Year 2017

VCE

Mathematical Methods

Trial Examination 1

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Victorian Certificate of Education
2017

STUDENT NUMBER

Figures									Letter
Words									

MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes
Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.
In all questions where a numerical answer is required an exact value must be given unless otherwise specified.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $f(x)=e^{\tan(3x)}$, find $f'\left(\frac{\pi}{9}\right)$ 2 marks

b. Consider the function $g:(0,\pi)\to R$, $g(x)=\cos(2x)$. If the tangent to the graph of g at the point where $x=p$ makes an angle of 120^{0} with the positive x-axis, find the possible value(s) of p.

2 marks

Question 2 (3 marks)

Sam is a frequent flyer and occasionally gets a free upgrade. In a sample of ten flights, he found that once he got a free upgrade.
Let $\hat{P}$ represent the distribution of the sample proportion, on flights when he gets a free upgrade.

i. Sam wants to find the smallest value of the sample size n, such that the standard deviation of $\hat{P}$ is less than or equal to $\frac{1}{50}$. Determine the value of n.

2 marks

ii. Find the probability that when Sam takes five flights, he gets three upgrades.

1 mark

Question 3 (4 marks)

a. State the amplitude, period and range of the function $h: R \to R$, $h(x) = 3 - 6\cos\left(\frac{\pi x}{3}\right)$

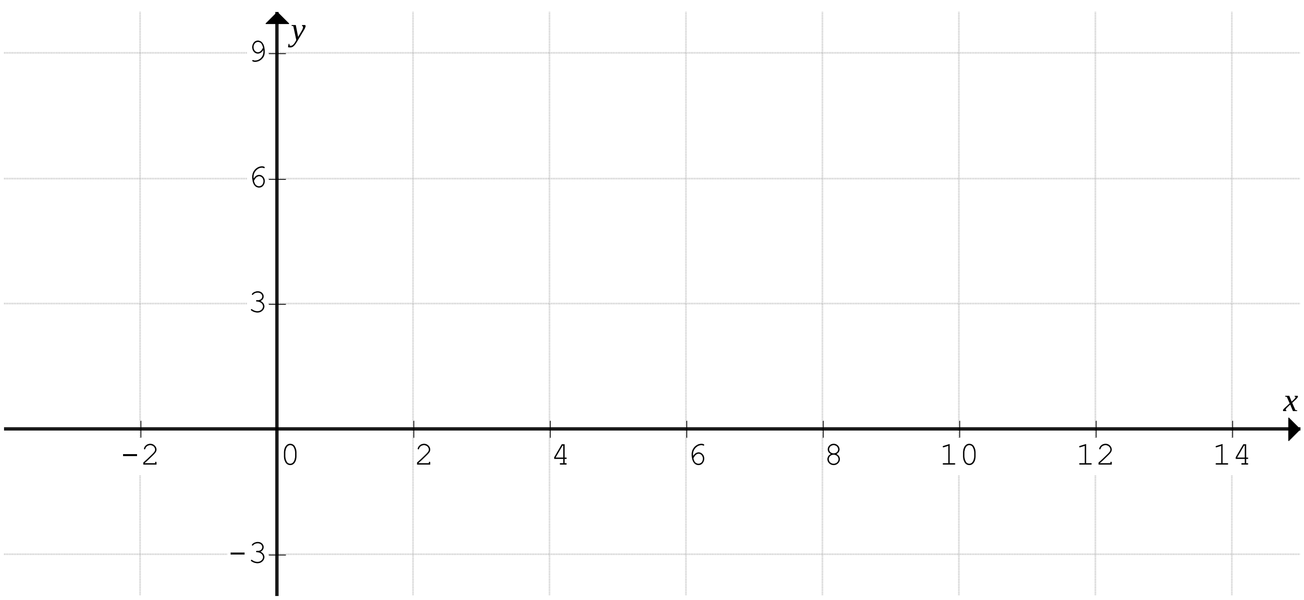
1 mark

b. Find the general solution of $3 - 6\cos\left(\frac{\pi x}{3}\right) = 0$

1 mark

c. Sketch the graph of the function $f:[0,12] \to R$, $f(x) = 3 - 6\cos\left(\frac{\pi x}{3}\right)$, on the axes below, stating coordinates of all axial intercepts and endpoints.

