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Mathematical Methods

2018

Trial Examination 1 (1 hour)

Instructions

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1 The graph of $f(x) = mx^2 + 1$ and the graph of its inverse touch at a point.

a. Find the value(s) of m .

2 marks

b. Find the coordinates of the point where the two graphs are in contact.

2 marks

Question 2 Consider $f'(x) = 3(x-1)^2 + m$ where $m \in \mathbb{R}$.

a. Find the values of m for $f(x)$ to have two stationary points, one stationary point and no stationary point.

2 marks

b. Write down a possible $f(x)$ with no stationary point in $ax^3 + bx^2 + cx + d$ form.

2 marks

Question 3 Consider polynomial function $f(x) = 16x^4 + 8x^3 + 7$.

- a. Find the remainder when $f(x)$ is divided by $2x - 1$. 1 mark

- b. Specify a translation of $f(x)$ so that the resulting graph intersects the x -axis at $\left(\frac{1}{2}, 0\right)$. 1 mark

- c. Let $g(x) = 2(2x - 1)(4x^3 + px^2 + qx + 1)$ be the function representing the resulting graph in part b. Determine the values of p and q .

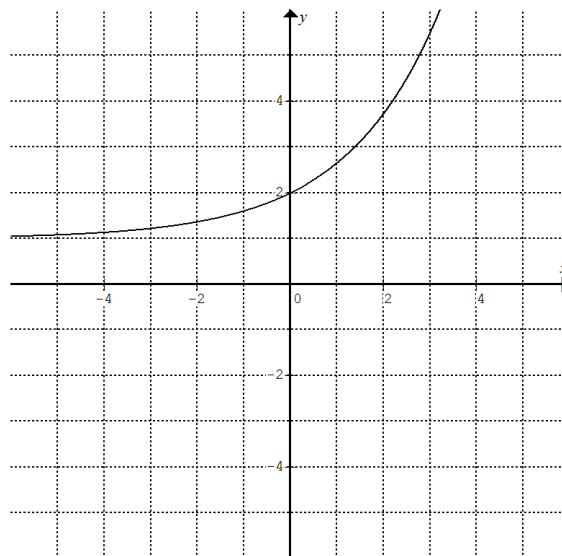
2 marks

Question 4 Consider $f(x) = \cos(\sin x) - 1$.

- a. Find a general solution to equation $f(x) = 0$. 2 marks

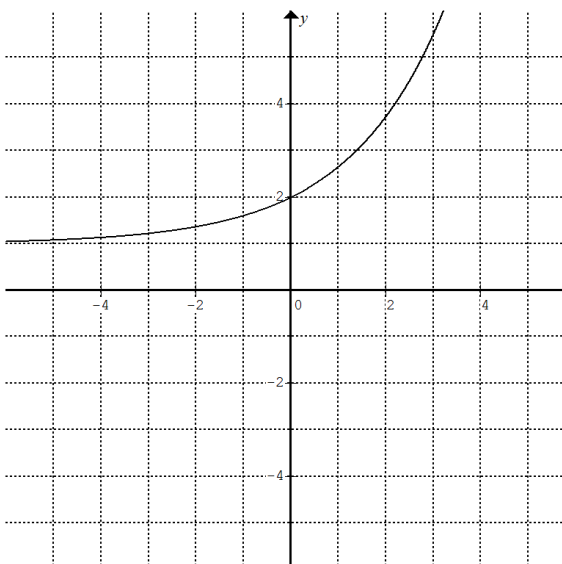
- b. Find the coordinates of the stationary points of $f(x)$ in general form. 2 marks

Question 5 The graph of $f(x) = 1 + \sqrt{e^x}$ is shown below.



- a. Accurately sketch the graph of the derivative function of $f(x)$ on the diagram above. 2 marks

- b. Accurately sketch the graph of the derivative function of $f^{-1}(x)$ on the diagram below. 3 marks



Question 6 Consider $y = \log_e(a \tan x)$ for $0 < x < \frac{\pi}{2}$ and $a \in \mathbb{R}^+$.

- a. Given $\sec x = \frac{1}{\cos x}$, show that $\frac{dy}{dx} = \frac{1}{(\sin x)(\cos x)}$. 2 marks

- b. Hence or otherwise, find the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{(\sin x)(\cos x)} dx$. 2 marks

Question 7 The speed v (m s^{-1}) of a particle at time t (s) is given by $v = \frac{t+6}{(t+1)^2}$ where $t \geq 0$.

- a. Starting from $v = \frac{t+6}{(t+1)^2}$ show that it can be expressed in the form $v = \frac{1}{t+1} + \frac{5}{(t+1)^2}$. 1 mark

- b. Find the exact value of the average speed (m s^{-1}) from $t = 0$ to $t = 4$. 3 marks

Question 8 A city has a population of 1000000 people. Out of a random sample of 2500 people, 900 were obese. Estimate the proportion of obese people in the city, and its approximate 95% confidence interval. Correct answers to 2 decimal places.

3 marks

Question 9 Out of 1200 students at a high school 540 are male students.

a. A random sample of 25 students is selected from the school.

There are 10 male students and 8 VCE students in the sample. Among the 10 male students, 6 are VCE students. Find the probability that a student is male among the VCE students in the sample.

2 marks

b. Another random sample of 25 students is selected from the school. Find the probability (evaluation is not required) that 12 or 13 students are male.

2 marks

Question 10 $f(x) = \begin{cases} 1.5a, & 1 \leq x \leq 3 \\ a, & 4 \leq x \leq 6 \\ 0, & \text{elsewhere} \end{cases}$

is a probability density function of random variable X .

a. Show that $a = 0.2$.

1 mark

b. Find the median of X .

1 mark

c. Calculate $\bar{X}$.

2 marks

End of exam