YEAR 12 *Trial Exam Paper*

2017

MATHEMATICAL METHODS

Written examination 2

Worked solutions

This book presents:

- worked solutions, giving you a series of points to show you how to work through the questions
- mark allocations
- tips on how to approach the exam

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SECTION A – Multiple-choice questions

Question 1

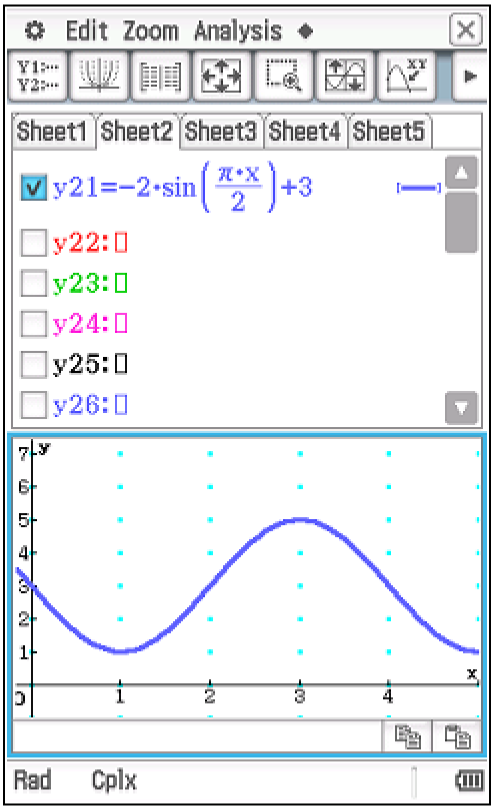
Answer: C

Explanatory notes

The period is given by $\dfrac{2\pi}{n} = \dfrac{2\pi}{\frac{\pi}{2}} = 4$.

The median is 3 and the amplitude is 2. Hence, the range is 3 – 2 to 3 + 2; that is, $[1,5]$.

This can be readily checked by sketching the graph using CAS.



Question 2

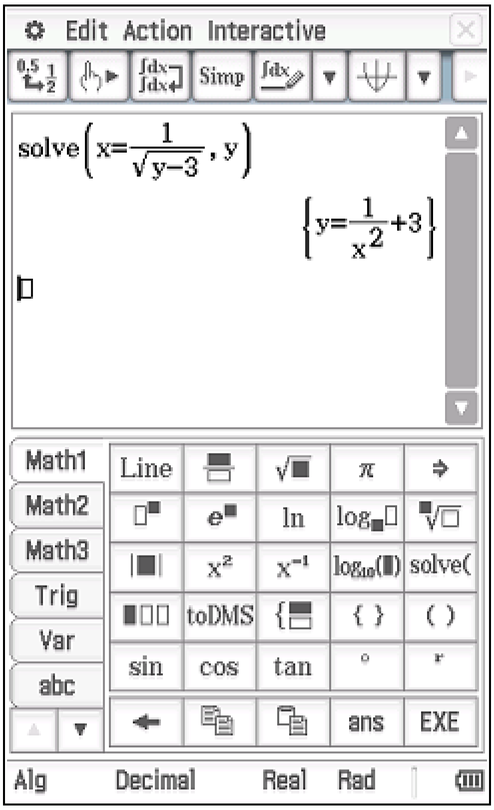
Answer: B

Explanatory notes

Using CAS to obtain the inverse function gives the inverse function as $y = \frac{1}{x^2} + 3$.

And since the range of the original becomes the domain of the inverse, the domain is R^+.

So the answer is option B.



Tip

- *You could also use the draw inverse function on the calculator to sketch the inverse.*

Question 3

Answer: C

Explanatory notes

The roots (x-intercepts) are at $x = b$, $x = c$ and $x = d$, with a repeated root at $x = d$.
So, the factors are $(x-b)$, $(x-c)$ and $(x-d)^2$.

