

The Mathematical Association of Victoria

Trial Examination 2017

MATHEMATICAL METHODS

Trial Written Examination 2 - SOLUTIONS

SECTION A: Multiple Choice

Question	Answer	Question	Answer
1	C	11	B
2	B	12	D
3	D	13	C
4	E	14	D
5	C	15	E
6	B	16	C
7	E	17	A
8	D	18	A
9	E	19	E
10	C	20	B

Question 1

Answer C

$$f : \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}, f(x) = -3\cos(4x + \pi) + 1$$

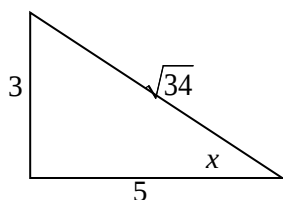
Amplitude = 3 translated vertically by 1 unit giving range = $[-2, 4]$

$$\text{period} = \frac{2\pi}{4} = \frac{\pi}{2}$$

Question 2

Answer B

$$\tan(x) = \frac{3}{5} \text{ and } \pi \leq x \leq \frac{3\pi}{2}$$



$$\text{In 3rd quadrant } \sin(x) = -\frac{3}{\sqrt{34}}$$

Question 3**Answer D**

$$h(x) = x^4$$

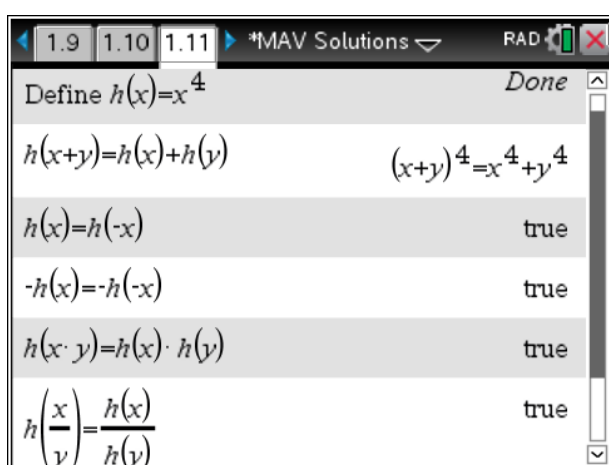
$$h(x+y) = h(x) + h(y) \text{ is false}$$

$$\text{LHS} = (x+y)^4$$

$$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

$$\text{RHS} = x^4 + y^4$$

$$\text{LHS} \neq \text{RHS}$$

**Question 4****Answer E**

$$h: (-\infty, 4) \rightarrow \mathbb{R}, h(x) = 2(4-x)^2 - 1$$

$$y = 2(4-x)^2 - 1$$

Swap x and y for inverse

$$x = 2(4-y)^2 - 1$$

$$4-y = \pm \sqrt{\frac{x+1}{2}}$$

$$y = 4 \pm \sqrt{\frac{x+1}{2}}$$

solve($x=2 \cdot (4-y)^2-1, y$)

$$\left\{ y=4-\frac{\sqrt{2 \cdot (1+x)}}{2}, y=4+\frac{\sqrt{2 \cdot (1+x)}}{2} \right\}$$

Domain of $h^{-1} = \text{range } h = (-1, \infty)$

$$h^{-1}: (-1, \infty) \rightarrow \mathbb{R}, h^{-1}(x) = 4 - \frac{\sqrt{2x+2}}{2}$$

Question 5**Answer C**

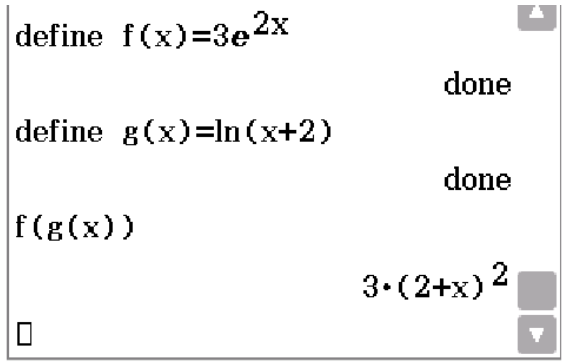
$$f(x) = 3e^{2x} \text{ and } g(x) = \log_e(x+2)$$

For $f(g(x))$, test that the range $g \subseteq \text{domain } f$

Giving $R \subseteq R$

So $f(g(x))$ exists with

$$\text{dom } f(g(x)) = \text{dom } g(x) = (-2, \infty)$$



$h = f(g(x))$ can be defined as

$$h: (-2, \infty) \rightarrow R, h(x) = 3(x+2)^2$$

Question 6**Answer B**

$$mx + y = 2$$

$$2x - 3y = k$$

Using ratios

For infinite solutions or no solution

$$\frac{m}{2} = -\frac{1}{3}, m = -\frac{2}{3}$$

For no solution

$$-3 \neq \frac{k}{2}, k \neq -6$$

$$m = -\frac{2}{3}, k \in R \setminus \{-6\}$$

Question 7**Answer E**

$$\text{Range} = 42 - 37 = 5$$

$$\text{Amplitude} = 2.5$$

$$\text{Vertical translation} = 39.5$$

$$\text{Period} = \frac{2\pi}{n} = 24, \text{ giving } n = \frac{\pi}{12}$$

$$y = 2.5 \sin\left(\frac{\pi t}{12}\right) + 39.5$$

Question 8**Answer D**

$$f(x) = \sqrt{2x-1} + 3 \text{ and } g(x) = \frac{1}{(x-2)^2} + 4$$

Domain of f is $\left[\frac{1}{2}, \infty\right)$

Domain of g is $\mathbb{R} \setminus \{2\}$

The intersection of both is

$$\left[\frac{1}{2}, 2\right) \cup (2, \infty)$$

which is the domain of $f + g$

Question 9**Answer E**

$$f'(x) = e^{x+1} + x + 1$$

$$f(x) = e^{x+1} + \frac{x^2}{2} + x + c$$

$$f(0) = 2 \text{ giving } 2 = e^{0+1} + \frac{0^2}{2} + 0 + c \therefore c = 2 - e$$

then

$$\begin{aligned} f(x) &= e^{x+1} + \frac{x^2}{2} + x + 2 - e \\ &= \frac{x^2}{2} + x + 2 + e(e^x - 1) \end{aligned}$$

