

2017 Trial Examination

STUDENT NUMBER

Figures

Words

Letter

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MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes

Writing time: 60 minutes

QUESTION & ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape, notes of any kind, or a calculator of any kind.

Materials supplied

- Question and answer book of 12 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = (x^2 + 1)e^{4x}$.

Find $\frac{dy}{dx}$, expressing your answer in factorised form.

2 marks

b. Let $f(x) = \frac{\log_e(x^2)}{x^2}$

i. Find $f'(x)$

1 mark

ii. Find $f'(e)$

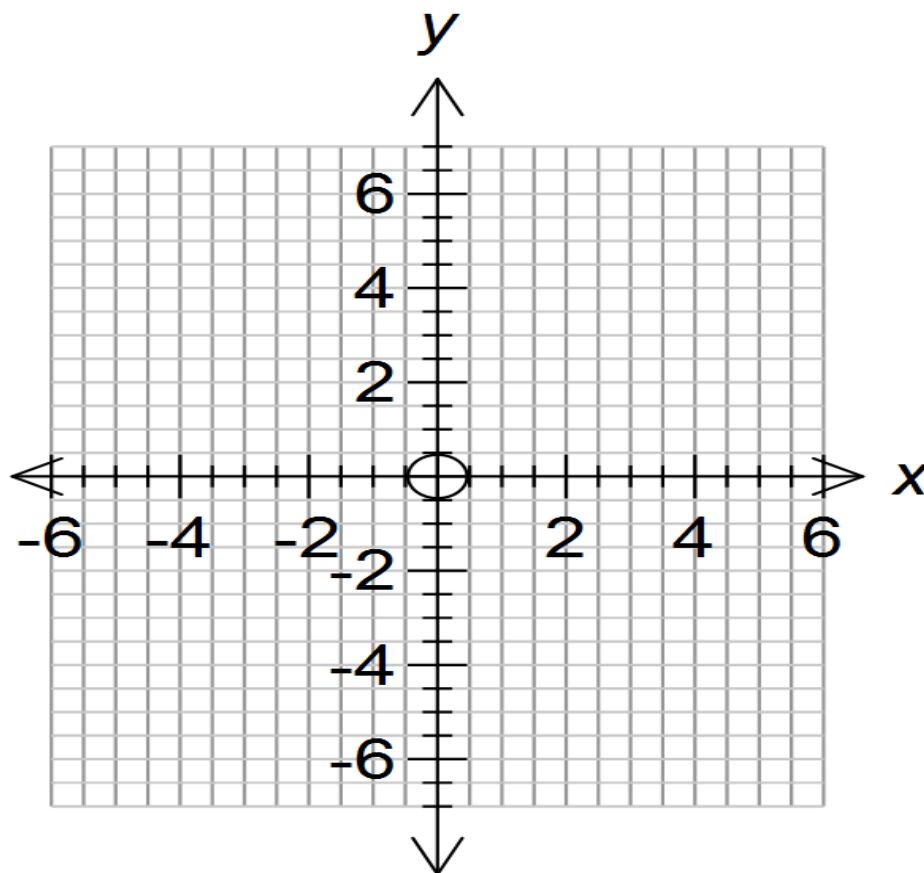
1 mark

TURN OVER

Question 2 (5 marks)

Let $f: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R}$, where $f(x) = 3 + \frac{2}{(x-1)^2}$.

- a.** Sketch the graph of $f(x)$. Label the axis intercepts with their coordinates and label any equation(s) of asymptotes. 3 marks



- b.** Find the area enclosed by the graph of $f(x)$, the lines $x = 2$ and $x = 5$, and the x-axis. 2 marks

Question 3 (4 marks)

Let $f: [0, 2\pi] \rightarrow \mathbb{R}$, where $f(x) = \log_e(\cos x)$.

- a.** Find the equation of the tangent to the graph of $f(x)$ at $x = \frac{\pi}{4}$. 2 marks

- b.** Hence, find the angle θ from the positive direction of the x -axis to the tangent to the graph of $f(x)$ at $x = \frac{\pi}{4}$, measured in the anticlockwise direction. 1 mark

- c.** Find the x values of the stationary points of the function f . 1 mark

TURN OVER

Question 4 (4 marks)

A class contains 5 boys and 10 girls. The teacher runs a four question quiz at the end of each lesson and selects a student at random to answer these questions. The teacher can select the same student to answer any number of questions.