Tip

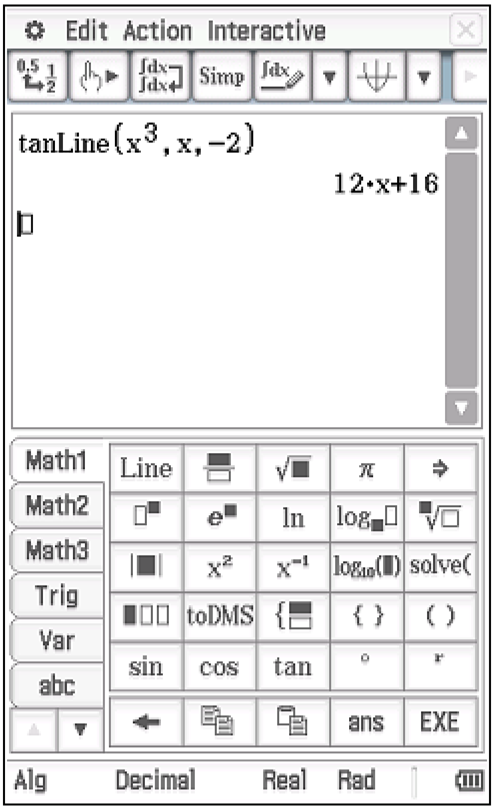
- *Don't be confused by the fact that the roots at b and c are negative.*

Question 4

Answer: B

Explanatory notes

Using CAS to find the equation of the tangent line to the curve at $x = -2$ gives $y = 12x + 16$, whereby $(-1, 4)$ satisfies this equation.



Question 5

Answer: B

Explanatory notes

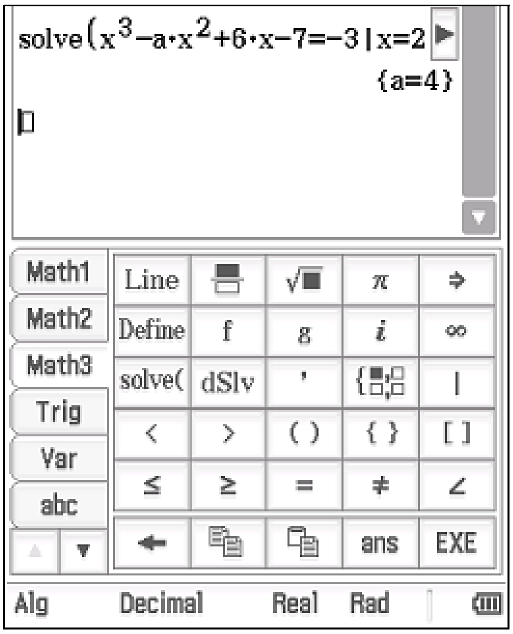
Reflect the graph in the line $y = x$ to get the graph of the inverse and then reflect in the x-axis to get the graph of $y = -f^{-1}(x)$.

Question 6

Answer: A

Explanatory notes

Using CAS gives

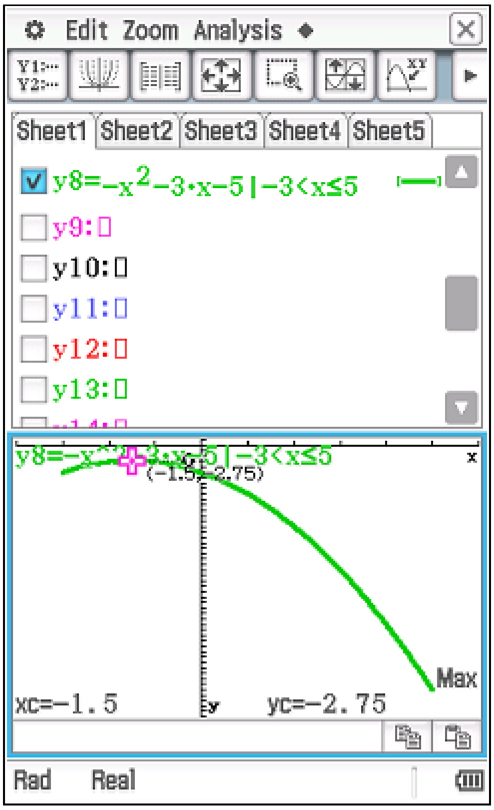


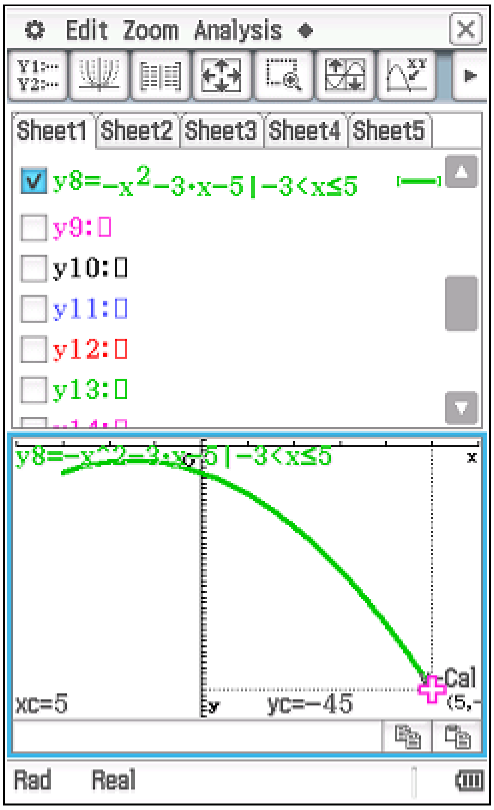
Question 7

Answer: D

Explanatory notes

Sketching the graph using CAS shows that the range is from the right endpoint to the maximum turning point.





