



Trial Examination 2017

VCE Mathematical Methods Units 3&4

Written Examination 1

Suggested Solutions

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Question 1 (4 marks)

a. $\frac{dy}{dx} = 6x(3x^2 - 4)^{-\frac{1}{2}}$

$$= \frac{3x}{\sqrt{3x^2 - 4}}$$

A1

b. $f'(x) = 2x^2 \sec^2(2x) + 2x \tan(2x)$

M1

$$f'\left(\frac{\pi}{2}\right) = \frac{\left(\frac{\pi}{2}\right)^2 \times 2}{(\cos(\pi))^2} + 2\left(\frac{\pi}{2}\right) \tan(\pi)$$

$$= \frac{\pi^2}{2} + 0$$

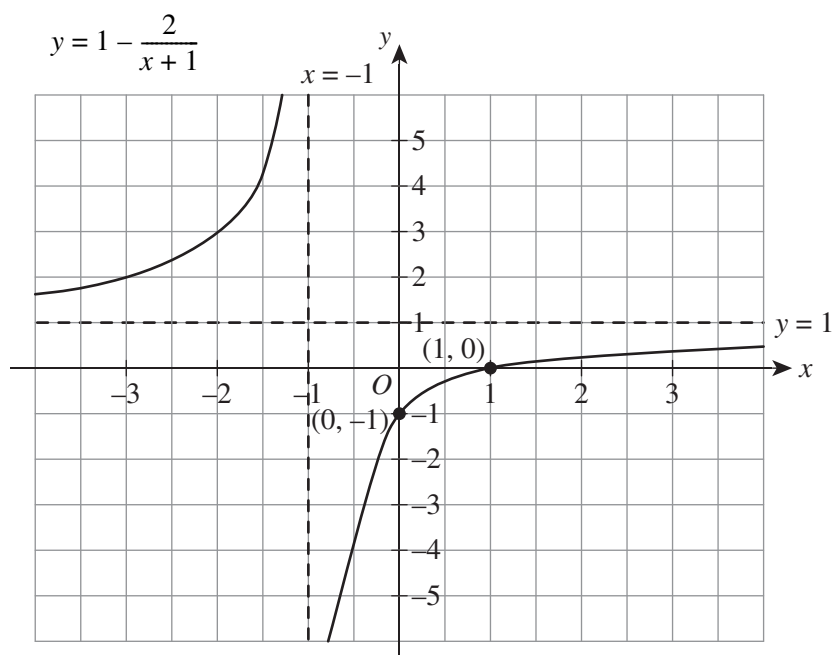
$$= \frac{\pi^2}{2}$$

M1

A1

Question 2 (5 marks)

a.

*correct shape* A1*correct asymptotes labelled at $x = -1$, $y = 1$* A1*correct intercepts labelled at $(0, -1)$ and $(1, 0)$* A1

$$\text{b. area} = - \int_0^1 1 - \frac{2}{x+1} dx \quad \text{M1}$$

$$= -[x - 2\log_e|x+1|]_0^1$$

$$= -[1 - 2\log_e(1+1)] - [0 - 2\log_e(0+1)]$$

$$= -[1 - \log_e(4)] - [0]$$

$$= \log_e(4) - 1 \quad \text{A1}$$

Question 3 (3 marks)

$$2(2^{4b}) + 7(2^{2b}) = 4 \quad \text{let } u = 2^{2b}$$

$$2u^2 + 7u - 4 = 0 \quad \text{M1}$$

$$(2u - 1)(u + 4) = 0$$

$$u = \frac{1}{2}, -4$$

$$\therefore 2^{2b} = \frac{1}{2} \text{ since } 2^{2b} \neq -4 \quad \text{A1}$$

$$2^{2b} = 2^{-1}, b = -\frac{1}{2} \quad \text{A1}$$

Note: Students must disregard the incorrect solution to get full marks.

Question 4 (2 marks)

Let $y = e^{3x} + 4$ (for inverse swap x and y).

$$x = e^{3y} + 4 \quad \text{M1}$$

$$x - 4 = e^{3y}$$

$$\log_e(x - 4) = 3y$$

$$f^{-1}(x) = \frac{\log_e(x - 4)}{3} \quad \text{A1}$$

Alternatively:

$$f^{-1}(x) = \frac{1}{3} \log_e(x - 4)$$

$$= \log_e(x - 4)^{\frac{1}{3}} \quad \text{A1}$$

Question 5 (3 marks)

