

# 2018 Mathematical Methods Trial Exam 2 Solutions

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## SECTION A – Multiple-choice questions

|   |   |   |   |   |   |   |   |   |    |
|---|---|---|---|---|---|---|---|---|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| B | C | A | A | C | C | D | D | E | E  |

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| E  | A  | D  | E  | D  | C  | D  | A  | B  | E  |

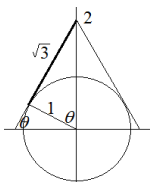
Q1  $x(x+1)-(x+1)=0$ ,  $(x+1)(x-1)=0$ ,  $x=\pm 1$

Q2  $\log_e 0$  is undefined.

Q3 The domain of the inverse is the range of the function.

Q4  $x = e^{\log_e(\frac{y}{2})}$ ,  $x = \frac{y}{2}$ ,  $y = 2x$

Q5  $m = \tan \theta = \sqrt{3}$



Q6  $f(x) - f(-x) = 0$ ,  $f(x)$  is even.  $\tan\left(x + \frac{\pi}{2}\right)$  is not even.

Q7  $f'(x) = 2x(2x^2 + b - a) = 0$ , one solution if  $a = b$ , three solutions if  $a > b$ .

Q8 When  $y = 2$ ,  $x = 1$ ; when  $y = 4$ ,  $x = 3$ .

Q9 The function oscillates about 3.

Q10 Average rate

$$= \frac{1}{10000-100} \left( \log_{10} \left( \sqrt{10000} + \frac{1}{\sqrt{10000}} \right) - \log_{10} \left( \sqrt{100} + \frac{1}{\sqrt{100}} \right) \right)$$

$$= \frac{1}{9900} \left( \log_{10} \frac{100.01}{10.1} \right) \approx 0.0001$$

Q11  $m_1 m_2 = -1$ ,  $\frac{a}{b} \times \frac{a^2}{b^2} = -1$ ,  $\frac{a^3}{b^3} = -1$ ,  $a^3 = -b^3$ ,  $a^3 + b^3 = 0$

Q12  $\int_0^{2a} \left( 2f\left(\frac{x}{2}\right) + 1 \right) dx = 2 \int_0^{2a} f\left(\frac{x}{2}\right) dx + \int_0^{2a} 1 dx = 2(2b) + 2a$

Q13  $y = f(x) \rightarrow y = f(x-b) \rightarrow y = f(-x-b) \rightarrow y = f\left(-\frac{x}{a}-b\right)$

$$y = f\left(-\left(\frac{x}{a}+b\right)\right)$$

Q14



Q15 The difference between the two numbers is a possible random variable with values of 0 or 1.

Q16  $\sum \Pr = 1$ ,  $a^2 + 0.6a - 0.07 = 0$ ,  $(a-0.1)(a+0.7) = 0$ ,  $a = 0.1$

$$\bar{X} = 0.50a + a^2(a+1) + 0.43(a+2) + 0.6a(a+3) = 1.15$$

Q17  $\int_0^a e^{x+a} dx = 1$ ,  $[e^{x+a}]_0^a = 1$ ,  $e^{2a} - e^a - 1 = 0$ ,  $a \approx 0.4812$

Q18  $\Pr((A \cup B)') = 1 - \Pr(A \cup B)$ ,  $\Pr(A \cup B) = 1 - 0.2 = 0.8$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B),$$

$\Pr(A \cap B) = 0.6 + 0.3 - 0.8 = 0.1$ ,  $\Pr(A|B) = \frac{0.1}{0.3} \approx 0.33$

Q19  $n = 9$ ,  $p = 0.6$  and  $q = 0.4$

$\therefore \mu = np = 5.4$  and  $\sigma = \sqrt{npq} \approx 1.47$

Q20 The mean of  $\hat{P} = p = 0.525$ ,  $n = 500$

The standard deviation of  $\hat{P} = \sqrt{\frac{p(1-p)}{n}} \approx 0.022333$

$\Pr(\hat{P} > 0.60) \approx 0.00039$  (normal approx)

Number of samples  $\approx 10 \times 0.00039 \approx 0$

## SECTION B

Q1a  $y^2 = 225$ ,  $0 \leq x \leq 15$

Q1b  $x^2 + y^2 = 225$ ,  $-15 \leq x \leq 0$

Q1c Dilate parallel to the y-axis by a factor of  $\frac{2}{3}$ .

