

Year 2018

VCE

Mathematical Methods

Trial Examination 1

Solutions



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Question 1

a. $y = \frac{\cos(2x)}{2x}$ using the quotient rule

$$u = \cos(2x) \quad v = 2x$$

$$\frac{du}{dx} = -2\sin(2x) \quad \frac{dv}{dx} = 2 \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{-4x\sin(2x) - 2\cos(2x)}{4x^2} \quad \text{A1}$$

b. Let $y = e^{\sqrt{x}}$

$$y = e^u \quad u = \sqrt{x} = x^{\frac{1}{2}} \quad \text{chain rule}$$

$$\frac{dy}{du} = e^u \quad \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = g'(x) = \frac{1}{2\sqrt{x}} e^{\sqrt{x}} \quad \text{M1}$$

$$g'(4) = \frac{1}{2\sqrt{4}} e^{\sqrt{4}}$$

$$g'(4) = \frac{e^2}{4} \quad \text{A1}$$

Question 2

$$X \sim \text{Bi}(n=4, p="p")$$

$$\Pr(X=1) = \binom{4}{1} p(1-p)^3 = 2\Pr(X=2) = 2\binom{4}{2} p^2(1-p)^2 \quad \text{A1}$$

$$4p(1-p)^3 = 2 \times 6p^2(1-p)^2$$

$$4p(1-p)^3 - 12p^2(1-p)^2 = 0$$

$$4p(1-p)^2(1-p-3p) = 0$$

$$4p=1 \quad \text{since } 0 < p < 1 \quad \text{M1}$$

$$p = \frac{1}{4} \quad \text{A1}$$

Question 3

a. $y = \sqrt{16 - x^2} = u^{\frac{1}{2}}$, $u = 16 - x^2$ chain rule

$$\frac{dy}{du} = \frac{1}{2} u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}} \quad , \quad \frac{du}{dx} = -2x$$

A1

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{-2x}{2\sqrt{u}} = \frac{-x}{\sqrt{16 - x^2}}$$

$$\text{at } x = p \quad m_T = \left. \frac{dy}{dx} \right|_{x=p} = \frac{-p}{\sqrt{16 - p^2}} = \tan(60^\circ) = \sqrt{3}$$

M1

$$-p = \sqrt{3} \times \sqrt{16 - p^2} \quad \text{so } p < 0$$

$$p^2 = 3(16 - p^2) = 48 - 3p^2$$

$$4p^2 = 48$$

$$p^2 = 12$$

$$p = -2\sqrt{3}$$

A1

b.i. $A = 2ab$ however $b = \sqrt{16 - a^2}$

A1

$$A = 2a\sqrt{16 - a^2}$$

ii. Using the product rule and **a.**

$$\frac{dA}{da} = \frac{d}{da}(2a)\sqrt{16 - a^2} + 2a \frac{d}{da}(\sqrt{16 - a^2})$$

$$= 2\sqrt{16 - a^2} - \frac{2a^2}{\sqrt{16 - a^2}} = 0 \quad \text{for maximum}$$

M1

$$2\sqrt{16 - a^2} = \frac{2a^2}{\sqrt{16 - a^2}}$$

$$16 - a^2 = a^2$$

$$2a^2 = 16$$

$$a^2 = 8 \quad \text{since } a > 0$$

$$a = \sqrt{8} = 2\sqrt{2}$$

A1

Question 4

- a.** $\log_8(x+5) + \log_8(3x-1) = 2$
 $\log_8(x+5)(3x-1) = 2$
 $(x+5)(3x-1) = 8^2 = 64$ M1
 $3x^2 + 14x - 5 = 64$
 $3x^2 + 14x - 69 = 0$
 $(3x+23)(x-3) = 0$
 $x = -\frac{23}{3}, 3$ but $x > \frac{1}{3}$
 $x = 3$ as the only answer A1
- b.** Let $u = 8^x$, $64^x = (8^2)^x = 8^{2x} = (8^x)^2 = u^2$
 $32 \times 64^x - 12 \times 8^x + 1 = 0$
 $32u^2 - 12u + 1 = 0$
 $(8u-1)(4u-1) = 0$ M1
 $u = 8^x = \frac{1}{8}, \frac{1}{4}$
 $x = -1, -\frac{2}{3}$ A1