Question 10**Answer C**

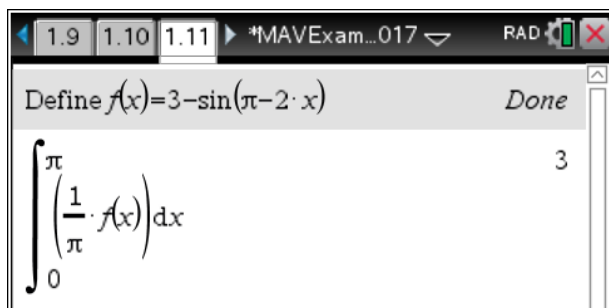
$$\text{Given } \int_1^3 g(x) dx = 2$$

$$\begin{aligned} 2 \int_3^1 (g(x) + 1) dx &= 2 \int_3^1 g(x) dx + 2 \int_3^1 1 dx \\ &= -2 \int_1^3 g(x) dx - 2 \int_1^3 1 dx \\ &= -2 \times 2 - 2[x]_1^3 \\ &= -4 - 4 \\ &= -8 \end{aligned}$$

Question 11**Answer B**

$$\text{Average value} = \frac{1}{b-a} \int_a^b f(x) dx = \frac{1}{\pi-0} \int_0^{\pi} f(x) dx$$

where $f(x) = -\sin(\pi - 2x) + 3$



Average value = 3

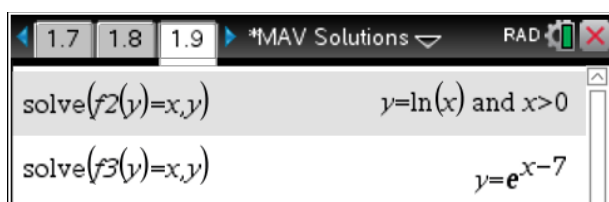
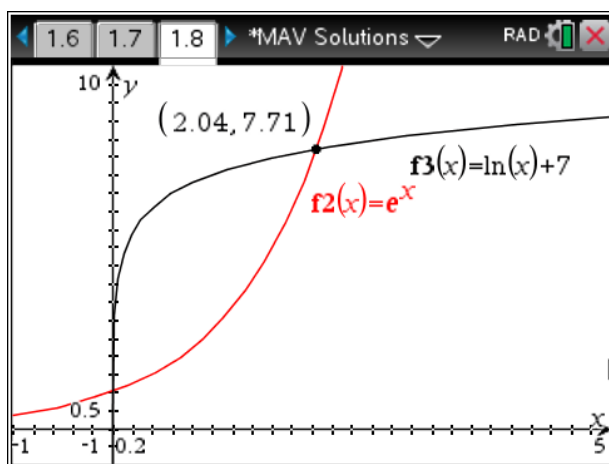
Question 12**Answer D**

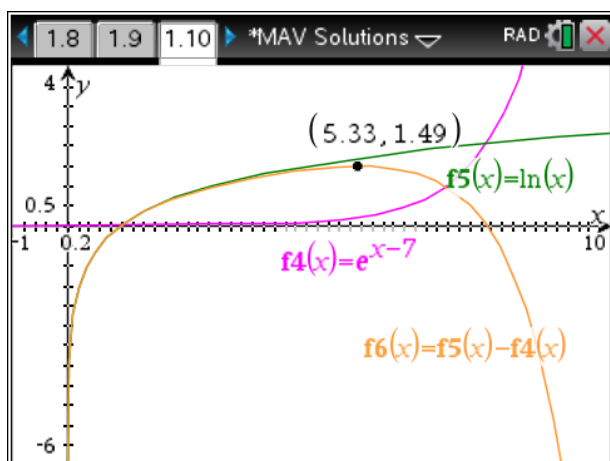
$$f(x) = e^x \text{ and } g(x) = \log_e(x) + 7$$

$$f^{-1}(x) = \log_e(x) \text{ and } g^{-1}(x) = e^{x-7}$$

Let $k(x) = f^{-1}(x) - g^{-1}(x)$

Maximum value is approximately 1.49.



**Question 13****Answer C**

$$f(x) = \log_e(3-x)$$

Intercepts

$$(0, \log_e(3)), (2, 0)$$

$$\text{Gradient} = -\frac{\log_e(3)}{2}$$

$$\text{Gradient of the perpendicular line} = \frac{2}{\log_e(3)}$$

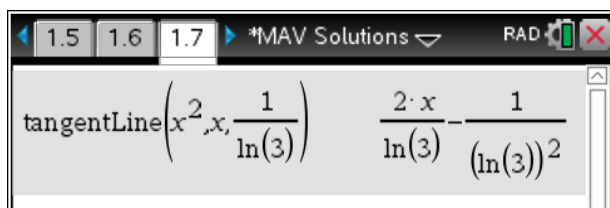
$$g(x) = x^2$$

$$g'(x) = 2x = \frac{2}{\log_e(3)}, x = \frac{1}{\log_e(3)}$$

$$\left(\frac{1}{\log_e(3)}, \frac{1}{(\log_e(3))^2} \right)$$

$$y - \frac{1}{(\log_e(3))^2} = \frac{2}{\log_e(3)} \left(x - \frac{1}{\log_e(3)} \right)$$

