

**Year 12 Trial Exam Paper**  
**2018**  
**MATHEMATICAL METHODS**  
**Written examination 1**

**Reading time: 15 minutes**

**Writing time: 1 hour**

**STUDENT NAME:**

**QUESTION AND ANSWER BOOK**

**Structure of book**

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 9                          | 9                                         | 40                     |

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring blank sheets of paper, notes of any kind or correction fluid/tape into the examination.
- Calculators are NOT permitted in this examination.

**Materials provided**

- Question and answer book of 11 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

**Instructions**

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- You must answer the questions in English.

**Students are NOT permitted to bring mobile phones or any other unauthorised electronic devices into the examination.**

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**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

**Question 1** (4 marks)

- a.** Let  $y = \cos(x) \cos(4x)$ .

Find  $\frac{dy}{dx}$ .

2 marks

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- b.** Let  $f(x) = 2e^{2x}$ .

Evaluate  $f'(\log_e(2))$ .

2 marks

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**Question 2** (5 marks)

Let  $y = \frac{\log_e(x)}{x}$ .

**a.** Find  $\frac{dy}{dx}$ .

2 marks

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**b.** Hence, calculate  $\int_1^2 \frac{\log_e(x)}{x^2} dx$ .

3 marks

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**Question 3** (3 marks)

Let  $f: D \rightarrow R$ ,  $f(x) = \log_e(x) + \log_e(5 - 2x)$ .

- a.** State the maximal domain of  $f$ .

1 mark

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- b.** Solve the equation  $f(x) = 0$ .

2 marks

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**Question 4** (5 marks)

Lily enjoys seeing birds in a tree outside her house. The random variable  $X$  represents the number of birds in the tree when she looks outside each morning.

The distribution of  $X$  is given in the table below.

|              |      |     |      |     |
|--------------|------|-----|------|-----|
| $X$          | 0    | 1   | 2    | 3   |
| $\Pr(X = x)$ | 0.45 | 0.3 | 0.15 | 0.1 |

- a.** Find  $\Pr(X = 0 | X < 2)$ .

1 mark

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- b.** Find  $E(X)$ .

1 mark

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- c.** Lily records how many birds she sees each morning before school for a week. Let  $\hat{P}$  represent the sample proportion of days that Lily sees at least two birds in a particular five day week.

- i.** Find  $E(\hat{P})$ .

1 mark

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- ii.** Find  $\Pr(\hat{P} = 0.2)$ .

Express your answer in the form  $\frac{a}{b^c}$ , where  $a$ ,  $b$  and  $c$  are positive integers.

2 marks

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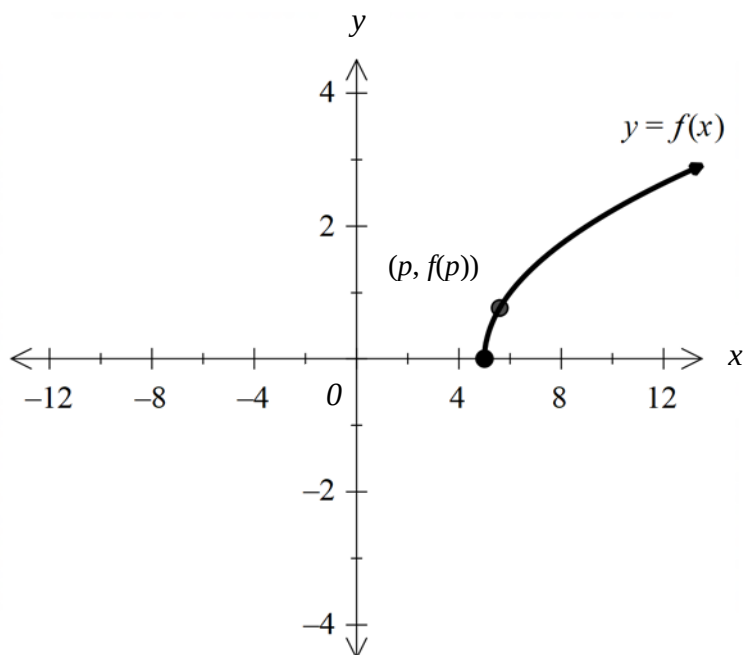
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**Question 5** (4 marks)

Let  $f: [5, \infty) \rightarrow \mathbb{R}$ , where  $f(x) = \sqrt{x-5}$ .



The point,  $P$ , with coordinates  $(p, f(p))$ , is on the curve defined by  $y = f(x)$ .

- a.** Find the gradient of the tangent to the curve at the point  $P$  in terms of  $p$ .

