

MATHEMATICAL METHODS

Written Examination 2



2017 Trial Examination

SOLUTIONS

SECTION 1

Question 1

Answer: C

Explanation:

Sketch on CAS over the restricted domain

Question 2

Answer: B

Explanation:

Sketch on CAS or solve $\left(\frac{dy}{dx} = 0\right)$

Question 3

Answer: E

Explanation:

solve on CAS over restricted domain

Question 4

Answer: A

Explanation:

Period = $\frac{2\pi}{2/\pi}$, range: $[-3 - 1, -3 + 1]$

Question 5

Answer: C

Explanation:

$\frac{f(5)-f(0)}{5}$ on CAS

Question 6

Answer: B

Explanation:

Find inverse rule on CAS. Domain of $g^{-1} = \text{Range of } g = [3, \infty)$

Question 7

Answer: E

Explanation:

$$1 - \Pr(\text{same}) = 1 - (0.2^2 + 0.25^2 + 0.45^2 + 0.1^2)$$

Question 8

Answer: D

Explanation:

$$\text{Period} = 16 \rightarrow n = \frac{\pi}{8}, \text{ amp} = 3$$

Question 9

Answer: B

Explanation:

$$\frac{1}{2} \frac{d}{dx} \left(x^2 \log_e(kx) \right) - \frac{x}{2} = x \log_e(kx)$$

$$\frac{x^2}{2} \log_e(kx) - \int \frac{x}{2} dx + c = \int x \log_e(kx) dx$$

Question 10

Answer: A

Explanation:

tangentline($cx^2 - 5, x, -1$) on CAS and then sub in $(-4, -1)$

Question 11

Answer: B

Explanation:

$$x' = \frac{1}{2}x, \quad y' = -y \rightarrow -y = e^{2\left(\frac{1}{2}x\right)} - 4$$

Question 12

Answer: A

Explanation:

$$\begin{aligned} \text{Area} &= \int_{-b}^b (y_1 - y_2) dx \\ &= \int_{-b}^b (a - f(x)) dx \end{aligned}$$

Question 13

Answer: B

Explanation:

Sketch on CAS

Note that option A is incorrect because $x = 7$ is outside the domain of the function.

Question 14

Answer: D

Explanation:

$$\Pr(X > 23.3) = \Pr(Z > -2) = \Pr(Z < 2)$$

Question 15

Answer: E

Explanation:

$$\int_0^1 f(x)dx = 1 \rightarrow k = \frac{15}{14}, \text{ solve } \int_a^1 f(x)dx = \frac{256-11\sqrt{2}}{256} \text{ for } a$$

Question 16

Answer: C

Explanation:

$$x' = 2x \text{ and } y' = -3y + 1$$

$$\rightarrow x = \frac{x'}{2} \text{ and } y = \frac{y'-1}{-3}$$

Substituting into $f(x) = \cos(2x)$ gives

$$g(x) = 1 - 3 \cos(x)$$

Question 17

Answer: D

Explanation:

$$\Pr\left(\hat{p} \geq \frac{5}{32}\right) = \Pr(X \geq 5) = \text{binomcdf}(32, 0.2, 5, 32)$$

Question 18

Answer: A

Explanation:

Check for solution that satisfies both equations- $0.15 + a = 0.58$ and $a + b = 0.55$

Question 19

Answer: B

Explanation:

$$(x - 2)(3 - x) = \frac{k}{2} - x, \Delta > 0$$

Question 20

Answer: D

Explanation:

$$\frac{d}{du} \left(\sqrt{u^2 + (u^2 + 1 - 2)^2} \right) = 0 \rightarrow u = 0, \pm 0.5\sqrt{2}$$

When $u = 0$ the distance is 1, when $u = 0.5\sqrt{2}$ the distance is $\frac{\sqrt{3}}{2}$ that is < 1 .

Minimal distance occurs when $x = 0.5\sqrt{2}$, distance = $\frac{\sqrt{3}}{2}$

SECTION 2

Question 1

a. $Period = \frac{2\pi}{\frac{\pi}{24}} = 48,$ 1 mark

$Range: [-1, 11]$

1 mark

b. $f'(x) = -\frac{\pi}{4} \sin\left(\frac{\pi}{24}x\right)$

1 mark

c. $y = -\frac{\pi}{4}x + 5(3\pi + 1)$ (on CAS)

1 mark

d. $\frac{\pi}{4} = -\frac{\pi}{4} \sin\left(\frac{\pi}{24}x\right) \rightarrow x = 36, 84$

1 mark

$tangentline(f(x), x, 36): y = \frac{\pi x}{4} - 9\pi + 5$

$tangentline(f(x), x, 84): y = \frac{\pi x}{4} - 21\pi + 5$

1 mark

e. $x' = -x + b, y' = ay + 5$

1 mark

$x = -x' + b, y = \frac{y'-5}{a}$

$\frac{y'-5}{a} = -\frac{\pi}{4} \sin\left(\frac{\pi}{24}(-x' + b)\right)$

1 mark

$y' = -\frac{\pi}{4}a \sin\left(-\frac{\pi}{24}x' + \frac{b\pi}{24}\right) + 5$

$\frac{b\pi}{24} = \frac{\pi}{2} \rightarrow b = 12, \quad -\frac{\pi}{4}a = 6 \rightarrow a = -\frac{24}{\pi}$