$$y = \frac{2}{\log_e(3)} x - \frac{1}{(\log_e(3))^2}$$



Question 14**Answer D**

$$a = 2t^2 - 1$$

$$v = \int (2t^2 - 1) dt$$

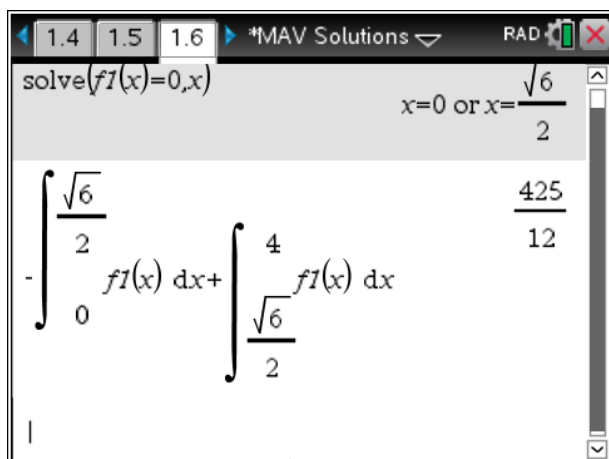
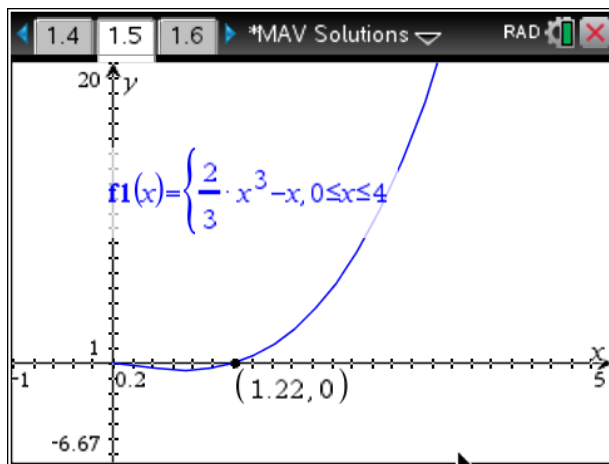
$$= \frac{2}{3}t^3 - t + c$$

$$c = 0$$

Solve $v(t) = 0$ for t

$$t = \frac{\sqrt{6}}{2}, t > 0$$

$$\text{Distance} = -\int_0^{\frac{\sqrt{6}}{2}} v(t) dt + \int_{\frac{\sqrt{6}}{2}}^4 v(t) dt = \frac{425}{12}$$



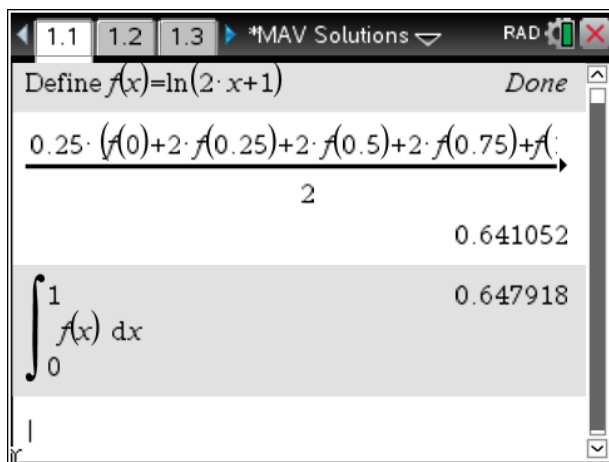
Question 15**Answer E**

$$\frac{(\text{left-endpoint rectangles of width } 0.25 + \text{right-endpoint rectangles of width } 0.25)}{2}$$

The smaller the width of the rectangle the better the estimate. In this case the right-endpoint rectangles estimate will give an over estimate and the left-endpoint rectangles will give an underestimate. The average of the two should give the best estimate.

$$\frac{0.25(f(0) + 2f(0.25) + 2f(0.5) + 2f(0.75) + f(1))}{2}$$

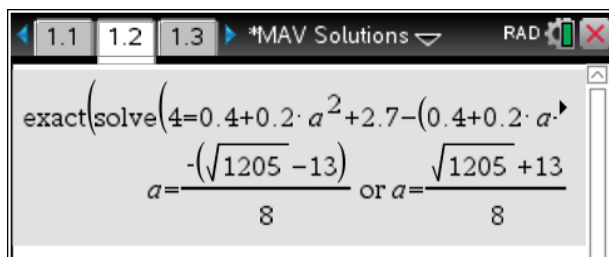
$$\approx 0.6411$$

**Question 16****Answer C**

$$0.1 + b + 0.2 + 0.3 = 1, \quad b = 0.4$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 \quad 4 = 0.4 + 0.2a^2 + 2.7 - (0.4 + 0.2a + 0.9)^2$$

$$a = \frac{13 \pm \sqrt{1205}}{8}$$

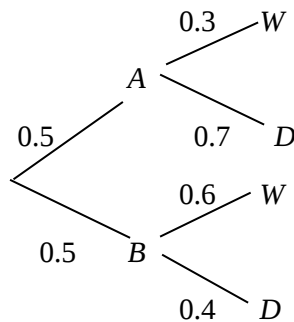
**Question 17****Answer A**

Let W represent a white chocolate and

A represent Box A.

$$\Pr(A|W) = \frac{\Pr(W \cap A)}{\Pr(W)}$$

$$= \frac{\frac{1}{2} \times \frac{3}{10}}{\frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{6}{10}} = \frac{1}{3}$$

**Question 18****Answer A**

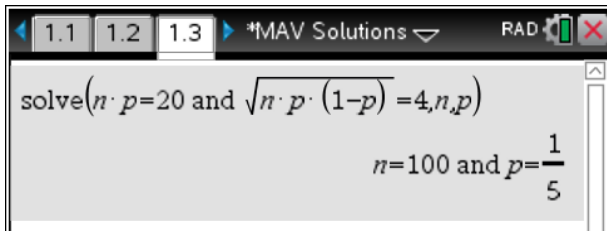
$$X \sim \text{Bi}(n, p)$$

$$E(X) = np = 20$$

$$SD(X) = \sqrt{npq} = 4$$

$$20q = 16, q = \frac{4}{5}, p = \frac{1}{5}$$

$$\frac{1}{5}n = 20, n = 100$$

**Question 19****Answer E**

Let W represent winning.

