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Student Name.....

MATHEMATICAL METHODS UNITS 3 & 4

TRIAL EXAMINATION 1

2018

Reading Time: 15 minutes

Writing time: 1 hour

Instructions to students

This exam consists of 9 questions.
All questions should be answered in the spaces provided.
There is a total of 40 marks available.
The marks allocated to each of the questions are indicated throughout.
Students may **not** bring any calculators or notes into the exam.
Where a numerical answer is required, an exact value must be given unless otherwise directed.
Where more than one mark is allocated to a question, appropriate working must be shown.
Diagrams in this trial exam are not drawn to scale.
A formula sheet can be found on pages 12 and 13 of this exam.

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Question 1 (4 marks)

- a. Differentiate $\sqrt{2x - x^3}$ with respect to x . 1 mark

- b. Let $f(x) = \frac{\sin(2x)}{e^{2x}}$.

Evaluate $f'\left(\frac{\pi}{2}\right)$. 3 marks

Question 2 (4 marks)

Let $y = x \cos(2x)$.

- a.** Find $\frac{dy}{dx}$. 2 marks

- b.** Hence, evaluate $\int_0^{\frac{\pi}{6}} 2x \sin(2x) dx$. 2 marks

Question 3 (2 marks)

The Year 12 leadership team at a school is made up of four female and three male students. Three of these students are rostered at random to be present at the Junior assembly each week.

Let $\hat{P}$ represent the sample proportion of female students who are rostered to be present at next week's Junior assembly.

- a.** Write down the possible values of $\hat{P}$. 1 mark

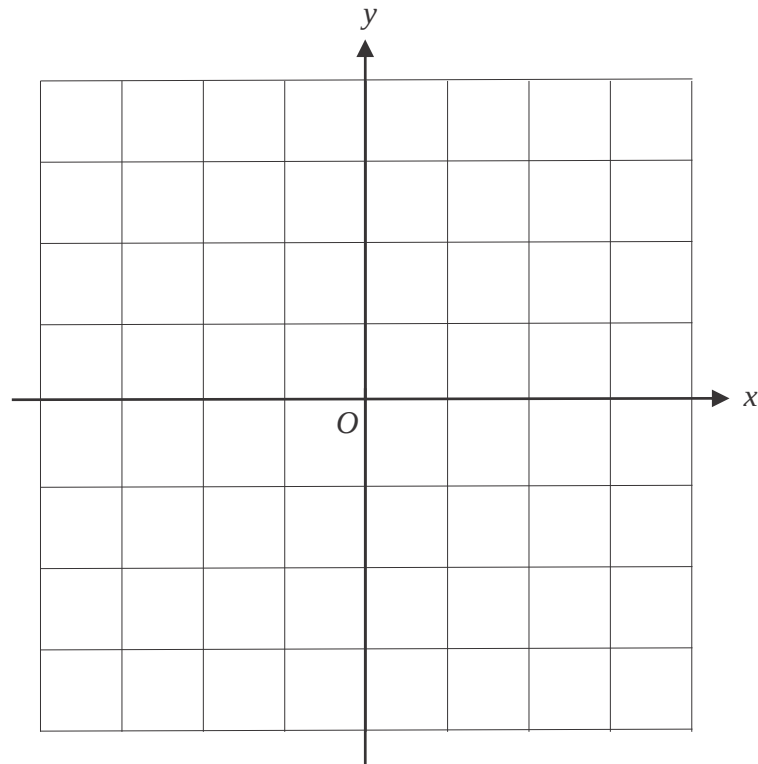
- b.** Find $\Pr(\hat{P} = 1)$. 1 mark

Question 4 (5 marks)

- a. Solve $\left(\sin(\theta) - \frac{1}{\sqrt{2}}\right)(\sqrt{3}\sin(\theta) + \cos(\theta)) = 0$ for θ , where $0 \leq \theta \leq \pi$. 2 marks

- b. Let $f: \left[-\frac{\pi}{8}, \frac{\pi}{4}\right) \rightarrow \mathbb{R}, f(x) = \tan(2x)$.

