



THE SCHOOL FOR EXCELLENCE (TSFX)
UNITS 3 & 4 MATHEMATICAL METHODS 2017
WRITTEN EXAMINATION 1 – SOLUTIONS

Errors and updates relating to this examination paper will be posted at
www.tsfx.com.au/examupdates

Note:

- $\left(A \frac{1}{2} \times 4 \downarrow\right)$ means four answer half-marks rounded **down** to the next integer.
Rounding occurs at the end of a part of a question.
- **M1** = 1 Method mark.
- **A1** = 1 Answer mark (it **must** be this or its equivalent).
- **H1** = 1 consequential mark (**H**is/**H**er mark...correct answer from incorrect statement or slip).

QUESTION 1

(a) $\frac{d}{dx}(x^3 \log_e(4x)) = \frac{d}{dx}(x^3) \times \log_e(4x) + x^3 \times \frac{d}{dx}(\log_e(4x))$ (Product Rule) **M1**

$$= 3x^2 \log_e(4x) + x^3 \times \frac{4}{4x}$$

$$= 3x^2 \log_e(4x) + x^3 \times \frac{1}{x}$$

$$= 3x^2 \log_e(4x) + x^2$$
 A1

(b) $f(x) = \cos^2(3x)$

Let $u = \cos(3x)$ $\therefore \frac{du}{dx} = -3\sin(3x)$ (Chain Rule) **M1**

Let $y = u^2$ $\therefore \frac{dy}{du} = 2u = 2\cos(3x)$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -6\sin(3x)\cos(3x)$ **A1**

$f'\left(\frac{\pi}{12}\right) = -6\sin\left(\frac{3\pi}{12}\right)\cos\left(\frac{3\pi}{12}\right) = -6 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$
 $= -3$ **A1**

QUESTION 2

$$(a) \int \left(4e^{-2x} + \frac{3}{\cos^2(x)} + 7 \right) dx = \frac{4}{-2} e^{-2x} + 3 \tan(x) + 7x + c$$

$$= -2e^{-2x} + 3 \tan(x) + 7x + c$$

A $\frac{1}{2} \times 4 \downarrow$

$$(b) \int_0^1 (1-2x)^4 dx = \left[\frac{1}{-2} \times \frac{1}{5} (1-2x)^5 \right]_0^1$$

A1

$$= \left(-\frac{1}{10} \times (-1)^5 \right) - \left(-\frac{1}{10} \right)$$

$$= \frac{1}{5}$$

A1

QUESTION 3

$$\frac{3 + \cos^2(\theta)}{\sin(\theta) - 2} = 3 \sin(\theta)$$

$$3 + \cos^2(\theta) = 3 \sin(\theta)(\sin(\theta) - 2)$$

$$3 + \cos^2(\theta) = 3 \sin^2(\theta) - 6 \sin(\theta)$$

$$3 + (1 - \sin^2(\theta)) = 3 \sin^2(\theta) - 6 \sin(\theta)$$

M1

$$0 = 4 \sin^2(\theta) - 6 \sin(\theta) - 4$$

$$2 \sin^2(\theta) - 3 \sin(\theta) - 2 = 0 \text{ and so } (2 \sin(\theta) + 1)(\sin(\theta) - 2) = 0$$

$$2 \sin(\theta) + 1 = 0 \text{ or } \sin(\theta) - 2 = 0$$

A1

$$\text{Hence } \sin(\theta) = -\frac{1}{2} \text{ only as } \sin(\theta) \neq -2$$

$$\theta = \frac{7\pi}{6} + 2n\pi \text{ or } \frac{11\pi}{6} + 2n\pi \text{ where } n \in \mathbb{Z}.$$

A1

QUESTION 4

- (a) State the transformations needed to change $f(x) = 2x^2 - 4x - 6$ back to the graph with equation $g(x) = x^2$. The answer will be the reverse of the transformations needed to change $g(x)$ to $f(x)$.

First re-write $f(x)$ in transformation form: $f(x) = 2x^2 - 4x - 6 = 2(x-1)^2 - 8$

The transformations required to change $g(x)$ to $f(x)$ are:

- A dilation from the x axis by a factor of 2
- Translations of 1 unit to the right and 8 units down

Reversing this gives the answer required:

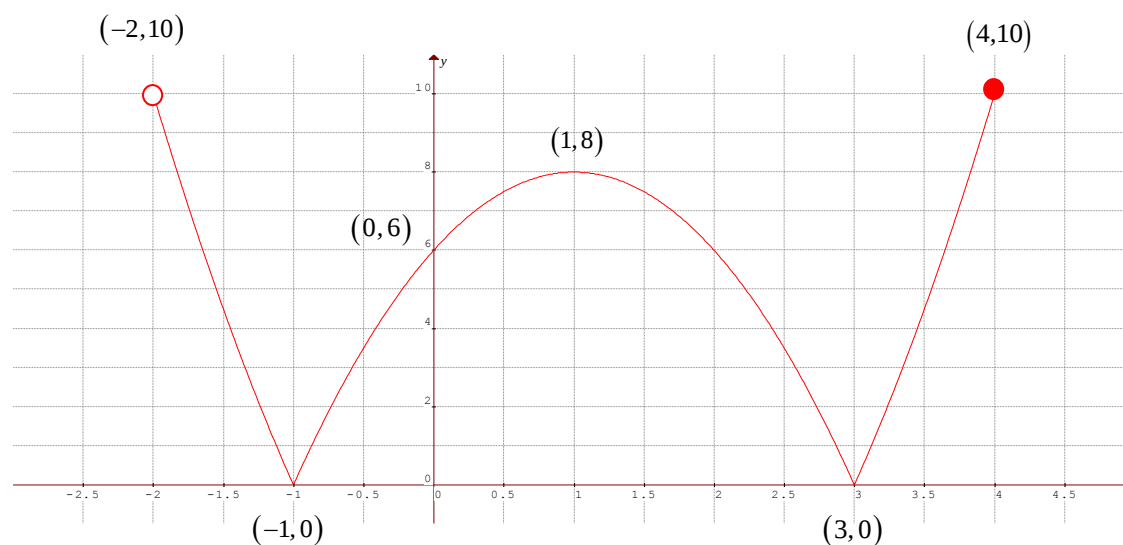
A translation of 1 unit to the right and 8 units down followed by a dilation by a factor $\frac{1}{2}$ from the x axis.

A2 all correct
A1 one correct

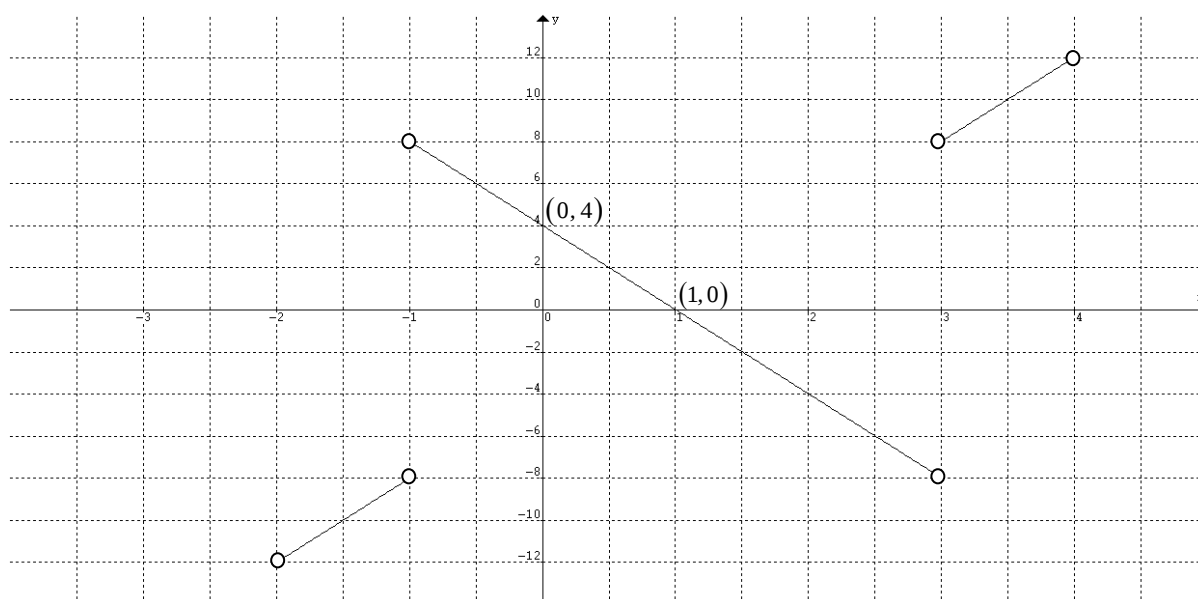
- (b) The following information is required on the graph:

- $(-2, 10)$ empty dot
- $(4, 10)$ full dot
- Maximum turning point coordinates $(1, 8)$
- Intercepts $(0, 6)$, $(-1, 0)$ and $(3, 0)$

A $\frac{1}{2} \times 4 \downarrow$



(c)



Lines correct irrespective of whether endpoints are empty/full

A1

All six end-points need to be empty

A $\frac{1}{2}$

Intercepts at (0, 4) and (1, 0)

A $\frac{1}{2}$

Round answer down

QUESTION 5

(a) $(m+1)x + 9y = 21$
 $2x + (m-2)y = 7$

Setting these equations up in matrix form gives: $\begin{bmatrix} m+1 & 9 \\ 2 & m-2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21 \\ 7 \end{bmatrix}$

M1

The determinant of the 2×2 matrix is zero for non-unique solutions:

$$(m+1)(m-2) - 18 = 0$$

$$m^2 - m - 2 - 18 = 0$$

$$m^2 - m - 20 = (m-5)(m+4) = 0$$

$\therefore m = 5$ or $m = -4$ for non-unique solutions.