$$\Pr(W) = 0.65$$

$$W \sim \text{Bi}(n, 0.65)$$

$$\Pr(W \geq 2) > 0.95$$

$$1 - (\Pr(W = 0) + \Pr(W = 1)) > 0.95$$

$$\Pr(W = 0) + \Pr(W = 1) < 0.05$$

$$0.35^n + 0.65n(0.35)^{n-1} < 0.05$$

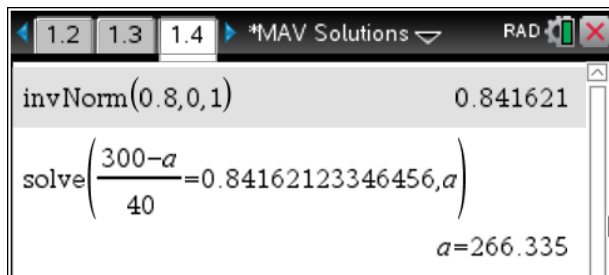
Question 20**Answer B**

$$X \sim N(\mu, 40^2)$$

$$\Pr(X > 300) = 0.2$$

$$\text{Solve } \frac{300 - \mu}{40} = 0.8416... \text{ for } \mu$$

$$\mu = 266 \text{ to the nearest integer}$$

**SECTION B****Question 1**

$$f: \left[-\pi, \frac{7\pi}{2}\right] \rightarrow \mathbb{R}, f(x) = -a \sin\left(\frac{x}{3} + \frac{\pi}{3}\right) - \pi$$

$$\text{a. period} = \frac{2\pi}{\frac{1}{3}} = 6\pi \quad \mathbf{1A}$$

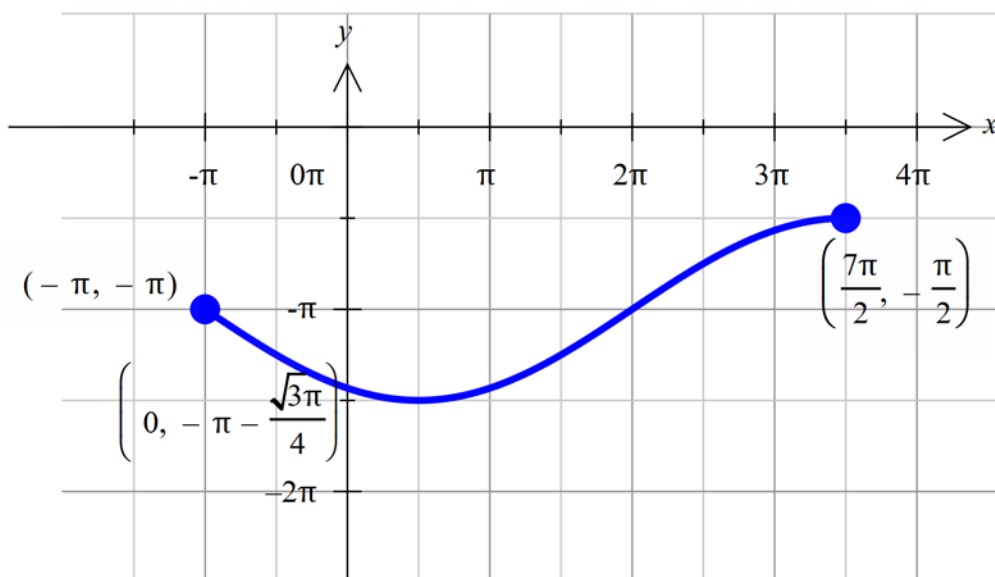
$$\text{b. range of } f \text{ is } \left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$$

$$\text{So } \frac{-\frac{3\pi}{2} + \frac{\pi}{2}}{2} = -\frac{\pi}{2}$$

$$\text{As there is already a negative sign in front of } a, a = \frac{\pi}{2} \quad \mathbf{1M \text{ show that}}$$

$$\text{c. shape } \mathbf{1A}$$

$$\text{intercepts } \left(0, -\pi - \frac{\sqrt{3}\pi}{4}\right), \text{ endpoints } (-\pi, -\pi) \text{ and } \left(\frac{7\pi}{2}, -\frac{\pi}{2}\right) \quad \mathbf{1A}$$



d. domain of f is restricted to $[-\pi, b]$

First stationary point after $x = -\pi$ is $\left(\frac{\pi}{2}, -\frac{3\pi}{2}\right)$

Maximum possible $b = \frac{\pi}{2}$ **1A**

e. For $b = \frac{\pi}{2}$, the domain of f_1^{-1} is $\left[-\frac{3\pi}{2}, -\pi\right]$. **1A**

f. The equation of tangent to f at $x = 0$ is $y = -\frac{\pi x}{12} - \frac{\sqrt{3}\pi}{4} - \pi$. **1A**

The equation of tangent to f at $x = \pi$ is $y = \frac{\pi x}{12} - \frac{\sqrt{3}\pi}{4} - \pi - \frac{\pi^2}{12}$. **1A**

```
define f(x)=-π/2 sin(x/3+π/3)-π | -π≤x≤7π/2
done
tanLine(f(x), x, 0)
      -√3·π/4 - π - x·π/12
tanLine(f(x), x, π)
      -√3·π/4 - π + x·π/12 - π²/12
```

g. point of intersection of the tangents = $\left(\frac{\pi}{2}, -\frac{\sqrt{3}\pi}{4} - \pi - \frac{\pi^2}{24}\right)$ **1A**

```
solve(-√3·π/4 - π - x·π/12 = -√3·π/4 - π + x·π/12 - π²/12, x)
      {x=π/2}
```

Define	$g(x) = \frac{-\sqrt{3} \cdot \pi}{4} - \pi - \frac{x \cdot \pi}{12}$	done
$g\left(\frac{\pi}{2}\right)$	$\frac{-\sqrt{3} \cdot \pi}{4} - \pi - \frac{\pi^2}{24}$	

h. Sketch the graph to view the required area.



Area =

$$\int_0^{\frac{\pi}{2}} \left(-\frac{\pi}{2} \sin\left(\frac{x}{3} + \frac{\pi}{3}\right) - \pi \right) - \left(-\frac{\pi x}{12} - \frac{\sqrt{3}\pi}{4} - \pi \right) dx + \int_{\frac{\pi}{2}}^{\pi} \left(-\frac{\pi}{2} \sin\left(\frac{x}{3} + \frac{\pi}{3}\right) - \pi \right) - \left(\frac{\pi x}{12} - \frac{\sqrt{3}\pi}{4} - \pi - \frac{\pi^2}{12} \right) dx$$

OR

Alternatively define the functions as $f(x) = -\frac{\pi}{2} \sin\left(\frac{x}{3} + \frac{\pi}{3}\right) - \pi$

And $g(x) = -\frac{\pi x}{12} - \frac{\sqrt{3}\pi}{4} - \pi$ and $h(x) = \frac{\pi x}{12} - \frac{\sqrt{3}\pi}{4} - \pi - \frac{\pi^2}{12}$.

