THE SCHOOL FOR EXCELLENCE (TSFX)

UNITS 3 & 4 MATHEMATICAL METHODS 2017

WRITTEN EXAMINATION 1

Reading Time: 15 minutes
Writing Time: 1 hour

QUESTION AND ANSWER BOOKLET

Student Name:

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are **NOT** permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials Supplied

- Question and answer book of 9 pages.
- A separate sheet of miscellaneous formulas.
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are **NOT** permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

VIC: Level 3, 99 William Street, Melbourne VIC 3000
Phone: (03) 9663 3311 Fax: (03) 9663 3939

www.tsfx.com.au
admin@tsfx.com.au

NSW: PO Box r1407, Royal Exchange, Sydney NSW 1225
Phone: 1300 364 173 Fax: 1300 364 065

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Instructions

- Answer **all** questions in the spaces provided.
- In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

QUESTION 1 (5 marks)

(a) Find $\frac{dy}{dx}$ if $y = x^3 \log_e(4x)$.

2 marks

(b) If $f(x) = \cos^2(3x)$ find $f'\left(\frac{\pi}{12}\right)$.

3 marks

QUESTION 2 (4 marks)

(a) Find the anti-derivative of $4e^{-2x} + \frac{3}{\cos^2(x)} + 7$.

2 marks

(b) Evaluate $\int_0^1 (1-2x)^4 dx$.

2 marks

QUESTION 3 (3 marks)

Find the value(s) of θ if $\frac{3+\cos^2(\theta)}{\sin(\theta)-2} = 3\sin(\theta)$.

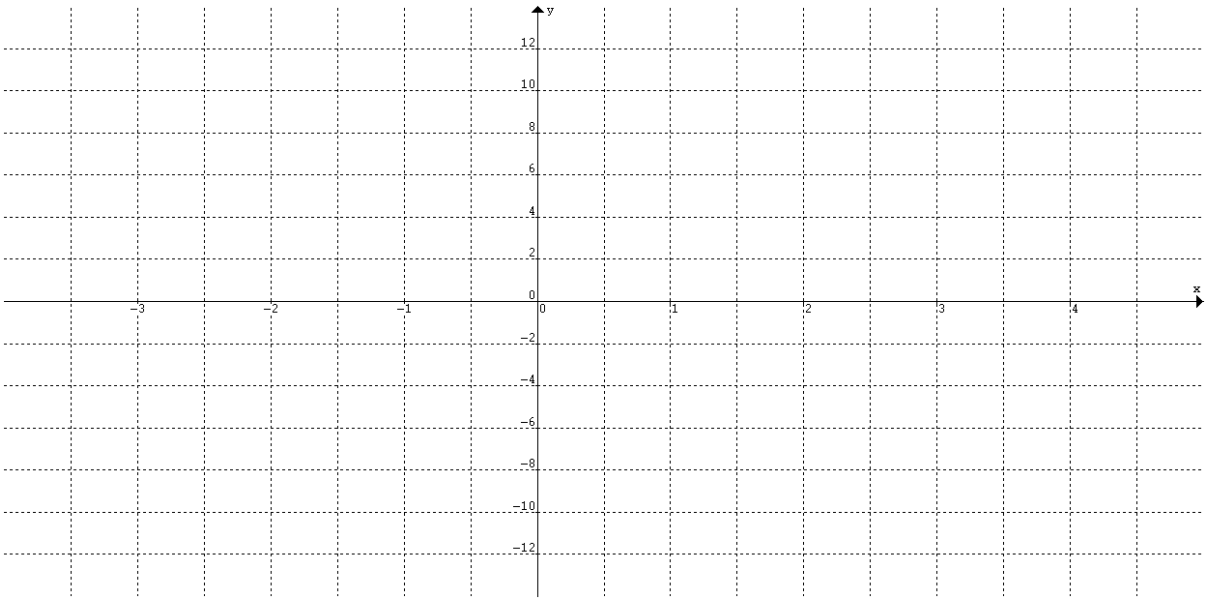
3 marks

QUESTION 4 (6 marks)

(a) Describe the transformations required (in the correct order) to change the graph with equation $f(x) = 2x^2 - 4x - 6$ to the graph with equation $g(x) = x^2$.

