

The Mathematical Association of Victoria

Trial Examination 2017

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 10 pages,
- Formula Sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Question 1 (5 marks)

a. Find $\frac{dy}{dx}$ when $y = \tan(x^2 + 2)$.

2 marks

b. Find $f'(-2)$ when $f(x) = \frac{\log_e(x^2 - 1)}{x^2 - 1}$.

3 marks

Question 2 (4 marks)

The depth of water in a wave pool during a particular hour is given by $d(t) = -5\cos\left(\frac{\pi t}{8}\right) + 5$ metres, where t is the time in minutes and $0 \leq t \leq 60$.

a. Find the value of t for which the depth of water is first at its maximum.

1 mark

b. Find the fraction of time for the particular hour when the depth of water is more than 2.5 metres.

3 marks

TURN OVER

Question 3 (7 marks)

Consider the polynomial $P(x) = 5x^3 - x^2 + x + 7$.

- a. Find a linear factor of $P(x)$. 1 mark

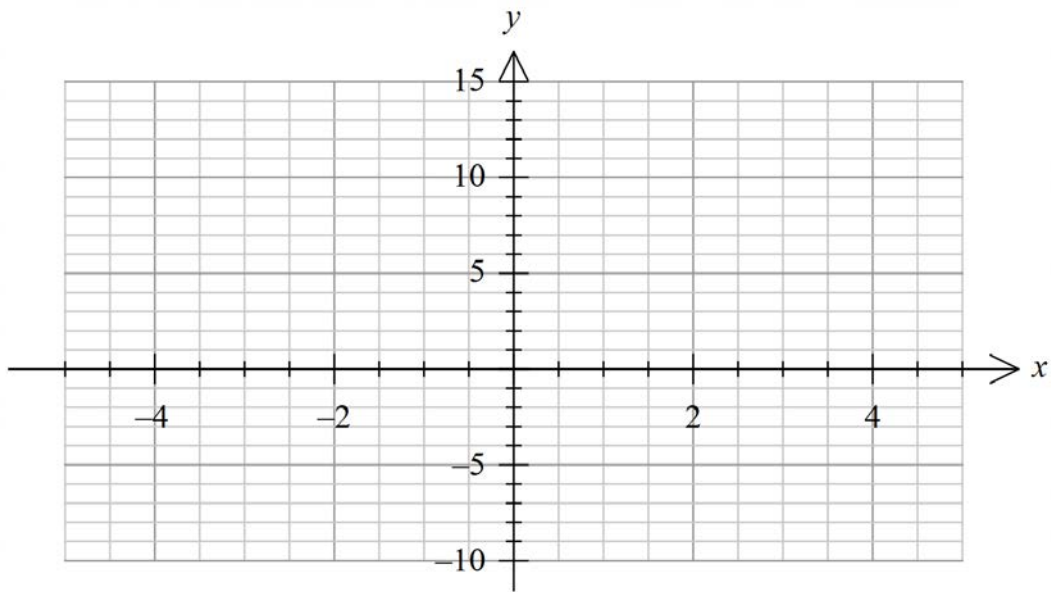
- b. Find $Q(x)$, the quadratic factor of $P(x)$. 1 mark

- c. Show that there are no linear factors for $Q(x)$. 1 mark

- d. The graph of $y = P(x)$ has no stationary points. Find the x coordinate of the point where the gradient of the graph of P is a minimum. 2 marks

- e. Hence sketch the graph of $y = P(x)$ labelling any axial intercepts with coordinates.

2 marks



TURN OVER

Question 4 (6 marks)

Let $f(x) = x \sin(x)$.

- a. Find the average rate of change of f over the interval $\left[\frac{\pi}{2}, \pi\right]$. 2 marks

- b. Find $f'(x)$. 1 mark

- c. Hence find $\int_0^{\frac{\pi}{2}} x \cos(x) dx$. 3 marks

Question 5 (3 marks)

Solve $\log_2((2x-2)^2) - 4\log_2(1-x) = 1$ for x .

Question 6 (3 marks)

The government is concerned about the number of teenagers in Australia who smoke on a regular basis. They randomly selected 10 000 teenagers and found 2000 of them smoke regularly. Find an approximate 95% confidence interval for the proportion of teenagers in Australia who smoke on a regular basis. Use an integer multiple of the standard deviation in your calculation.

TURN OVER

Question 7 (5 marks)

A random variable X has a probability density function given by

$$g(x) = \begin{cases} x & 0 \leq x \leq 1 \\ \frac{1}{x^2} & 1 < x \leq a, \text{ where } a \text{ is a real constant.} \\ 0 & \text{elsewhere} \end{cases}$$

a. Show that $a = 2$.

2 marks

b. Find $E(X)$.

3 marks

Question 8 (7 marks)

Consider the functions f and g with rules $f(x) = 3\sqrt{4-2x} + 1$ and $g(x) = \sqrt{x} + 4$.

- a. If the graph of f is mapped to the graph of g using the transformation

$$T: R^2 \rightarrow R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right), \text{ find } a, b, c \text{ and } d, \text{ where } a, b, c \text{ and } d \text{ are}$$

real constants.

3 marks

- b. Find the rule for f^{-1} and state the domain.

2 marks

TURN OVER

- c. Find the rule for $g(f(x))$ and state the range.

2 marks

END OF QUESTION AND ANSWER BOOK