Gives Area = $\int_0^{\frac{\pi}{2}} (f(x) - g(x)) dx + \int_{\frac{\pi}{2}}^{\pi} (f(x) - h(x)) dx$

OR

By symmetry Area = $2 \int_0^{\frac{\pi}{2}} (f(x) - g(x)) dx$

correct terminals 1A

correct functions 1A

i. Area enclosed between f and the tangents, correct to 2 decimal = 0.21 square units **1A**

The screenshot shows a TI-Nspire calculator interface. At the top, there is a toolbar with icons for editing, actions, and interactive tools. Below the toolbar, the main workspace contains the following text:

Define $g(x) = \frac{-\sqrt{3} \cdot \pi}{4} - \pi - \frac{x \cdot \pi}{12}$

done

Define $h(x) = \frac{-\sqrt{3} \cdot \pi}{4} - \pi + \frac{x \cdot \pi}{12} - \frac{\pi^2}{12}$

done

Below this, there is a calculation of the area between a function $f(x)$ and the tangents $g(x)$ and $h(x)$:

$$\int_0^{\frac{\pi}{2}} f(x) - g(x) dx + \int_{\frac{\pi}{2}}^{\pi} f(x) - h(x) dx$$

The result of the calculation is displayed as 0.2072391854.

j. Transformations to get from f to f_1 where

$$f(x) = -\frac{\pi}{2} \sin\left(\frac{x}{3} + \frac{\pi}{3}\right) - \pi = -\frac{\pi}{2} \sin\left(\frac{1}{3}(x + \pi)\right) - \pi$$

There are many solutions. Three possibilities have been given.

Translate π units up and π units to the right: $y_1 = -\frac{\pi}{2} \sin\left(\frac{x}{3}\right)$ **1A**

Dilate by a factor of $\frac{2}{\pi}$ from the x -axis: $y_2 = -\sin\left(\frac{x}{3}\right)$

Reflect in the x -axis: $y_2 = \sin\left(\frac{x}{3}\right)$ **1A**

Dilate by a factor of $\frac{1}{3}$ from the y -axis: $f_2(x) = \sin(x)$ **1A**

OR

Dilate by a factor of $\frac{2}{\pi}$ from x -axis: $y_1 = \frac{2}{\pi} \left[-\frac{\pi}{2} \sin\left(\frac{x}{3} + \frac{\pi}{3}\right) - \pi \right] = -\sin\left(\frac{x}{3} + \frac{\pi}{3}\right) - 2$ **1A**

Dilate by a factor of $\frac{1}{3}$ from y -axis: $y_2 = -\sin\left(\frac{3x}{3} + \frac{\pi}{3}\right) - 2 = -\sin\left(x + \frac{\pi}{3}\right) - 2$

Reflect in the x -axis: $y_3 = \sin\left(x + \frac{\pi}{3}\right) + 2$ **1A**

Translate $\frac{\pi}{3}$ units right and 2 units down: $f_2(x) = \sin\left(\left(x - \frac{\pi}{3}\right) + \frac{\pi}{3}\right) + 2 - 2 = \sin(x)$ **1A**

OR

Translate up by π units: $y_1 = -\frac{\pi}{2} \sin\left(\frac{x}{3} + \frac{\pi}{3}\right)$

Reflect over x -axis: $y_2 = \frac{\pi}{2} \sin\left(\frac{x}{3} + \frac{\pi}{3}\right)$ **1A**

Dilate by a factor of $\frac{2}{\pi}$ from the x -axis: $y_3 = \sin\left(\frac{x}{3} + \frac{\pi}{3}\right) = \sin\left(\frac{1}{3}(x + \pi)\right)$ **1A**

Translate to the right by π units: $y_4 = \sin\left(\frac{x}{3}\right)$

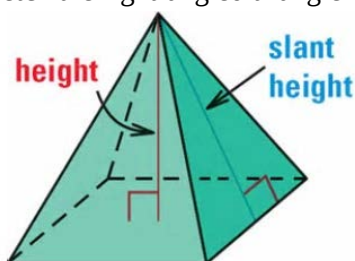
Dilate by a factor $\frac{1}{3}$ from the y -axis: $f_2(x) = \sin(x)$ **1A**

Question 2

Square based pyramid with vertical height, h metres, and length of the sides of the base, x metres.

$$\text{TSA} = 60 \text{ m}^2.$$

a. Sketch the right-angled triangle required.



This gives the Pythagoras formula $y^2 = \left(\frac{x}{2}\right)^2 + h^2$

Rearrange to get

$$y^2 = \frac{x^2}{4} + h^2$$

$$y = \pm \sqrt{\frac{x^2}{4} + h^2}$$

For $y > 0$, $y = \sqrt{h^2 + \frac{x^2}{4}}$ as required. **1M show that**

b. TSA = square base + 4 slanting faces.

$$\text{TSA} = x^2 + 4 \times \frac{1}{2}xy$$
 1A

$$\text{Using } y = \sqrt{h^2 + \frac{x^2}{4}}$$

$$\text{TSA} = x^2 + 2x\sqrt{h^2 + \frac{x^2}{4}}$$
 1A

c. To find the relationship between x and h we know that $\text{TSA} = 60 \text{ m}^2$.

Letting $x^2 + 2x\sqrt{h^2 + \frac{x^2}{4}} = 60$ and solve for h

For $h > 0$, $h = \frac{\sqrt{900 - 30x^2}}{x}$ **1A**

Volume = $\frac{1}{3} \times \text{area of base} \times \text{height}$.

Volume = $\frac{1}{3}x^2 \times \frac{\sqrt{900 - 30x^2}}{x}$

Giving $V = \frac{1}{3}x\sqrt{900 - 30x^2}$ as required **1M show that**

solve $\left(x^2 + 2 \cdot x \cdot \sqrt{h^2 + \frac{x^2}{4}} = 60, h \right)$
 $\left\{ h = -\sqrt{\frac{900 - 30 \cdot x^2}{x^2}}, h = \sqrt{\frac{900 - 30 \cdot x^2}{x^2}} \right\}$

d. We need $900 - 30x^2 > 0$ giving implied domain $x \in (0, \sqrt{30})$ **1A**

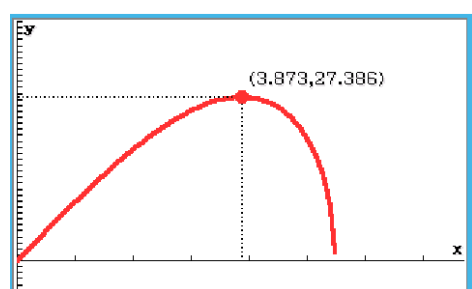
e. Solve $\frac{dV}{dx} = 0$

Considering the graph of V , for the given domain we have a local maximum.