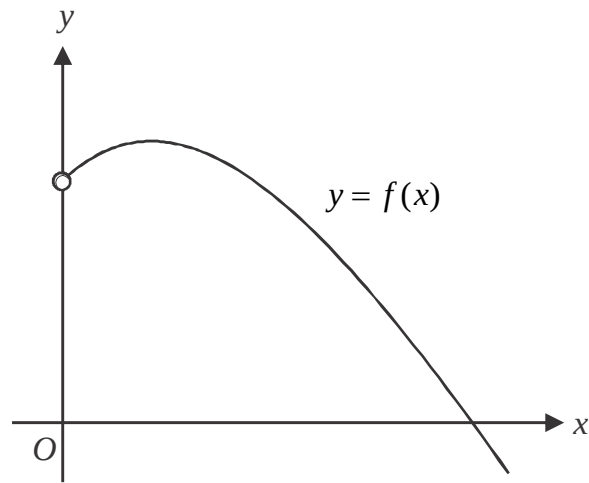
Sketch the graph of f on the axes below. Label any asymptotes with the appropriate equation, and label any endpoints with their coordinates. 3 marks



Question 5 (4 marks)

Let $f : (0, \infty) \rightarrow \mathbb{R}$, $f(x) = 1 - x \log_e(x)$.

The graph of f is shown below.



- a.** Find the coordinates of the stationary point of f , expressed in simplest form. 3 marks

- b.** Find the values of x for which f is strictly decreasing. 1 mark

Question 6 (5 marks)

Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x}$ where f and g have maximal domains.

- a. i.** Find the rule for $f(g(x))$. 1 mark

- ii.** State the domain and range of $f(g(x))$. 2 marks

The domain of f is to be restricted to $x \in [a, \infty)$ where a is the smallest possible value such that $g(f(x))$ exists.

- b.** Find the value of a . 2 marks

Question 7 (5 marks)

In a sample space with two events A and B , $\Pr(A) = 3q$, $\Pr(A' \cap B') = q$ and $\Pr(A' | B') = \frac{1}{3}$.

- a.** Find $\Pr(B)$ in terms of q . 1 mark

- b.** Find $\Pr(A \cup B)$ in terms of q . 2 marks

- c.** If $\Pr(A \cap B') \leq \frac{1}{2}$, find the possible values for q . 2 marks

Question 8 (4 marks)

A continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} \frac{1}{3}e^{-\frac{x}{3}} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

The median of X is m .

- a.** Find the value of m .

2 marks

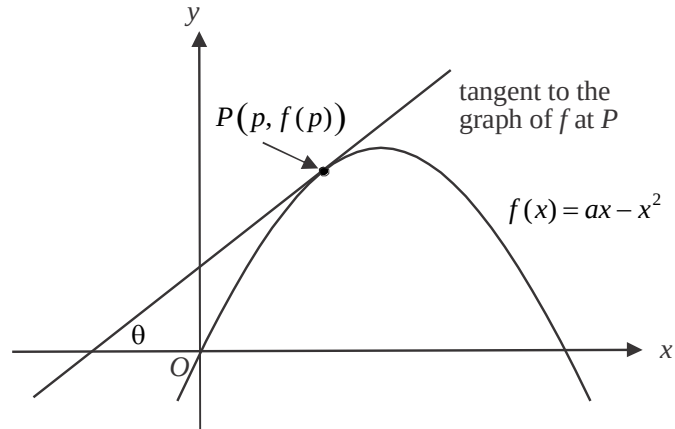
- b.** Given that $m > 2$, find $\Pr(X < 2 | X \leq m)$.

2 marks

Question 9 (7 marks)

Let $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = ax - x^2$, where a is a positive constant.

A tangent is drawn to the graph of f at the point $P(p, f(p))$, where $0 \leq p < \frac{a}{2}$, as shown below.



The tangent makes an angle of θ with the positive direction of the horizontal axis as shown.

- a.** Show, without using calculus, that the turning point of the graph of f is located at the point where $x = \frac{a}{2}$. 1 mark

- b.** Find, in terms of a , the maximum value of θ . 2 marks

- c. If $p = \frac{a}{4}$, find in terms of a , the area enclosed by the tangent, the graph of f and the y -axis.

4 marks

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

Calculus			
$\frac{d}{dx}(x^n)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c, n\neq -1$	
$\frac{d}{dx}((ax+b)^n)=an(ax+b)^{n-1}$		$\int (ax+b)^n dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c, n\neq -1$	
$\frac{d}{dx}(e^{ax})=ae^{ax}$		$\int e^{ax} dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}(\log_e(x))=\frac{1}{x}$		$\int \frac{1}{x} dx=\log_e(x)+c, x>0$	
$\frac{d}{dx}(\sin(ax))=a\cos(ax)$		$\int \sin(ax) dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}(\cos(ax))=-a\sin(ax)$		$\int \cos(ax) dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}(\tan(ax))=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

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Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{p}) = p$
standard deviation	$\text{sd}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$