Question 5

- a.i** number of red, 0, 1, 2, 3, in a total of 10, so $\hat{P} = \frac{x}{10}$
 $\hat{P} = \{0, \frac{1}{10}, \frac{1}{5}, \frac{3}{10}\}$ A1
- ii.** $\Pr\left(\hat{P} = \frac{1}{5}\right) = \Pr(2R) = RRO + ROR + ORR = 3 \times \frac{3}{10} \times \frac{2}{9} \times \frac{7}{8}$
 $\Pr\left(\hat{P} = \frac{1}{5}\right) = \frac{7}{40}$ A1
- b.** $n = 300$, $p = \frac{1}{4}$
 $sd(\hat{P}) = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{\frac{1}{4} \times \frac{3}{4}}{300}} = \frac{1}{40}$ A1
 $95\% \quad \hat{p} \pm 2sd(\hat{p}) = \frac{1}{4} \pm 2 \times \frac{1}{40} = \frac{5}{20} \pm \frac{1}{20}$
 $\left(\frac{1}{5}, \frac{3}{10}\right)$ A1

Question 6**a.** completing the square

$$g(x) = 3 + 4x - x^2 = -(x^2 - 4x + 4) + 3 + 4 = 7 - (x - 2)^2$$

$$\text{range } g = (-\infty, 7] \quad \text{A1}$$

$$f(x) = \log_e(x+2) \quad \text{domain } x+2 > 0 \Rightarrow x > -2, \text{ range } R$$

	$f(x)$	$g(x)$
domain	$(-2, \infty)$	R
range	R	$(-\infty, 7]$

Since range $g \not\subset$ domain f , so $f(g(x))$ does not exist. A1

b. solving $g(x) = 3 + 4x - x^2 = -2$

$$\Rightarrow x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0 \quad \text{M1}$$

$$\Rightarrow x = -1, 5$$

now $g(-1) = g(5) = -2$, $g(2) = 7$, so if we now restrict the domain of g , as

domain $g(x) = D = (-1, 5) = \text{domain } f(g(x))$ now the range of $g = (-2, 7)$

so range $g \subset$ domain f , so now $f(g(x))$ exist. A1

$$f(g(x)) = f(3 + 4x - x^2) = \log_e(3 + 4x - x^2 + 2)$$

$$f(g(x)) = \log_e(5 + 4x - x^2) = \log_e((5-x)(x+1))$$

$$f(g(x)) : (-1, 5) \rightarrow R, \quad f(g(x)) = \log_e(5 + 4x - x^2) = \log_e((5-x)(x+1)) \quad \text{A1}$$

Question 7**a.** Since it is discrete probability distribution $\sum \Pr(X = x) = 1$

$$2\cos^2(k) + \cos(k) = 1$$

$$2\cos^2(k) + \cos(k) - 1 = 0$$

$$(2\cos(k) - 1)(\cos(k) + 1) = 0 \quad \text{M1}$$

$$\cos(k) = \frac{1}{2}, \quad \cos(k) = -1$$

$$k = \frac{\pi}{3}, 2\pi - \frac{\pi}{3} \quad k = \pi \quad \text{since } 0 \leq k \leq 2\pi$$

but $k = \pi$ is not valid as $\cos(\pi) = -1$ and each probability must be positive.

$$k = \frac{\pi}{3}, \frac{5\pi}{3} \text{ are the only answers in } 0 \leq k \leq 2\pi \quad \text{A1}$$

b. $E(X) = \sum x \Pr(X = x)$

$$E(X) = 2\cos^2(k) + 2\cos(k) = (2\cos^2(k) + \cos(k)) + \cos(k) = 1 + \frac{1}{2} = \frac{3}{2}$$

$$E(X^2) = \sum x^2 \Pr(X = x)$$

$$E(X^2) = 2\cos^2(k) + 4\cos(k) = (2\cos^2(k) + \cos(k)) + 3\cos(k) = 1 + \frac{3}{2} = \frac{5}{2}$$

$$\text{var}(X) = E(X^2) - (E(X))^2 = \frac{5}{2} - \left(\frac{3}{2}\right)^2 = \frac{5}{2} - \frac{9}{4}$$

$$\text{var}(X) = \frac{1}{4}$$

A1

Question 8

- a. The sine wave part has a length of π , and is one quarter of a cycle,

therefore one cycle $\frac{2\pi}{n} = 4\pi \quad n = \frac{1}{2}$

A1

- b. Since the function is continuous at $x = \pi$, $k\pi = a \sin\left(\frac{\pi}{2}\right) \Rightarrow a = k\pi$