$$-2x - 1 = x'$$

$$x = \frac{x' + 1}{-2}$$

$$3y - 2 = y'$$

$$y = \frac{y' + 2}{3}$$

M1

substitute into original equation $y = \frac{1}{x} + 3$

$$\frac{y' + 2}{3} = \frac{1}{\frac{x' + 1}{-2}} + 3$$

$$y' + 2 = \frac{-6}{x' + 1} + 9$$

$$y' = \frac{-6}{x' + 1} + 7$$

$$y_{\text{new}} = \frac{-6}{x + 1} + 7$$

A1

$$a = -6, b = 1, c = 7$$

must state values for all of a, b, and c A1

Question 6 (3 marks)

$$E(X) = np$$

$$= 3$$

$$\text{var}(X) = np(1 - p)$$

$$= \frac{12}{5}$$

$$\frac{\text{var}(X)}{E(X)} = \frac{np(1 - p)}{np}$$

$$= \frac{12}{3}$$

M1

$$1 - p = \frac{12}{15}$$

$$p = \frac{3}{15}$$

$$= \frac{1}{5}$$

A1

$$n \times \frac{1}{5} = 3$$

$$n = 15$$

A1

Question 7 (4 marks)

a. $f'(x) = \cos(2x) - 2x \sin(2x)$

A1

$$\text{b.} \quad \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(2x) - 2x \sin(x) dx = [x \cos(2x)]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \quad \text{M1}$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(2x) dx - [x \cos(2x)]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x \sin(2x) dx \quad \text{M1}$$

$$\left[\frac{\sin(2x)}{2} \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} - [x \cos(2x)]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x \sin(2x) dx$$

$$\left[\frac{\sin(\pi)}{2} - \frac{\sin\left(\frac{\pi}{3}\right)}{2} \right] - \left[\frac{\pi}{2} \cos(\pi) - \frac{\pi}{6} \cos\left(\frac{\pi}{3}\right) \right] = 2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x \sin(2x) dx$$

$$\left[0 - \frac{\sqrt{3}}{4} \right] - \left[-\frac{\pi}{2} - \frac{\pi}{12} \right] = 2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x \sin(2x) dx$$

$$\frac{7\pi}{24} - \frac{\sqrt{3}}{8} = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} x \sin(2x) dx \quad \text{A1}$$

Question 8 (4 marks)

$$\text{a.} \quad \Pr(Z > c) = \Pr(X < 2.3)$$

$$= \Pr(Z < -2)$$

$$\Pr(Z < -2) = \Pr(Z > 2)$$

$$c = 2 \quad \text{A1}$$

$$\text{b.} \quad \Pr(X < 2.7 | X < 3.5) = \frac{\Pr(X < 2.7 \cap X < 3.5)}{\Pr(X < 3.5)}$$

$$= \frac{\Pr(X < 2.7)}{\Pr(X < 3.5)}$$

$$= \frac{\Pr(Z < -1)}{\Pr(Z < 1)} \quad \text{equating } Z \text{ values to } X \text{ values M1}$$

$$= \frac{\frac{16}{100}}{\frac{84}{100}}$$

$$\text{calculating } \Pr(Z < 1) = 1 - \frac{16}{100} \quad \text{M1}$$

$$= \frac{16}{84}$$

$$= \frac{4}{21} \quad \text{A1}$$

Question 9 (12 marks)

$$\text{a. i. } A = 2 \left[(9 \times 9) - (3 \times 1) - \int_3^9 \frac{1}{9} x^2 dx \right] \quad \text{M1}$$

$$= 2 \left[81 - 3 - \left[\frac{x^3}{27} \right]_3^9 \right] \quad \text{M1}$$

$$= 2[78 - (27 - 1)]$$

$$= 2(78 - 26)$$

$$= 104 \text{ cm}^3 \quad \text{A1}$$

$$\text{ii. volume} = 104 \times 18$$

$$= 1872 \text{ cm}^3 \quad \text{A1}$$

$$\text{b. maximum rate when } R'(t) = 0 \quad \text{M1}$$

$$R'(t) = 6t - 4$$

$$= 0$$

$$t = \frac{2}{3} \Rightarrow t = 40 \text{ minutes} \quad \text{A1}$$

$$\text{c. } V(t) = \int R(t) dt \quad \text{M1}$$

$$= \int (3t^2 - 4t + 1) dt$$

$$= t^3 - 2t^2 + t$$

as $V(t) = 0$ at $t = 0$ as no water emptied M1

$$t^3 - 2t^2 + t = 1872 \quad \text{M1}$$

$$(t - 13)(t^2 + 11t + 144) = 0$$

$$\therefore t = 13 \text{ hours} \quad \text{A1}$$

$$\text{d. At } t = 10, V(10) = 10^3 - 2(10)^2 + 10$$

$$= 810 \text{ cm}^3 \quad \text{M1}$$

$$\therefore \text{yes, overflows as only } 810 \text{ cm}^3 \text{ removed} \quad \text{A1}$$