 marks)

A random variable X has the following probability distribution, where a and b are real constants.

x	0	1	2	3
$\Pr(X = x)$	0.1	a	b	0.1

- a. i.** Find b in terms of a .

1 mark

- ii.** Hence show that $E(X) = -a + 1.9$.

1 mark

- b.** If $\text{Var}(X) = 0.56$ find all possible sets of values for a and b .

3 marks

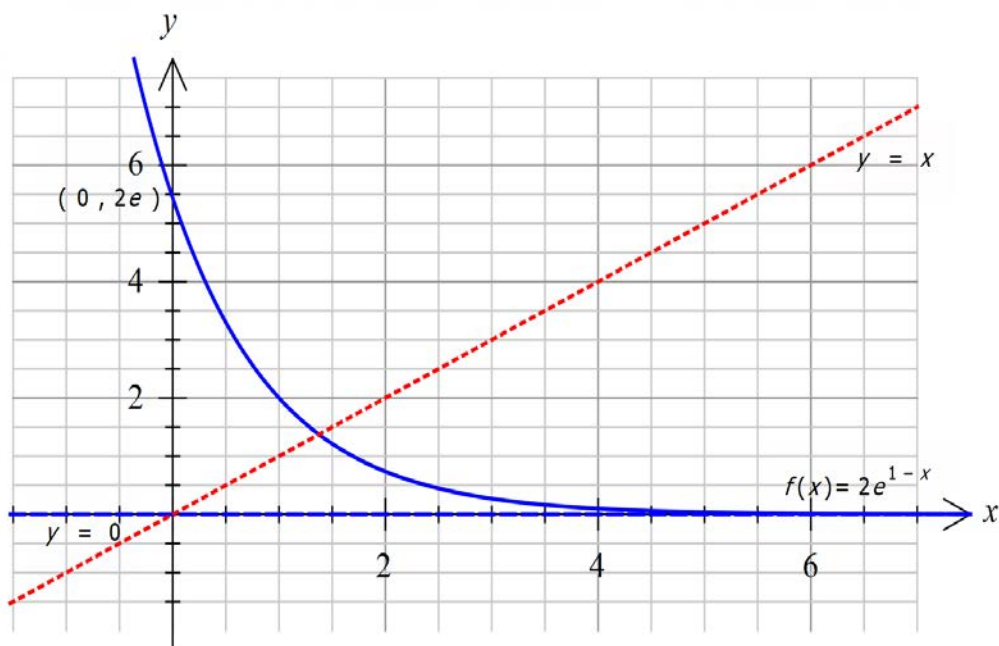
Question 9 (5 marks)Let $f(x) = 2e^{1-x}$.

- a. Find the rule and state the range for the inverse function, f^{-1} .

3 marks

- b. Part of the graph of f is shown below. Sketch the graph of f^{-1} on the same set of axes. Label the asymptote with its equation and axial intercept with its coordinates.

2 marks

**END OF QUESTION AND ANSWER BOOK**

