

MATHEMATICAL METHODS

Written examination 1



2017 Trial Examination

SOLUTIONS

Question 1

a. $y = (x^2 + 1)e^{4x} \rightarrow \frac{dy}{dx} = e^{4x}(4x^2 + 4 + 2x)$ 1 mark
 $\rightarrow \frac{dy}{dx} = 2e^{4x}(2x^2 + x + 2)$

b.

i. $f'(x) = \frac{x^2 \times \frac{2}{x} - 2 \log_e(x) \times 2x}{x^4}$ 1 mark
 $= \frac{x(2 - 4 \log_e(x))}{x^4}$ 1 mark

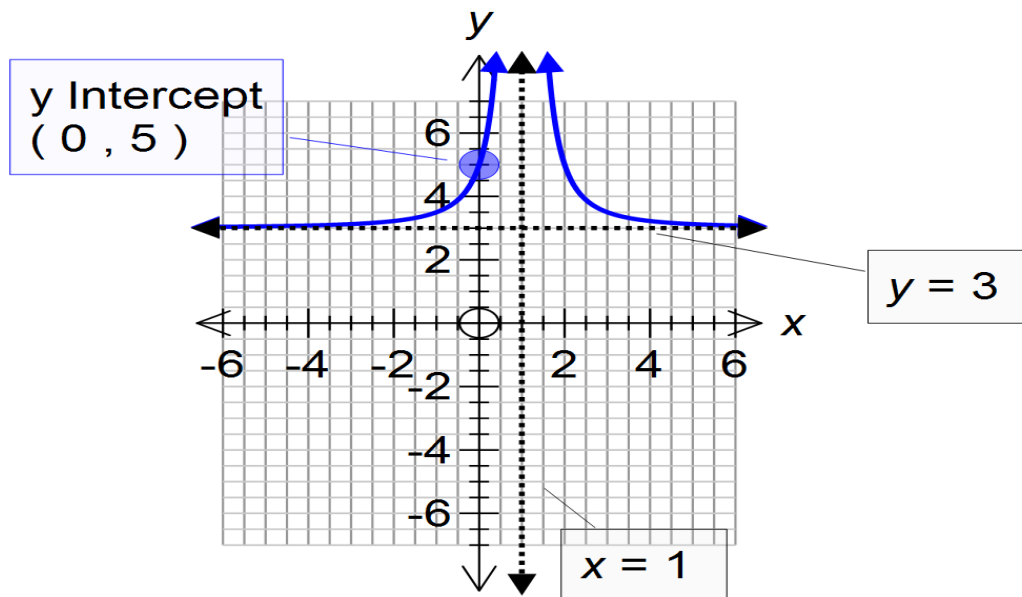
$$= \frac{2 - 4 \log_e(x)}{x^3}$$

ii. $f'(e) = \frac{2-4}{e^3} = -\frac{2}{e^3}$

1 mark

Question 2

a.



1 mark for equations of asymptotes, 1 mark for y-intercept, 1 mark for shape

b. $Area = \int_2^5 (3 + 2(x-1)^{-2}) dx$ 1 mark

$$Area = \left[3x - \frac{2}{x-1} \right]_2^5. Area = \left(15 - \frac{1}{2} \right) - (6 - 2) = 10.5 \text{ sq units}$$

1 mark

Question 3

a. $\frac{dy}{dx} = -\tan(x) \rightarrow m_T = -1$ 1 mark

$$y - \ln\left(\frac{\sqrt{2}}{2}\right) = -1\left(x - \frac{\pi}{4}\right)$$

$$y = -x + \left(\frac{\pi}{4} + \ln\left(\frac{\sqrt{2}}{2}\right)\right)$$
 1 mark

b. $\tan\theta = -1$

$$\theta = \frac{3\pi}{4}$$

1 mark

c. $-\tan x = 0 \rightarrow x = 0, \pi, 2\pi$

1 mark

Question 4

a. $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$

1 mark

b. $\Pr(B \geq 1) = 1 - \Pr(B = 0) = 1 - \frac{16}{81} = \frac{65}{81}$

1 mark

c. $C(5, 3) \times \left(\frac{16}{81}\right)^3 \times \left(\frac{65}{81}\right)^2$
 $= 10 \left(\frac{16}{81}\right)^3 \left(\frac{65}{81}\right)^2$

1 mark

1 mark

Question 5

a.