2 marks

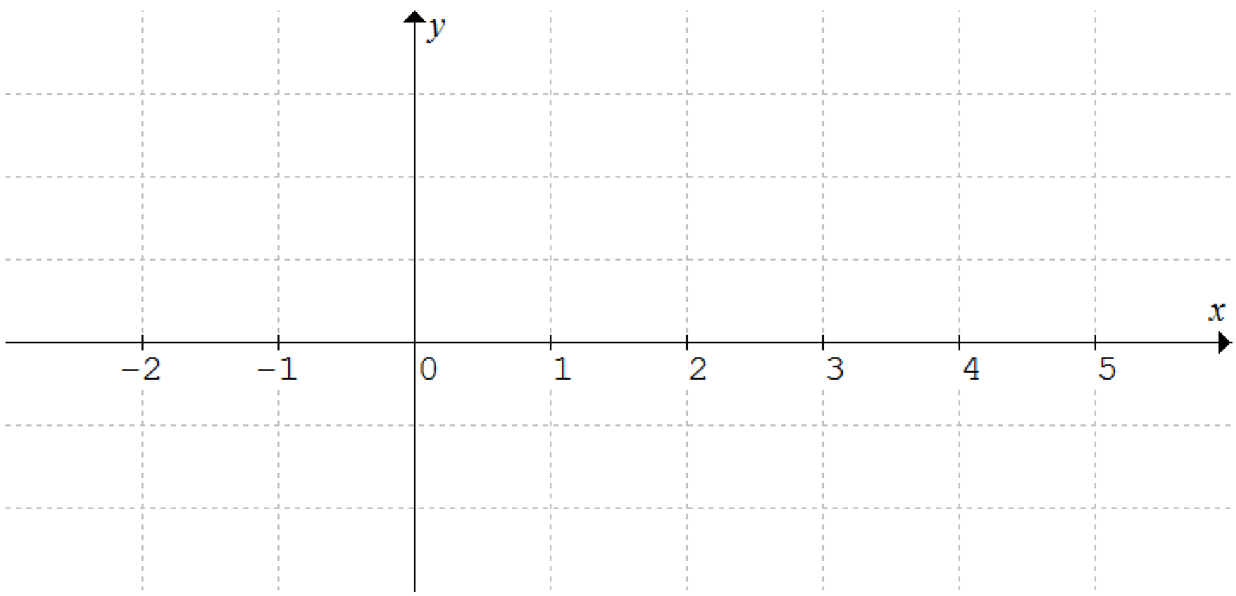


Question 4 (9 marks)

Consider the function $f : D \to R$, $f(x) = 3\log_e(x+1)$

a. Sketch the graph of the function on the axes below, clearly showing axial intercepts, and stating the equations of any asymptotes. State the maximal domain D and range of the function.

2 marks



Mia wants to calculate the area bounded by the graph of $y = 3\log_e(x+1)$, the *x*-axis and the ordinates $x = 0$ and $x = 4$.

b.i. As a first approximation, she decides to use four equally spaced upper rectangles. Draw these rectangles on the diagram above.

1 mark

ii. The area using these four upper rectangles can be expressed as $b\log_e(c)$, find the values of b and c.

1 mark

c. Find the rule for the inverse function f^{-1}.

2 marks

d.i. Evaluate $\int_{0}^{\log_e(125)} \left(e^{\frac{x}{3}} - 1\right) dx$

2 marks

ii. Hence find the area bounded by the graph of $y = 3\log_e(x+1)$, the x-axis and the ordinates $x = 0$ and $x = 4$.

1 mark

Question 5 (4 marks)

Consider the function $f:[0,\infty)\to R,\ f(x)=k\sqrt{3x+4}$

a. If the average rate of change of f between $x=0$ and $x=4$ is equal to 9, find the value of k. 2 marks

b. Find the average value of f between $x=0$ and $x=4$. 2 marks

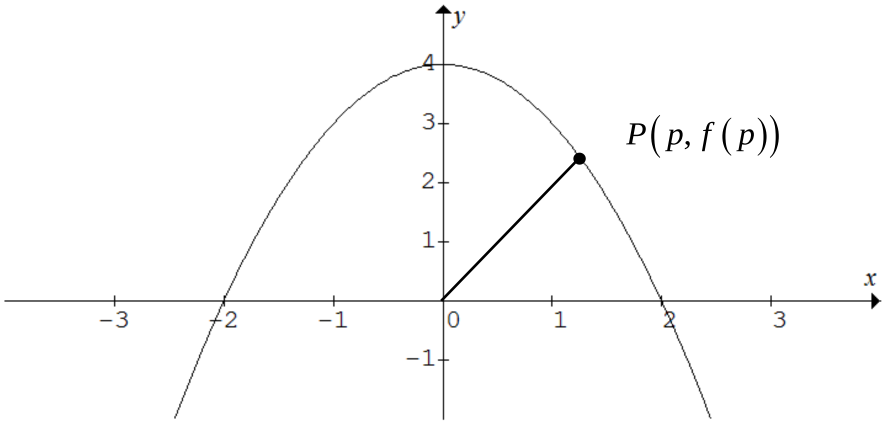
Question 6 (3 marks)

A transformation $T: R^2 \to R^2$ is defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$

The image of the curve $y = \sqrt{16 - x^2}$ under the transformation *T*, has the equation $y = 5 - 2\sqrt{55 - 6x - x^2}$. Find the values of a, b, h, and k.