A1

(b) (i) When $m = 5$, the given equations become:

$$6x + 9y = 21 \quad (1)$$

$$2x + 3y = 7 \quad (2)$$

Dividing equation (1) by 3 gives $2x + 3y = 7$, which is the same as equation (2).

Therefore, an infinite set of solutions exist when $m = 5$.

A1

(ii) Substitute $x = \lambda$ into equation (2):

$$2\lambda + 3y = 7$$

$$\therefore y = \frac{7 - 2\lambda}{3}$$

A1

QUESTION 6

$$\int_{1.5}^5 \left(2 + \frac{1}{x-1} \right) dx = [2x + \log_e(x-1)]_{1.5}^5$$

(Anti-derivative) A1

$$= (10 + \log_e 4) - (3 + \log_e 0.5)$$

$$= 7 + 2\log_e 2 - \log_e \left(\frac{1}{2} \right)$$

A1

$$= 7 + 2\log_e 2 - (\log_e 1 - \log_e 2)$$

$$\left[\log_e \left(\frac{1}{2} \right) = -\log_e(2) \right] \quad \text{M1}$$

$$= 7 + 3\log_e 2$$

Equating gives: $A + B\log_e 2 = 7 + 3\log_e 2$

Therefore: $A = 7$ and $B = 3$

(Both values correct) A1

QUESTION 7

$$(a) \int_0^1 a(x^4 - x^5) dx = a \left[\frac{1}{5}x^5 - \frac{1}{6}x^6 \right]_0^1 = 1$$

(Let anti-deriv = 1) M1

$$a \left(\frac{1}{5} - \frac{1}{6} \right) = 1$$

$$\therefore \frac{a(6-5)}{30} = 1$$

$$\therefore a = 30$$

(b) $\int_0^1 30x(x^4 - x^5) dx = 30 \left[\frac{1}{6}x^6 - \frac{1}{7}x^7 \right]_0^1$ (Correct integrand) M1

$$= 30 \left(\frac{1}{6} - \frac{1}{7} \right)$$

$$= \frac{30}{42}, \frac{5}{7}$$
 A1

(c) The mode occurs at the maximum value of $f(x)$.

$$\frac{d}{dx}(30x^4 - 30x^5) = 0$$

$$30(4x^3 - 5x^4) = 0$$

$$30x^3(4 - 5x) = 0$$

$$x = \frac{4}{5} \text{ or } x = 0$$
 M1

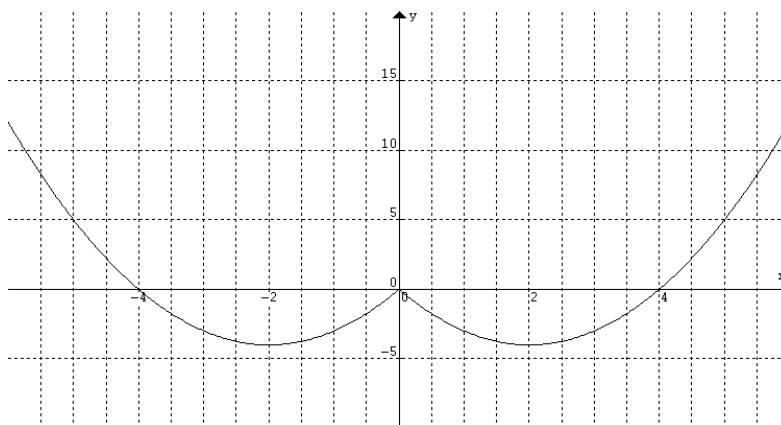
When $x = 0$: $y = 0$

When $x = \frac{4}{5}$: $y = 30 \times \left(\frac{4}{5}\right)^4 \left(\frac{1}{5}\right) = \frac{4}{5}$

The mode is $\frac{4}{5}$. A1

QUESTION 8

(a)



(left of origin) A1
(right of origin) A1

(b) $R \setminus \{0\}$ A1

QUESTION 9

(a) $\hat{p} = \frac{9}{15} = \frac{3}{5}$

A1

Value represents the proportion of heart surgery patients that died.

A1

(b) $\left(p - z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}, p + z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}} \right)$

For a 95% confidence interval, 5% will be in the tails. i.e. 2.5% in each tail. Therefore, the area under the normal distribution curve to the left of z is 0.975.

This means that $z = 1.96$

$$\left(\frac{3}{5} - 1.96 \sqrt{\frac{0.6(0.4)}{15}}, \frac{3}{5} + 1.96 \sqrt{\frac{0.6(0.4)}{15}} \right) = (0.352, 0.848)$$

A2

(c) The Normal distribution can be used when $np \geq 10$ and $n(1-p) \geq 10$.

$$np = 15 \times 0.6 = 9 \text{ which is less than } 10.$$

M1

This means that the Normal model cannot be used in this case.

A1