1 mark

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- b.** Find the value of  $p$  such that the tangent to the curve at  $P$  passes through the point  $(0, 0)$ .

3 marks

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**Question 6** (2 marks)

The events  $A$  and  $B$  from a sample space are independent. If  $\Pr(A' \cap B) = 0.2$  and  $\Pr(A) = 0.4$ , find  $\Pr(A \cap B)$ .

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**Question 7** (3 marks)

Let  $X$  be a continuous random variable with probability density function

$$f(x) = \begin{cases} kx^3(1-x)^2 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

where  $k$  is a real number.

Find the value of  $k$ .

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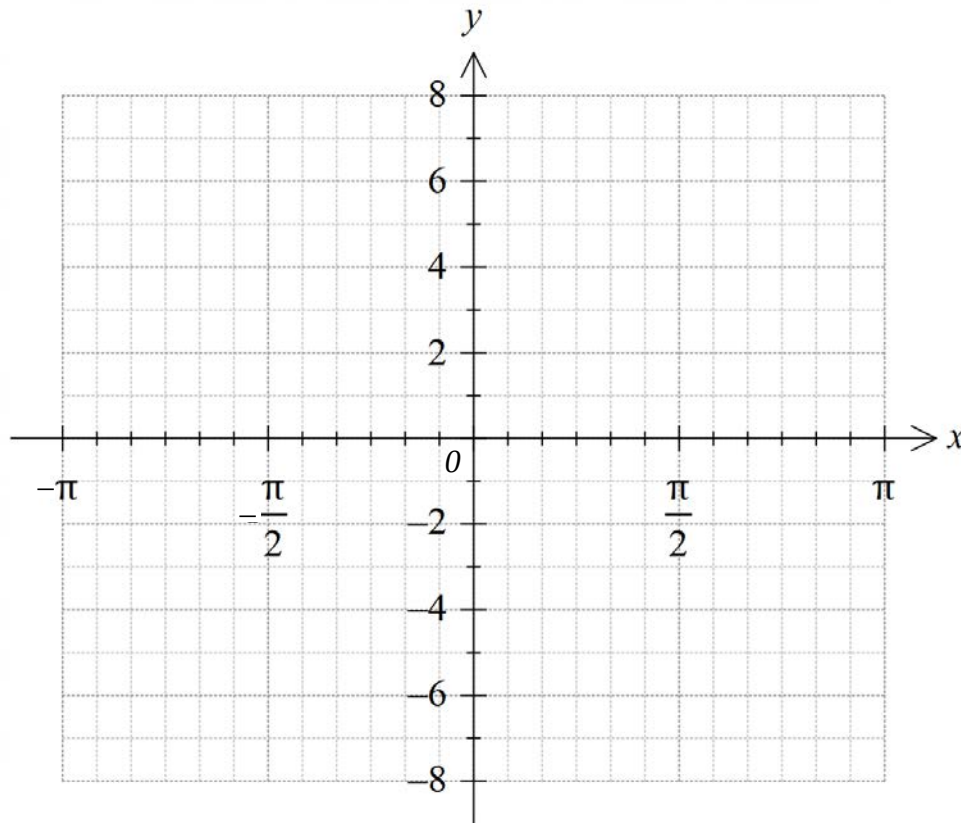


**Question 8** (7 marks)

Let  $f: \left[-\frac{7\pi}{12}, \frac{\pi}{4}\right] \rightarrow \mathbb{R}$ , where  $f(x) = 2 - 4\sin(2x)$ .

- a.** Sketch the graph of  $y = f(x)$  on the axes below. Label all end points and axes intercepts with their coordinates.

4 marks



- b.** Find the area enclosed by the graph of  $y = f(x)$  and the  $x$ -axis over the interval  $x \in \left[-\frac{7\pi}{12}, \frac{\pi}{12}\right]$ .

3 marks

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**Question 9** (7 marks)

Let  $f: R \setminus \{b\} \rightarrow R$ ,  $f(x) = \frac{a}{x-b}$ , where  $a$  and  $b$  are real numbers.

- a.** Find the domain and rule of the inverse function  $f^{-1}$ .

2 marks

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The transformation  $T: R^2 \rightarrow R^2$  with rule  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & g \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$ , where  $g, h$  and  $k$  are integers, maps the graph of  $y = f(x)$  onto the graph of  $y = f^{-1}(x)$ .

- b.** Find the values  $g, h$  and  $k$  in terms of  $a$  and  $b$ .

2 marks

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- c. Find the values of  $a$ , in terms of  $b$ , for which the equation  $f(x) = f^{-1}(x)$  has no real solutions.

3 marks

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**END OF QUESTION AND ANSWER BOOK**