A1

Since the total area under the curve is one.

$$\int_0^\pi kx \, dx + \int_\pi^{2\pi} a \sin\left(\frac{x}{2}\right) dx = 1$$

$$\left[\frac{1}{2}kx^2\right]_0^\pi + \left[-2a \cos\left(\frac{x}{2}\right)\right]_\pi^{2\pi} = 1$$

M1

$$\frac{1}{2}k\pi^2 - 2a \cos(\pi) + 2a \cos\left(\frac{\pi}{2}\right) = 1$$

$$\frac{1}{2}k\pi^2 + 2a = 1 \quad \text{substitute } a = k\pi, \text{ solve for } k$$

M1

$$\frac{1}{2}k\pi^2 + 2k\pi = 1$$

$$k\pi^2 + 4k\pi = 2$$

$$k\pi(\pi + 4) = 2$$

$$k = \frac{2}{\pi(\pi + 4)}, \quad a = \frac{2}{\pi + 4}$$

A1

Question 9

- a. Let $y = x^2 \log_e(2x)$ using the product rule

$$\begin{aligned}\frac{dy}{dx} &= x^2 \frac{d}{dx}[\log_e(2x)] + \log_e(2x) \frac{d}{dx}(x^2) \\ &= x^2 \times \frac{1}{x} + 2x \log_e(2x) \\ \frac{d}{dx}[x^2 \log_e(2x)] &= x + 2x \log_e(2x)\end{aligned}\quad \text{A1}$$

- b. $f(x) = x \log_e(2x)$ using the product rule

$$\begin{aligned}f'(x) &= x \frac{d}{dx}[\log_e(2x)] + \log_e(2x) \frac{d}{dx}(x) \\ &= x \times \frac{1}{x} + \log_e(2x) \\ &= 1 + \log_e(2x)\end{aligned}$$

$$\text{for turning points } f'(x) = 0 \Rightarrow 1 + \log_e(2x) = 0 \quad \text{M1}$$

$$\log_e(2x) = -1 \quad 2x = e^{-1} \quad x = \frac{1}{2e} \quad f\left(\frac{1}{2e}\right) = \frac{1}{2e} \log_e\left(\frac{1}{e}\right) = -\frac{1}{2e}$$

$$\text{the minimum turning point is } \left(\frac{1}{2e}, -\frac{1}{2e}\right) \quad \text{A1}$$

- c. $A = \int_1^2 x \log_e(2x) dx$

$$\text{from a. } \frac{d}{dx}[x^2 \log_e(2x)] = x + 2x \log_e(2x)$$

$$\int (x + 2x \log_e(2x)) dx = \int x dx + 2 \int x \log_e(2x) dx = x^2 \log_e(2x)$$

$$2 \int x \log_e(2x) dx = x^2 \log_e(2x) - \int x dx = x^2 \log_e(2x) - \frac{1}{2} x^2 \quad \text{M1}$$

$$\int x \log_e(2x) dx = \frac{x^2}{4} (2 \log_e(2x) - 1)$$

$$A = \left[\frac{x^2}{4} (2 \log_e(2x) - 1) \right]_1^2 \quad \text{A1}$$

$$A = (2 \log_e(4) - 1) - \left(\frac{1}{4} (2 \log_e(2) - 1) \right) = 4 \log_e(2) - \frac{1}{2} \log_e(2) - \frac{3}{4}$$

$$A = \frac{7}{2} \log_e(2) - \frac{3}{4} \quad \text{A1}$$

END OF SUGGESTED SOLUTIONS