Question 7 (4 marks)

The diagram shows part of the function $f : R \to R$, $f(x) = 4 - x^2$. A point P with coordinates $(p, f(p))$ where $0 \le p \le 2$ is shown.



Determine the value of p, for which the distance from the origin O to the point P is a minimum.

Question 8 (9 marks)

a. Determine $\frac{d}{dx}\left[x\cos(2x)\right]$

1 mark

Let X be a continuous random variable with probability density function

$$f(x)=\begin{cases} k\,x\sin(2x) & 0\le x\le\frac{\pi}{2} \\ 0 & \text{elsewhere}\end{cases}$$

b.i Show that $k=\frac{4}{\pi}$

2 marks

ii. The median of X is *m*, and satisfies the equation $\sin(2m) - 2m\cos(2m) = p$.
Find the value of *p*.

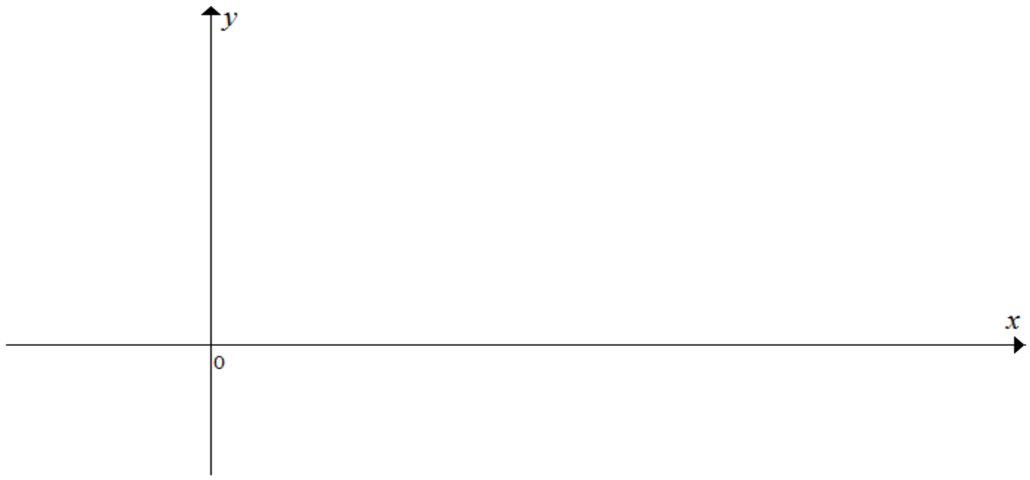
2 marks

iii. The mode of X is *M*, and satisfies the equation $\dfrac{\tan(2M)}{2M} = q$. Find the value of *q*.

2 marks

iv. Sketch the graph of f, on the axis below, clearly showing the relative positions of the median *m*, the mode *M*.

2 marks



END OF QUESTION AND ANSWER BOOKLET

END OF EXAMINATION

EXTRA WORKING PAGE

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc\sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$		$\int x^n dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq-1$	
$\frac{d}{dx}\left((ax+b)^n\right)=na(ax+b)^{n-1}$		$\int(ax+b)^n\,dx=\frac{1}{a(n+1)}(ax+b)^{n+1}+c\ ,\ n\neq-1$	
$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$		$\int e^{ax}dx=\frac{1}{a}e^{ax}+c$	
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$		$\int\frac{1}{x}dx=\log_e(x)+c,\ x>0$	
$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$		$\int\sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$	
$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$		$\int\cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$	
$\frac{d}{dx}\left(\tan(ax)\right)=\frac{a}{\cos^2(ax)}=a\sec^2(ax)$			
product rule	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A \mid B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\operatorname{var}(X) = \sigma^2 = E\left((X-\mu)^2\right) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)\,dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2\, f(x)\,dx$

Sample proportions

$\hat{P} = \dfrac{X}{n}$		mean	$E(\hat{P}) = p$
standard deviation	$\operatorname{sd}(\hat{P}) = \sqrt{\dfrac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\ ,\ \hat{p} + z\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}\right)$

END OF FORMULA SHEET