$\left(\frac{y}{\frac{2}{3}}\right)^2 = 225$ ,  $y^2 = 100$

Q1d Dilate parallel to both axes by a factor of  $\frac{2}{3}$ .

$\left(\frac{x}{\frac{2}{3}}\right)^2 + \left(\frac{y}{\frac{2}{3}}\right)^2 = 225$ ,  $x^2 + y^2 = 100$

Q1e  $x = -(b+10)$ ,  $y = \frac{a}{2}$ ,  $(b+10)^2 + \left(\frac{a}{2}\right)^2 = 225$ ,

$a^2 = 500 - 80b - 4b^2$

Q1f  $A = b\sqrt{500 - 80b - 4b^2}$ . Let  $\frac{dA}{db} = 0$ ,  $2b^2 + 30b - 125 = 0$

$b = \frac{-30 + \sqrt{1900}}{4} = \frac{-15 + 5\sqrt{19}}{2}$

$a = \sqrt{500 - 80b - 4b^2} = \sqrt{500 - 80b - (-60b + 250)}$

$$= 5\sqrt{16 - 2\sqrt{19}}$$



$$Q2a \quad \frac{dy}{dx} = \frac{(x^2 \left(\frac{20}{x}\right) - (20 \log_e x)(2x))}{x^4} = \frac{20}{x^3} (1 - 2 \log_e x)$$

$$Q2b \quad 1 - 2 \log_e x = 0, \quad x = e^{\frac{1}{2}} = \sqrt{e}, \quad y = \frac{20 \log_e x}{x^2} = \frac{10}{e}$$

$$\left(\sqrt{e}, \frac{10}{e}\right) \text{ or approx. } (1.6487, 3.6788)$$

$$Q2c \quad x = 10, \quad y \approx 0.4605, \quad \frac{dy}{dx} \approx -0.0721$$

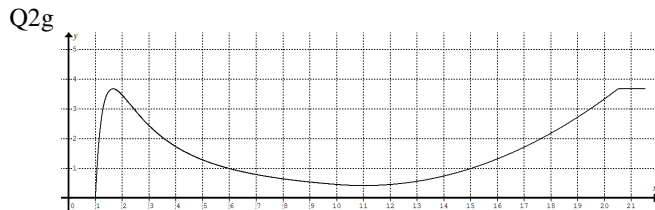
$$Q2d \quad y = a(x-11)^2 + b, \quad \frac{dy}{dx} = 2a(x-11)$$

$$2a(10-11) \approx -0.0721 \text{ and } a(10-11)^2 + b \approx 0.4605$$

$$a \approx 0.0361 \text{ and } b \approx 0.4245$$

$$Q2e \quad 0.0361(c-11)^2 + 0.4245 \approx 3.6788, \quad c \approx 20.4946$$

$$Q2f \quad y \approx 3.6788, \quad 20.4946 \leq x \leq 21.5$$



$$Q2h \quad \text{Total area} = 13.3927 + 14.7666 + 3.6987 \approx 32 \text{ m}^2$$

$$Q3a \quad x = \sin(2\pi t) + \cos(2\pi t) = \sin(0.30\pi) + \cos(0.30\pi) \approx 1.40$$

$$Q3b \quad \text{Let } \frac{dx}{dt} = 0. \quad 2\pi \cos(2\pi t) - 2\pi \sin(2\pi t) = 0, \quad \tan(2\pi t) = 1,$$

$$2\pi t = \frac{\pi}{4}, \quad t = \frac{1}{8}, \frac{5}{8}, \quad \max x = \sqrt{2}, \quad \min x = -\sqrt{2}$$