- a.** What is the probability that the number of boys selected in a given lesson is zero? 1 mark

- b.** What is the probability that at least one of the students selected in the lesson is a boy? 1 mark

- c.** What is the probability that no boy is selected in exactly three of the five consecutive lessons? Give your answer in the form ap^mq^n , where a , m and n are positive integers and p and q are rational numbers. 2 marks

Question 5 (10 marks)

a. Let $f: D \rightarrow R$, where $f(x) = \log_e(1 - x^2)$

i. Find the maximal domain, D , of f .

1 mark

Let $g: (-1, 0] \rightarrow R$, where $g(x) = \log_e(1 - x^2)$

ii. Find the rule for g^{-1} .

2 marks

iii. Find the domain and range of g^{-1} .

2 marks

b. Let $h(x) = \sqrt{1 - e^x}$ and $k(x) = -1 - x^2$.

Given that the composite function $h(k(x))$ is defined,

i. Find the rule for $h(k(x))$

1 mark

Question 5 - continued
TURN OVER

ii. State the domain of $h(k(x))$.

1 mark

iii. Find the stationary point(s) of $h(k(x))$.

3 marks

Question 6 (5 marks)

Let $f: [0, 2\pi] \rightarrow \mathbb{R}$, where $f(x) = \sin(2x) + 1$

- a.** Find the x -intercepts of the graph of f .

2 marks

- b.** Calculate the average rate of change of f over the interval $[0, 2\pi]$.

1 mark

- c.** Calculate the average value of f between the x -intercepts of f .

2 marks

TURN OVER

Question 7 (3 marks)

A company produces batteries for calculators from two different machines A and B. 4% of the batteries produced by Machine A are faulty and 5% of batteries produced by Machine B are faulty. At the end of one day, Machine A produces 50 batteries and Machine B produces 80 batteries. The company owner selects one battery at random from all batteries produced in that day.

- a.** What is the probability that the battery selected by the manager is faulty? 1 mark

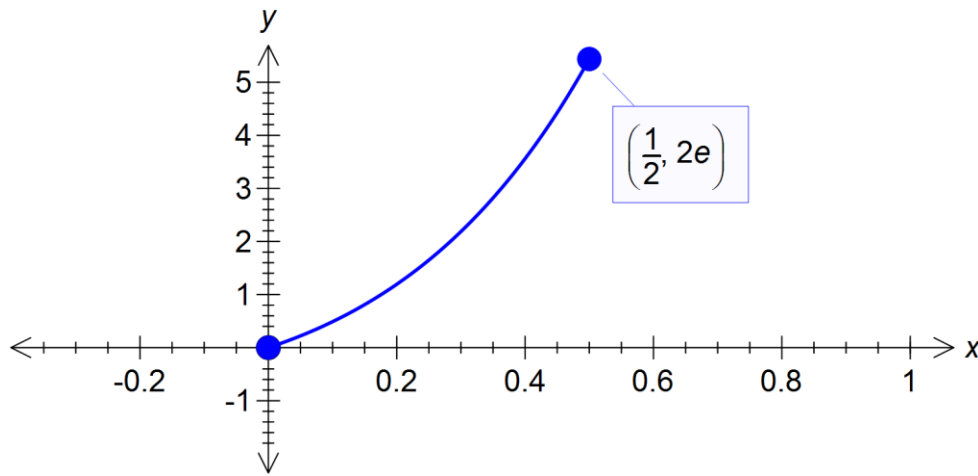
- b.** A battery is selected and found to be faulty.
What is the probability that this faulty battery was produced by Machine B? 2 marks

Question 8 (5 marks)

Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} 4xe^{2x} & 0 < x \leq \frac{1}{2} \\ 0 & \text{elsewhere} \end{cases}$$

Part of the graph of $f(x)$ is shown below.



- a. Show that $\frac{d}{dx}(e^{2x}(2 + bx)) = (4 + b)e^{2x} + 2bx e^{2x}$

1 mark

Question 8 – continued
TURN OVER

b. Hence, find $\Pr\left(X < \frac{1}{4}\right)$

2 marks

c. Show that the median m , of X is a solution of the equation $2e^{2m} - 4me^{2m} - 1 = 0$

2 marks

END OF QUESTION AND ANSWER BOOK