i. $1 - x^2 > 0 \rightarrow x^2 < 1 \rightarrow -1 < x < 1$

Domain: $(-1, 1)$

1 mark

ii. $x = \log_e(1 - y^2)$

$$e^x = 1 - y^2 \rightarrow y^2 = 1 - e^x \rightarrow y = \pm\sqrt{1 - e^x}$$

1 mark

$$g^{-1}(x) = -\sqrt{1 - e^x}$$

1 mark

iii. Domain: $(-\infty, 0]$

Range: $(-1, 0]$

1 mark each

b.

i. $h(k(x)) = \sqrt{1 - e^{-1-x^2}}$

1 mark

ii. $\text{Domain} = \text{Domain of } k(x) = \mathbb{R}$

1 mark

iii. $\frac{d(h(k(x)))}{dx} = \frac{1}{2\sqrt{1-e^{-1-x^2}}} (2xe^{-1-x^2})$

1 mark

For stationary point, numerator must equal zero

$$e^{-1-x^2} \times 2x = 0 \rightarrow x = 0$$

1 mark

$$(0, \sqrt{1 - e^{-1}})$$

1 mark

Question 6

a. $\sin(2x) + 1 = 0 \rightarrow 2x = \frac{3\pi}{2}, \frac{7\pi}{2}$

1 mark

$$\left(\frac{3\pi}{4}, 0\right) \text{ and } \left(\frac{7\pi}{4}, 0\right)$$

1 mark

b. $\text{Average ROC} = \frac{f(2\pi) - f(0)}{2\pi} = 0$

1 mark

c. $\text{Average value} = \frac{1}{\frac{7\pi}{4} - \frac{3\pi}{4}} \times \int_{3\pi/4}^{7\pi/4} (\sin(2x) + 1) dx$

1 mark

$$\text{Average value} = \frac{1}{\pi} \times \left(-\frac{\cos(2x)}{2} + x \right) \Big|_{\frac{3\pi}{4}}^{\frac{7\pi}{4}}$$

1 mark

$$\text{Average value} = \frac{1}{\pi} \times \left(\frac{7\pi}{4} - \frac{3\pi}{4} \right) = 1$$

1 mark

Question 7

a. $\Pr(\text{faulty}) = \frac{50 \times 0.04 + 80 \times 0.05}{130} = \frac{3}{65}$

1 mark

b. $\Pr(B|\text{Faulty}) = \frac{\Pr(B \cap \text{faulty})}{\Pr(\text{faulty})}$

1 mark

$$= \frac{\frac{4}{80} \times \frac{8}{13}}{\frac{4}{80} \times \frac{8}{13} + \frac{2}{50} \times \frac{5}{13}} = \frac{2}{3}$$

1 mark

Question 8

a. $\frac{d}{dx}(e^{2x}(2 + bx)) = 2e^{2x} \times (2 + bx) + be^{2x} = e^{2x}(4 + b) + 2bx e^{2x}$

1 mark

b. $\Pr\left(X < \frac{1}{4}\right) = \int_0^{1/4} 4xe^{2x} dx$

Use $b = 2$

$$\Pr\left(X > \frac{1}{4}\right) = (e^{2x}(2 + 2x)) \Big|_0^{1/4} - 6 \int_0^{1/4} e^{2x} dx$$

$$\Pr\left(X > \frac{1}{4}\right) = (e^{2x}(2 + 2x)) \Big|_0^{1/4} - (3e^{2x}) \Big|_0^{1/4}$$

1 mark

$$\Pr\left(X > \frac{1}{4}\right) = \frac{5}{2}e^{\frac{1}{2}} - 2 - (3e^{\frac{1}{2}} - 3) = 1 - \frac{e^{\frac{1}{2}}}{2}$$

1 mark

c. $\Pr(X < m) = \frac{1}{2}$

$$\int_0^m 4xe^{2x} dx = \frac{1}{2}$$

$$(e^{2x}(2 + 2x)) \Big|_0^m - (3e^{2x}) \Big|_0^m = \frac{1}{2}$$

$$e^{2m}(2 + 2m) - 2 - (3e^{2m} - 3) = \frac{1}{2}$$

1 mark

$$-e^{2m} + 2me^{2m} + 1 = \frac{1}{2}$$

$$e^{2m} - 2me^{2m} - \frac{1}{2} = 0 \rightarrow 2e^{2m} - 4me^{2m} - 1 = 0$$

$$2e^{2m} - 4me^{2m} - 1 = 0$$

1 mark