Question 8

Answer: D

Explanatory notes

For the average value of the function to be zero, the area bounded by the curve and the *x*-axis over the interval $[-p, 0]$ has to equal $\frac{81}{8}$.

Therefore

$$\frac{1}{2}p^2 = \frac{81}{8}$$

$$p^2 = \frac{81}{4}$$

$$p = \frac{9}{2}$$

Question 9

Answer: D

Explanatory notes

To be a probability density function, the value of *a* is 14 as the area under the curve must be 1.

Because the function is symmetrical, $E(X)$ occurs in the centre, so $E(X) = 11$.

Tip

- *Remember to look for symmetry.*

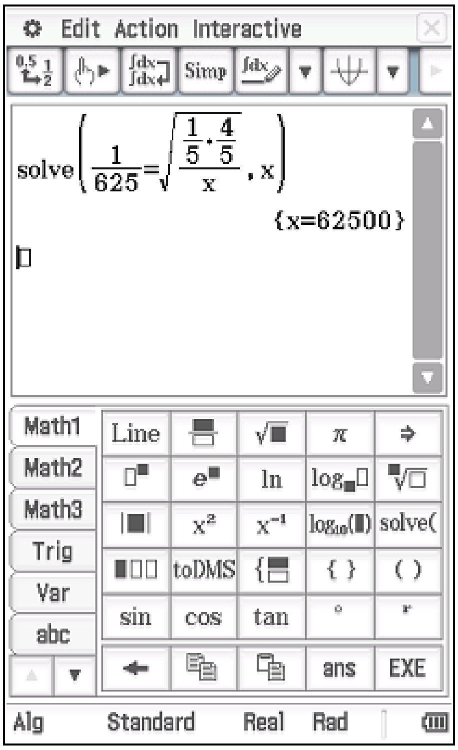
Question 10

Answer: E

Explanatory notes

$$\sigma = \sqrt{\frac{p(1-p)}{n}}$$

$$\frac{1}{625} = \sqrt{\frac{\frac{1}{5} \times \frac{4}{5}}{n}}$$



Question 11

Answer: D

Explanatory notes

The 95% confidence interval for p is

$$\left(\hat{p}-1.96\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+1.96\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

Tip

- *The confidence intervals for 90% and 95% are worth remembering:*

90% $\left(\hat{p}-1.65\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+1.65\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

95% $\left(\hat{p}-1.96\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p}+1.96\times\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

Question 12

Answer: D

Explanatory notes

$np=3$ and $npq=\frac{9}{4}$.

$\Rightarrow 3q=\frac{9}{4}$

So $q=\frac{3}{4}$

$\therefore\ p=\frac{1}{4}$

Hence, $\Pr(X=1)=\binom{12}{1}(p)^1(q)^{11}=12\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^{11}$.

Question 13

Answer: A

Explanatory notes

$$\begin{aligned}\Pr\left(X < 46\right) &= \Pr\left(Z < \frac{46-60}{7}\right)\\ &= \Pr\left(Z < -2\right)\\ &= \Pr\left(Z > 2\right), \text{ using symmetry}\end{aligned}$$

Question 14

Answer: D

Explanatory notes

$y = e^{2x+4} - 3$ can be written as $y + 3 = e^{2x+4}$ and therefore $y' = y + 3$ and $x' = 2x + 4$.

So $y = y' - 3$ and $x = \frac{1}{2}(x' - 4) = \frac{1}{2}x' - 2$. The matrix equation that corresponds to these equations is option A.

Question 15

Answer: D

Explanatory notes

Using the chain rule

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{\sqrt{f(2x)}} \times \frac{1}{2\sqrt{f(2x)}} \times f'(2x) \times 2\\ &= \frac{f'(2x)}{f(2x)}\end{aligned}$$

Or, using logarithmic laws, the equation for y can be written as $y = \frac{1}{2}\log_e(f(2x))$.

