

MATHEMATICAL METHODS UNIT 4

SAC 2 ANALYSIS TASK Tech Free

STUDENT NAME = _____

Time Allowed: 75 Minutes

Total Marks = 38

General instruction regarding the assessment of questions worth more than 1 mark:-

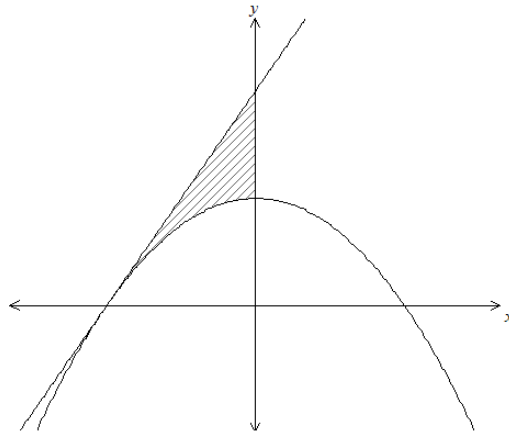
- Show all working clearly.
- Justify your answers when required.

All answers need to be given in exact form.

Technology Free with a Bound Book.

Question 1

The graph of $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 4 - x^2$ and the tangent to the graph of f , where it crosses the negative x -axis, are shown.



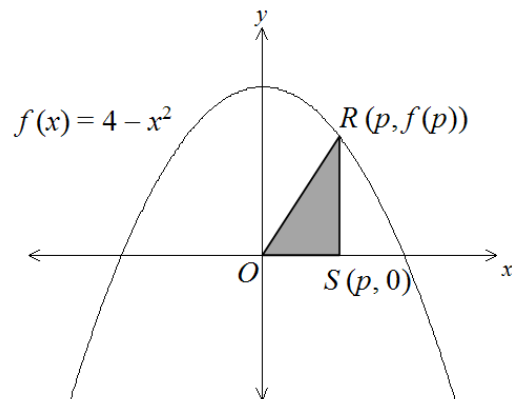
- a) Find the equation of the tangent to the graph of f where it crosses the negative x -axis.

2 marks

- b) Find the area of the shaded region.

3 marks

- c) Consider the triangle ORS , where vertex R is on the graph of f , with coordinates $(p, f(p))$, and vertices O and S are on the x -axis, with coordinates $(0, 0)$ and $(p, 0)$, respectively.

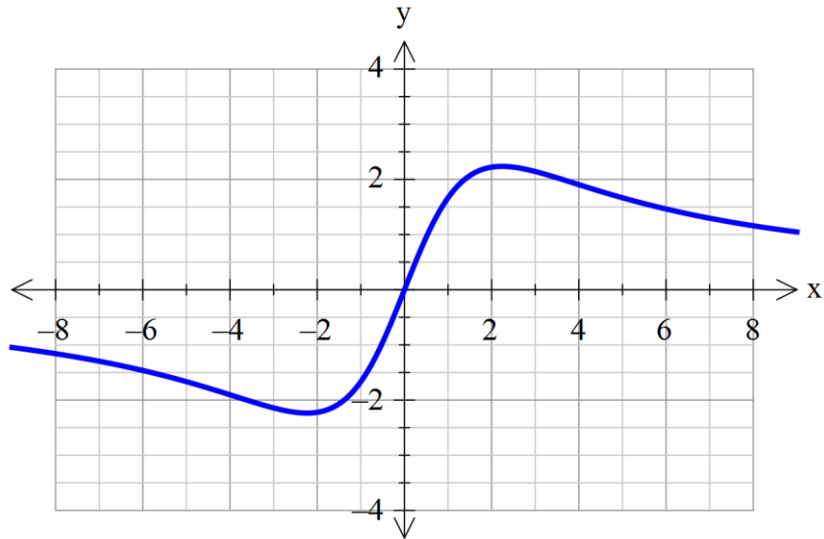


If $p \in [-2, 2]$, find the value(s) of p for which the area of triangle ORS is a maximum.

4 marks

Question 2

The graph of $h(x) = \frac{10x}{5+x^2}$ is shown.



a) Find $h'(x)$.

2 marks

b) Find the exact coordinates of the minimum and maximum turning points.

2 marks

- c) Find the derivative of $\log_e(5+x^2)$ and hence find an antiderivative for $\frac{10x}{5+x^2}$.

3 marks

- d) Find the area enclosed between the curve, the x -axis, the line where the x equals the x -coordinate of the maximum turning point, and the line $x = 6$.

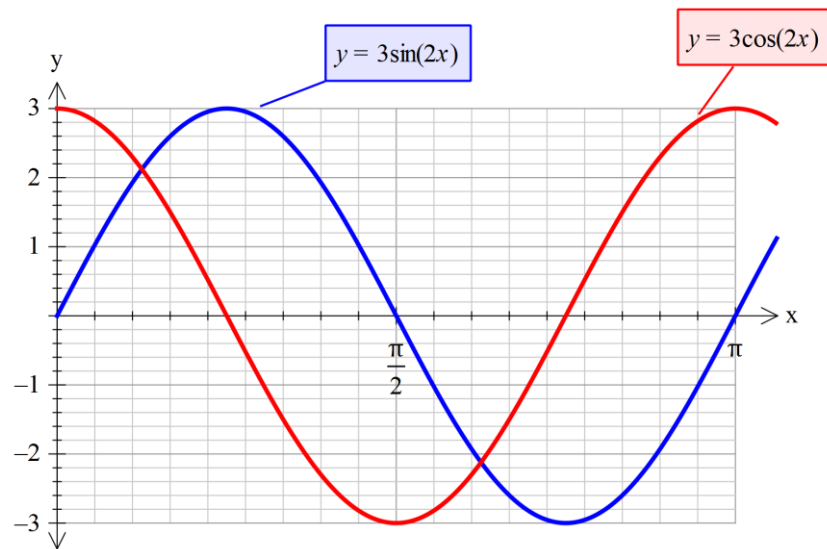
3 marks

e) Find the average value of the function $h(x)$ over the interval $[0,6]$.

2 marks

Question 3

The graphs of $f(x) = 3\sin(2x)$ and $g(x) = 3\cos(2x)$ are shown for $x \in [0, \pi]$.



- a) Find the coordinates of the points of intersection of f and g for the interval $\left[0, \frac{\pi}{2}\right]$.

3 marks

b) Determine the area enclosed between the two curves on the interval $\left[0, \frac{\pi}{2}\right]$.

4 marks

Question 4

Consider the function $f(x) = e^x$ and the point on the graph of $f(x)$ with the coordinates (a, e^a) , $a > 0$.

- a) Sketch the graph of f and mark the point (a, e^a) .

1 mark

- b) Draw the tangent to the graph at the point (a, e^a) .

1 mark

- c) Find the equation of the tangent above, in terms of a .

2 marks

- d) Write down the definite integral which determines the area of the region bound by the graph of f , the tangent at (a, e^a) and the coordinate axes.

2 marks

- e) Find the value of a if the area described above is equal to $e^a - 1$.

4 marks

END OF SAC

Additional Working Space.

Additional Working Space.