$$Q3c \quad \Delta t = \frac{5}{8} - \frac{1}{8} = \frac{1}{2} \text{ s}$$

$$Q3d \quad x = \sin(2\pi t) + \cos(2\pi t), \quad v = \frac{dx}{dt} = 2\pi(\cos(2\pi t) - \sin(2\pi t))$$

$$\text{Let } \frac{dv}{dt} = 0, \quad -(2\pi)^2(\sin(2\pi t) + \cos(2\pi t)) = 0, \quad \tan(2\pi t) = -1,$$

$$2\pi t = \frac{3\pi}{4}, \quad t = \frac{3}{8} \text{ s}$$

$$Q3e \quad v = 2\pi \left( \cos\left(\frac{3\pi}{4}\right) - \sin\left(\frac{3\pi}{4}\right) \right) = -2\pi\sqrt{2}, \quad \max$$

$$\text{speed} = 2\sqrt{2}\pi \text{ m s}^{-1}$$

$$Q3f \quad \text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{2\sqrt{2}}{\frac{1}{2}} = 4\sqrt{2} \text{ m s}^{-1}$$

Q3g Use CAS to sketch  $x = \sin(2\pi t) + \cos(2\pi t)$ . It is a sine graph with a period of 1 and amplitude of 1.41 approximately.

$$x \approx 1.41 \sin(2\pi t - \beta), \quad 1.41 \approx 1.41 \sin\left(\frac{2\pi}{8} - \beta\right), \quad \frac{2\pi}{8} - \beta = \frac{\pi}{2},$$

$$\beta = -\frac{\pi}{4} \approx -0.79 \text{ and } \alpha \approx 1.41$$

$$Q3h \quad x \approx 1.41 \sin(2\pi t + 0.79) = 1.41 \cos\left(\frac{\pi}{2} - (2\pi t + 0.79)\right)$$

$$\approx 1.41 \cos(0.78 - 2\pi t) \text{ or } 1.41 \cos(6.28t - 0.78)$$

$$Q3i \quad x \approx 1.41 \sin(2\pi t + 0.79) = 1.41 \sin(2\pi(t + 0.13))$$

Dilate  $x = \sin(2\pi t)$  parallel to the vertical axis by a factor of 1.41, and translate parallel to the horizontal axis by 0.13 (accept 0.12) to the left.

Q4a

| Length      | Very short | Short   | Medium  | Long    | Very long |
|-------------|------------|---------|---------|---------|-----------|
| Probability | 0.00003    | 0.02272 | 0.47725 | 0.47725 | 0.02275   |

Q4b Picking a bundle of firewood for sale is a random process, hence the price  $C$  is a random variable.

Note: It is incorrect to say the buyer's choice of firewood is random and hence  $C$  is a random variable.

$C$  is discrete because the prices are single values and not an interval (or intervals) of values.

Q4c

| $C$ (\$)    | 45      | 49      | 50      | 51      | 55      |
|-------------|---------|---------|---------|---------|---------|
| Probability | 0.02275 | 0.00003 | 0.02272 | 0.47725 | 0.47725 |

Q4d Total amount (\$)

$$= 600(45 \times 0.02275 + 49 \times 0.00003 + 50 \times 0.02272 + 51 \times 0.47725 + 55 \times 0.47725)$$

$$\approx 31650$$

$$Q4e \quad 600 \times 0.47725 \approx 286$$

$$Q4f \quad \text{Binomial: } n = 5, \quad p = \frac{0.47725 + 0.02275}{0.47725 + 0.47725 + 0.02275} \approx 0.51164$$

$$\Pr(X > 3) = \Pr(X = 4) + \Pr(X = 5) \approx 0.16733 + 0.03506 \approx 0.2$$

$$Q4g \quad 0.48 \times 25 = 12$$

Q4h The standard deviation of sample proportion

$$\approx \sqrt{\frac{0.48(1-0.48)}{25}} \approx 0.10$$

Q4i An approximate 95% confidence interval

$$\approx (0.48 - 1.96 \times 0.10, 0.48 + 1.96 \times 0.10) = (0.284, 0.676)$$

Please inform [mathline@itute.com](mailto:mathline@itute.com) re conceptual and/or mathematical errors