The maximum volume, $V = 5\sqrt{30} \text{ m}^3$, **1A**

at $x = \sqrt{15} \text{ m}$. **1A**

```
define v(x) = 1/3 * x * sqrt(900 - 30 * x^2)
done
solve( d/dx (v(x)) = 0, x )
{ x = -sqrt(15), x = sqrt(15) }
v(sqrt(15))
5 * sqrt(30)
```



f.i. TSA = $4 \times \text{area of equilateral triangle}$.

TSA = $4 \times \frac{1}{2}d^2 \sin(60^\circ)$ **1M**

= $2d^2 \times \frac{\sqrt{3}}{2} = \sqrt{3}d^2$ in the required form. **1A**

ii. TSA = 60 m^2 , so $\sqrt{3}d^2 = 60$

giving, for $d > 0$, $d = \sqrt{\frac{60}{\sqrt{3}}} = \sqrt{20\sqrt{3}}$ **1M**

$$\text{Vol} = V_T = \frac{d^3}{6\sqrt{2}}$$

giving $V_T = \frac{\left(\sqrt{\frac{60}{\sqrt{3}}}\right)^3}{6\sqrt{2}} = 24.028 \text{ m}^3$ correct to 3 decimal places. **1A**

g. $\text{TSA} = \pi r(r+s) = 60 \text{ m}^2$

Giving $s = \frac{60 - \pi r^2}{\pi r}$

Volume of cone $= \frac{1}{3}\pi r^2 l$ where l is height of cone.

To find l , for $l > 0$, use

$$l^2 + r^2 = s^2$$

$$l = \sqrt{s^2 - r^2}$$

Volume of cone $= \frac{1}{3}\pi r^2 \sqrt{s^2 - r^2}$ where $s = \frac{60 - \pi r^2}{\pi r}$

Therefore $V(r) = \frac{2}{3}\pi r^2 \sqrt{\frac{900}{\pi^2 r^2} - \frac{30}{\pi}}$ **1M**

For maximum volume, $V'(r) = 0$

Considering the graph of V , for the given domain we have a local maximum.

Maximum volume, correct to two decimal places $= 30.90 \text{ m}^3$ **1A**

`combine(solve( $\pi \cdot r \cdot (r+s)=60$ , s))`

$$\left\{ s = \frac{60 - r^2 \cdot \pi}{r \cdot \pi} \right\}$$

```

define v(r) = 1/3 * pi * r^2 * sqrt(s^2 - r^2)
done
60 - r^2 * pi -> s
60 - r^2 * pi
simplify(v(r))
2 * r^2 * sqrt(900 / (r * pi)^2 - 30 / pi) * pi

```

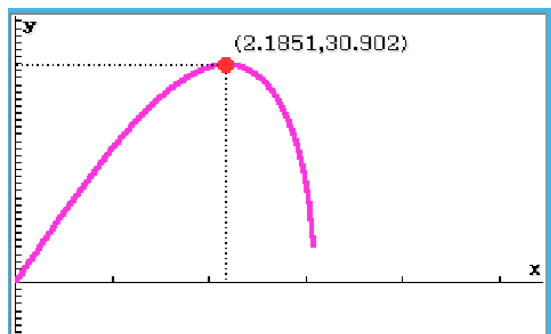
Define $f(r) = \frac{2 \cdot r^2 \cdot \sqrt{\frac{900}{(r \cdot \pi)^2} - \frac{30}{\pi} \cdot \pi}}{3}$

done

solve $\left(\frac{d}{dr}(f(r)) = 0, r \right)$

$\left\{ r = -\frac{\sqrt{15}}{\sqrt{\pi}}, r = \frac{\sqrt{15}}{\sqrt{\pi}} \right\}$

Alg Standard Real Rad



h. cone

1A

Question 3

a. (0.811, 0.893)

1A

1-Prop z Interval

Successes, x: 426

n: 500

C Level: 0.99

OK Cancel

zInterval_1Prop 426,500,0.99: stat.results

"Title"	"1-Prop z Interval"
"CLower"	0.8110944
"CUpper"	0.8929056
"p"	0.852
"ME"	0.0409056
"n"	500.

b. $0.99 \times 200 = 198$

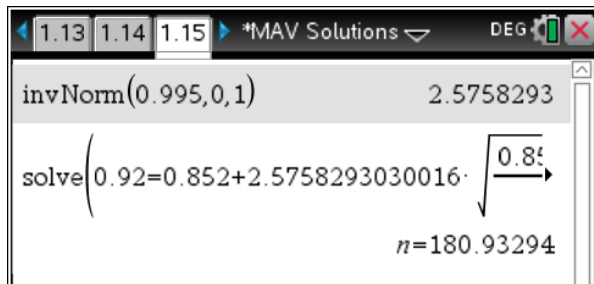
1A

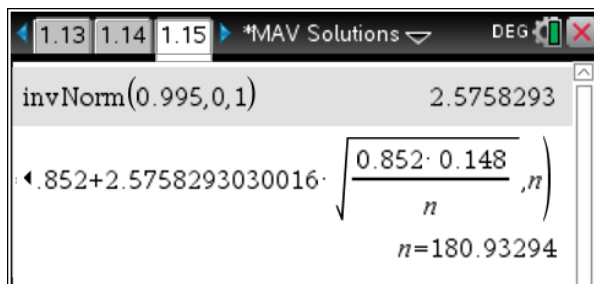
c. Yes, 0.92 is outside the 99% confidence interval.