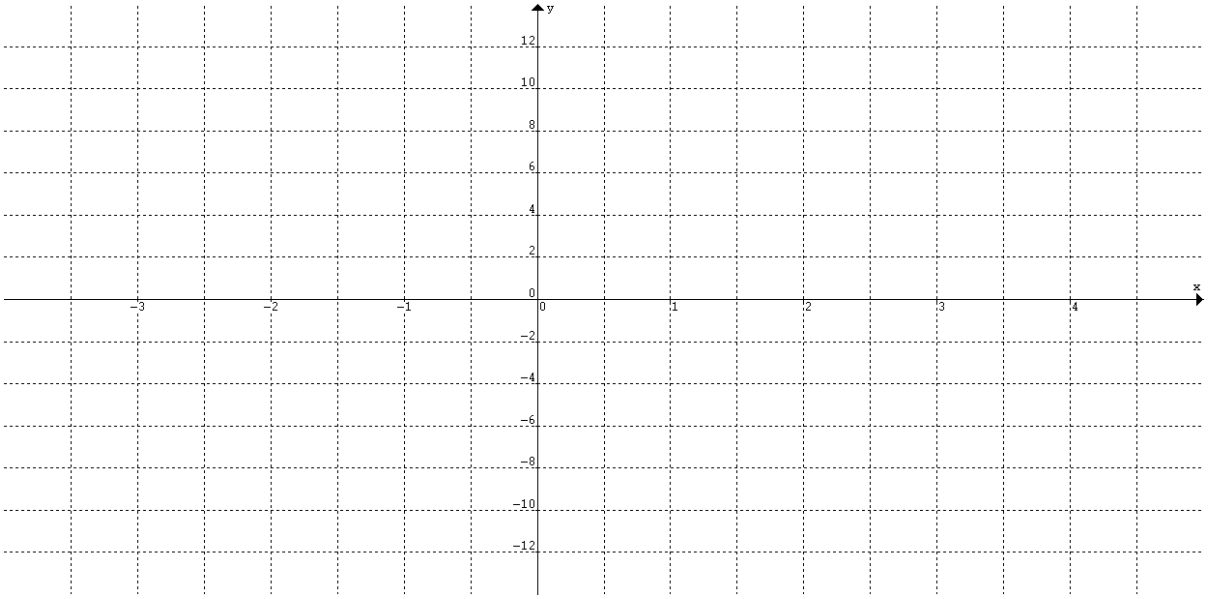
2 marks

(b) Sketch the graph of $y = h(x)$ where $h(x) = \begin{cases} f(x), & (-2,-1] \cup [3,4] \\ -f(x), & (-1,3) \end{cases}$ on the axes below, showing the coordinates of all of key points.



2 marks

(c) Sketch the graph of $y = h'(x)$ on the axes below, showing the coordinates of any intercepts with the axes.



2 marks

QUESTION 5 (4 marks)
Consider the simultaneous linear equations where $m \in R$.

$$(m+1)\,x+9y=21$$
$$2x+(m-2)\,y=7$$

(a) Find the value(s) of m for which there is no unique solution for x and y.

2 marks

(b) (i) State the value of m for which there is an infinite set of solutions.

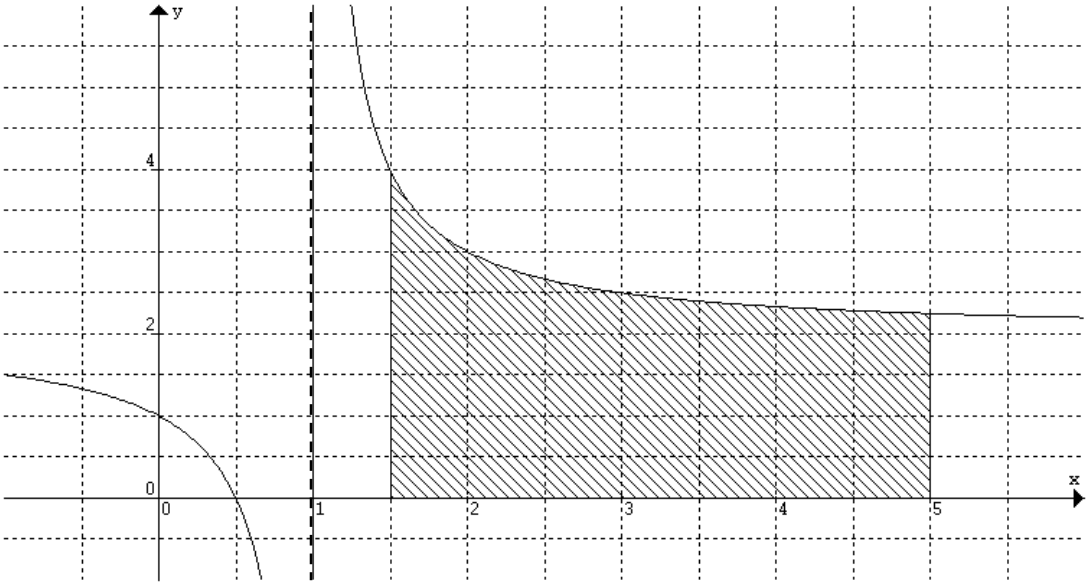
1 mark

(ii) If $x=\lambda$ (where $\lambda \in R$) is one solution for the value of m found in (b) part **i,** write down the corresponding value of y in terms of λ.