1 mark

f. *Solve on CAS:* $x = 6, 30, 54, 78$

2 marks

Question 2

a. $A(2, e^{-4} + 3)$ and $B(0, 4)$

2 marks

b. $Area = \int_0^2 (e^x - e^{-2x}) dx$

1 mark

$= \left(e^x + \frac{e^{-2x}}{2}\right) \Big|_0^2 = e^2 - 1 + \frac{e^{-4}}{2} - \frac{1}{2} = e^2 + \frac{1}{2e^4} - \frac{3}{2}$

1 mark

c. *Tangent line:* $y = ex + 3$ 1 mark
 $C(0, 3)$

1 mark

d. $\theta = \tan^{-1}(e^1) = 69.8 \approx 70^\circ$

2 marks

e. $\text{Area} = \int_1^2 ((e^x + 3) - (ex + 3)) dx$ 1 mark
 $= e^2 - \frac{5e}{2}$

1 mark

f. *Tangent passing through origin:* $y = ex$ 1 mark
 $ea = e^{-2a} + 3$ 1 mark
 $a = 1.41$ 1 mark

g. $\text{Length} = \sqrt{2^2 + (e^{-4} + 3 - 3)^2}$
 $= \sqrt{4 + e^{-8}}$

1 mark

Question 3

a. $f(x) = 2 - \frac{7}{x+4}$ (Use propfrac on CAS or long division)

1 mark for “2”

1 mark for “-7”

b. *Range:* $\mathbb{R} \setminus \{2\}$

1 mark

c. $x = 2 - \frac{7}{y+4} \rightarrow$

1 mark

$f^{-1}(x) = -4 - \frac{7}{x-2}$

1 mark

d. *solve* $-4 - \frac{7}{x-2} = x \rightarrow x = -1 \pm \sqrt{2}$

$(-1 + \sqrt{2}, -1 + \sqrt{2})$ and $(-1 - \sqrt{2}, -1 - \sqrt{2})$

2 marks

e. $Area = \int_{-1-\sqrt{2}}^{-1+\sqrt{2}} \left(\frac{2x+1}{x+4} - x \right) dx$

1 mark for correct integral and 1 mark for correct terminal values

f.

i. $[-5, -4) \cup (-4, \infty)$

1 mark

ii. $\frac{d}{dx}(h(x)) = 0 \rightarrow x = -4 \pm \sqrt{7}$

1 mark

$(-4 - \sqrt{7}, 2\sqrt{7} + 6)$ and $(-4 + \sqrt{7}, 6 - 2\sqrt{7})$

1 mark

iii. $Distance = \sqrt{x^2 + \left(\frac{2x+1}{x+4} - x \right)^2}$

1 mark

$\frac{d}{dx}(distance) = 0 \rightarrow x = -6.13, 0.11$

Min distance at $x = 0.11$ (0.11, 0.19)

1 mark

Minimum distance = 0.22

1 mark

Question 4

a. $binomcdf(24, 0.1, 1, 24) = 0.9202$

2 marks

b. $\Pr(X < 3 | X \geq 1) = \frac{binomcdf(24, 0.1, 1, 2)}{binomcdf(24, 0.1, 1, 24)}$
 $= 0.5265$

1 mark

1 mark

c. $normcdf(0, 40, 60, 12) = 0.0478$

2 marks

d. $invnorm(1 - 0.4062, 60, 12) = 62.85$

1 mark

$m = 63$

1 mark

e. $invnorm(0.28, 0, 1) = \frac{46-60}{c}$

1 mark

$-0.5828 = -\frac{14}{c}$

1 mark

$c = 24.02$

1 mark

f. $\text{binompdf}(24, 3, 0.28) = 0.0448$

1 mark

g. $\Pr(\hat{P} \geq 0.06 \mid \hat{P} \geq 0.04)$
 $= \Pr(X \geq 3 \mid X \geq 2) = \frac{\Pr(X \geq 3)}{\Pr(X \geq 2)} = 0.617$

1 mark

2 marks

h. $\left(\frac{5}{50} - 1.96\sqrt{\frac{\frac{5}{50}(1-\frac{5}{50})}{50}}, \frac{5}{50} + 1.96\sqrt{\frac{\frac{5}{50}(1-\frac{5}{50})}{50}} \right) = (0.02, 0.18)$

2 marks

i.

i. $\int_0^\infty f(x)dx = 1 \rightarrow k = \frac{e^{-\frac{1}{5}}}{4}$
 $\text{Mean} = \int_0^\infty x \times f(x)dx = 4 \text{ minutes}$

1 mark

1 mark

ii. $\int_0^m f(x)dx = \frac{1}{2} \rightarrow k = -1.54, 3.36$
 $\text{Median} = 3.36 \text{ minutes}$

1 mark