A suitable comment. **1A**

d. Margin of error $= 0.92 - 0.852 = 2.57 \dots \sqrt{\frac{0.852 \times 0.148}{n}}$ **1M**

$n = 181$ to the nearest integer **1A**

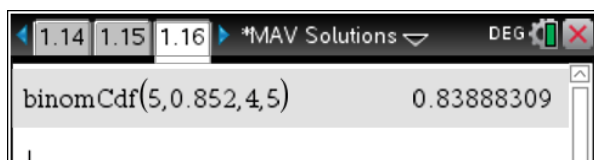




e. $X \sim \text{Bi}(5, 0.852)$ **1M**

$$\Pr(X > 3) = \Pr(X \geq 4)$$

$= 0.839$ correct to 3 decimal places **1A**



f. $\Pr(\text{first 3} | \text{only 3}) = \frac{\Pr(\text{first 3} \cap \text{only 3})}{\Pr(X = 3)}$ **1M**

$$\frac{(0.852)^3 \times (0.148)^2}{\Pr(X = 3)} = 0.1$$
 1A

OR

There are $\binom{5}{2} = 10$ ways for 3 globes only to last longer than 100 hours

and each way has the same probability. **1M**

Only in one of these ways will it be the first three. Answer 0.1 **1A**

$(0.852)^3 \cdot (0.148)^2$	0.01354697
$\text{binomPdf}(5, 0.852, 3)$	0.13546971
$\frac{0.013546971436032}{0.13546971436032}$	0.1

g. $Y \sim N(\mu, \sigma^2)$

$$\frac{150 - \mu}{\sigma} = 2.0537...$$

$$\frac{100 - \mu}{\sigma} = -1.045... \quad \mathbf{1M}$$

$$\mu = 116.86 \quad \mathbf{1A}$$

$$\sigma = 16.14 \quad \mathbf{1A}$$

$\text{invNorm}(0.98, 0, 1)$	2.0537489
$\text{invNorm}(0.148, 0, 1)$	-1.0450497
$\text{solve}\left(\frac{150-a}{b} = 2.053748910624 \text{ and } \frac{100-a}{b} = -1.0450497\right)$	$a = 116.86218 \text{ and } b = 16.135285$

h. Let J represent Jessica doing her homework and

D represent David doing his homework.

$$\Pr(J \cap D) = \Pr(J) \times \Pr(D) = 0.2q^2 \text{ for independent events} \quad \mathbf{1A}$$

$$\Pr(J' \cap D') = 1 - \Pr(J) - \Pr(D) + \Pr(J \cap D)$$

$$\Pr(J' \cap D') = 1 - 0.2 - q^2 + 0.2q^2 = 0.8 - 0.8q^2 \quad \mathbf{1A}$$

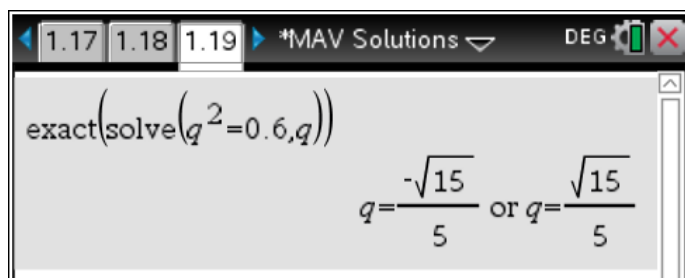
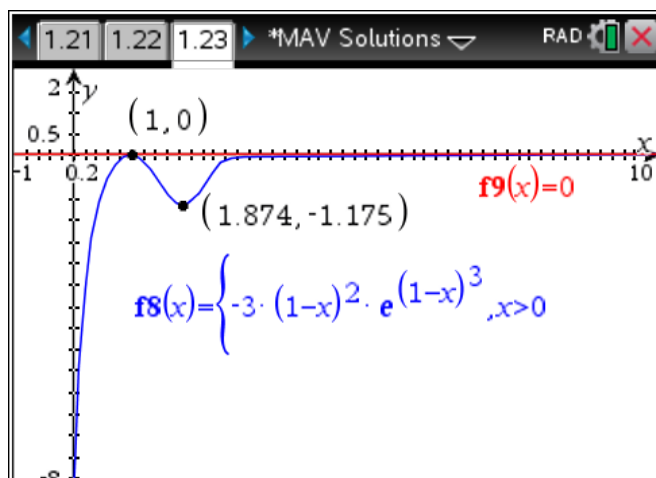
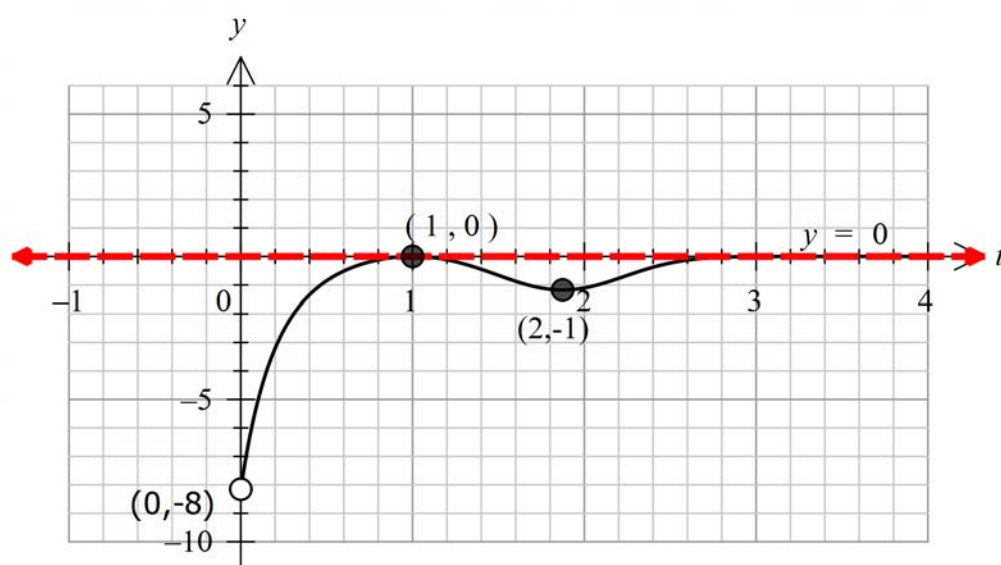
OR

$$\text{For independent events } \Pr(J' \cap D') = \Pr(J') \times \Pr(D') \quad \mathbf{1A}$$