1 mark

QUESTION 6 (4 marks)

Part of the graph of $y = 2 + \dfrac{1}{x-1}$ is shown below.



The shaded area between the curve from $x = \dfrac{3}{2}$ to $x = 5$ and the x axis is given by the value $A + B\log_e 2$, where A and B are positive integers. Find the values of A and B.

4 marks

QUESTION 7 (5 marks)
The proportion, X , of arsenic found in a randomly selected sample of a toxic chemical is a random variable having a probability distribution function given by

$$f(x) = \begin{cases} ax^4(1-x) & 0 \leq x \leq 1 \\ 0 & elsewhere \end{cases}$$

where a is a positive integer.

(a) Show that $a = 30$.

1 mark

(b) Find the mean of the distribution.

2 marks

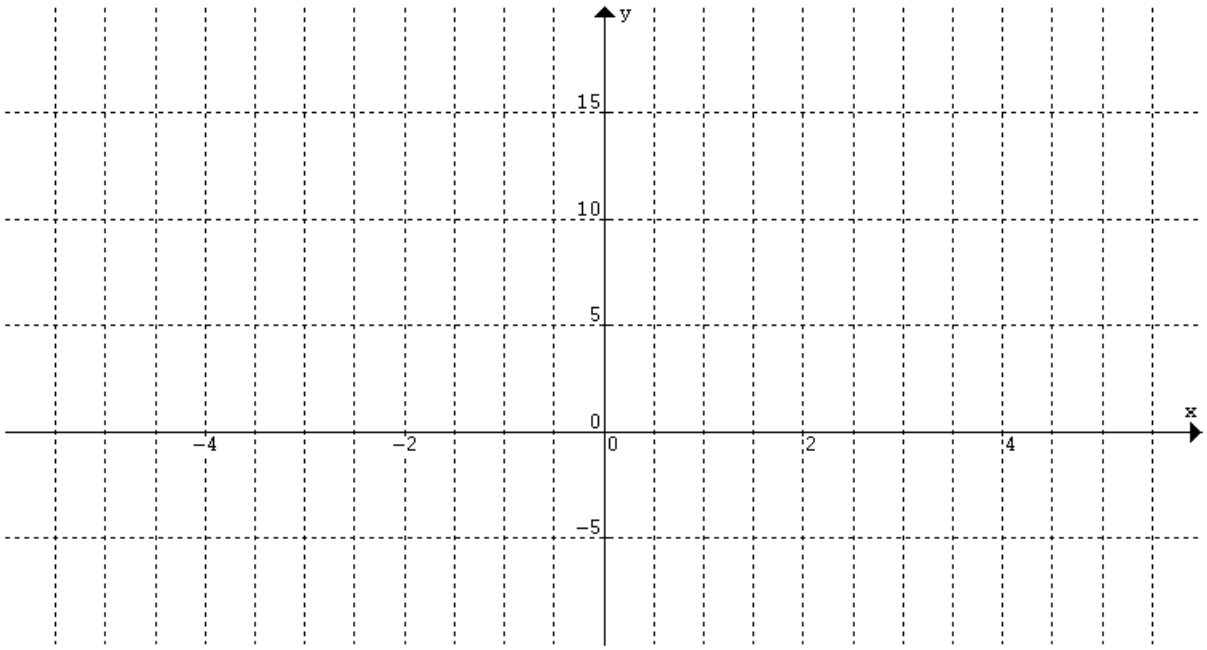
(c) Find the mode of the distribution.

2 marks

QUESTION 8 (3 marks)

Consider the functions $f(x) = x^2 - 4x$ and $g(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$. The domain for both functions is R.

(a) Sketch the graph of $f(g(x))$ on the axes below.



2 marks

(b) State the domain of $f'(g(x))$.

_____

_____

_____

1 mark

QUESTION 9 (6 marks)
The number of deaths following heart surgery performed by a particular surgeon in the previous month was 9 from a total of 15 operations. The national average death rate from the same type of heart surgery was 13%.

(a) Calculate the sample proportion and state what this value represents.

2 marks

(b) Calculate a 95% confidence interval to estimate the population proportion.

2 marks

(c) Can the Normal approximation be used for the distribution of the sample proportion? Give a reason for your answer.

2 marks

END OF QUESTION AND ANSWER BOOKLET