$$\Pr(J' \cap D') = 0.8(1 - q^2) \quad \mathbf{1A}$$

i. Solve $q^2 = 0.6$ for q , $q > 0$

$$q = \frac{\sqrt{15}}{5}$$

1A**Question 4****a. Shape 1A****Asymptote 1A****Points 1A**

$$\text{b. } \int_0^2 (g'(t)) dt = -2.35... \quad \text{1M}$$

Declined by 235 rabbits

1A

Calculator screen showing the definite integral:

$$\int_0^2 \left(-3 \cdot (1-x)^2 \cdot e^{(1-x)^3} \right) dx = -2.3504024$$

c. $g(t) = \int (g'(t)) dt + c$

$$= e^{(1-t)^3} + c$$

1A

$$g(1) = 2$$

$$e^{(1-1)^3} + c = 2, c = 1$$

$$g(t) = e^{(1-t)^3} + 1$$

1A

Calculator screen showing the indefinite integral and solving for c:

$$\int -3 \cdot (1-x)^2 \cdot e^{(1-x)^3} dx = e^{-x^3 + 3x^2 - 3x + 1}$$

solve $(e^{-x^3 + 3x^2 - 3x + 1} + c = 2, c) | x=1$ $c=1$

d. $g(0) = 3.718...$

372 rabbits

1A

Calculator screen showing the evaluation of the function at x=0:

$$e^{-x^3 + 3x^2 - 3x + 1} + 1 | x=0 = e + 1$$

$$e^{-x^3 + 3x^2 - 3x + 1} + 1 | x=0 = 3.7182818$$

e. Asymptote $y = 1$

100 rabbits

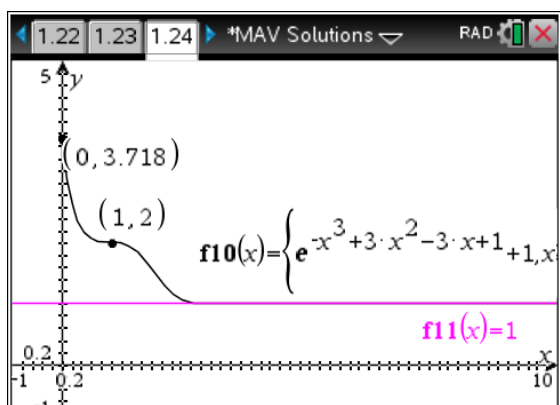
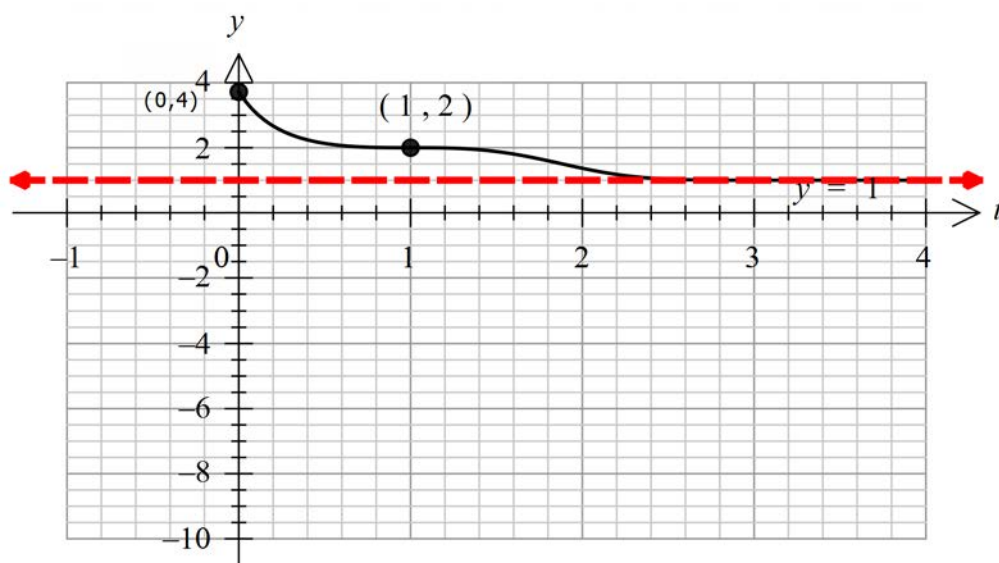
1A

f. Shape

1A

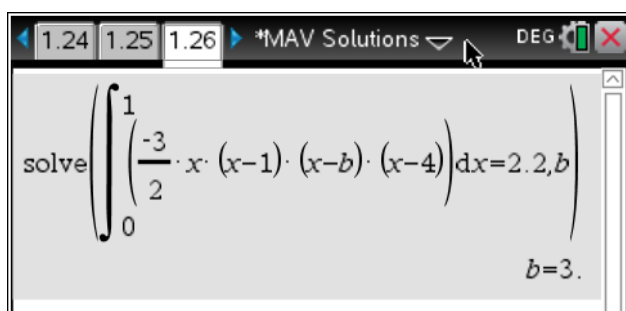
Points and asymptote

1A



g. Solve $\int_0^1 (v(t)) dt = 2.2$ for b **1M**

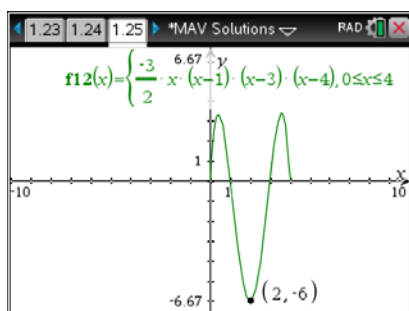
$b = 3$ **1A**



h. 6 ms^{-1} **1A**

$$\begin{aligned} \text{distance} &= 2.2 - \int_1^3 (v(t)) dt + \int_3^4 (v(t)) dt \\ &= 12 \text{ m} \end{aligned}$$

1A



Calculator screen showing the definite integral of $f12(x)$ from 1 to 3 plus the definite integral of $f12(x)$ from 3 to 4, resulting in 12.

$$2.2 - \int_1^3 f12(x) \, dx + \int_3^4 f12(x) \, dx = 12.$$

END OF SOLUTIONS