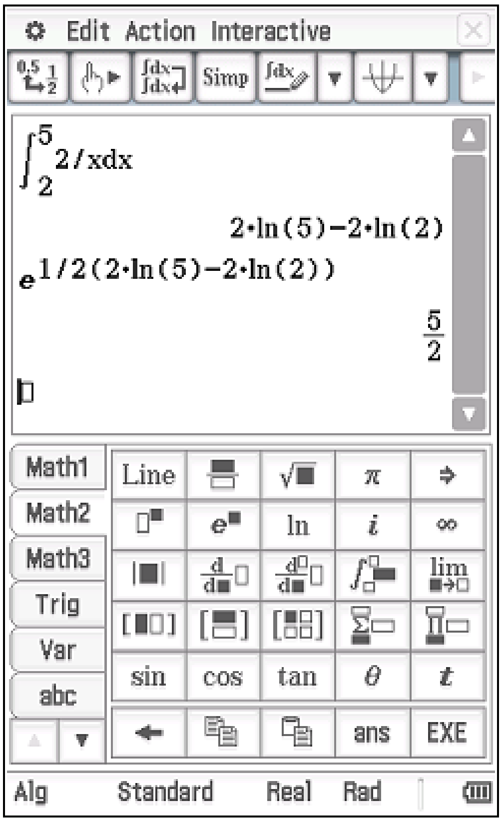
So $$\begin{aligned}\frac{dy}{dx} &= \frac{1}{2} \times \frac{1}{f(2x)} \times f'(2x) \times 2\\ &= \frac{f'(2x)}{f(2x)}\end{aligned}$$

Question 16

Answer: E

Explanatory notes

Using CAS gives



Question 17

Answer: B

Explanatory notes

$$\begin{aligned}\int_{-1}^{2}(3-f(x))\,dx &= \int_{-1}^{2}3\,dx-\int_{-1}^{2}f(x)\,dx\\ &=[3x]_{-1}^{2}-4\\ &=6+3-4\\ &=5\end{aligned}$$

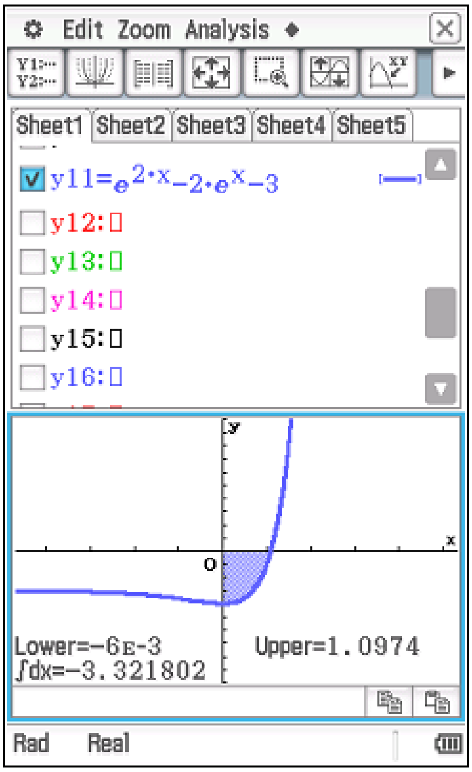
Question 18

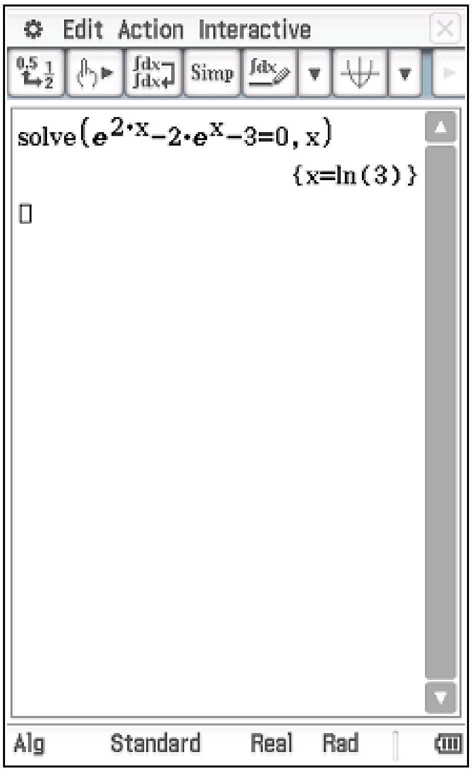
Answer: C

Explanatory notes

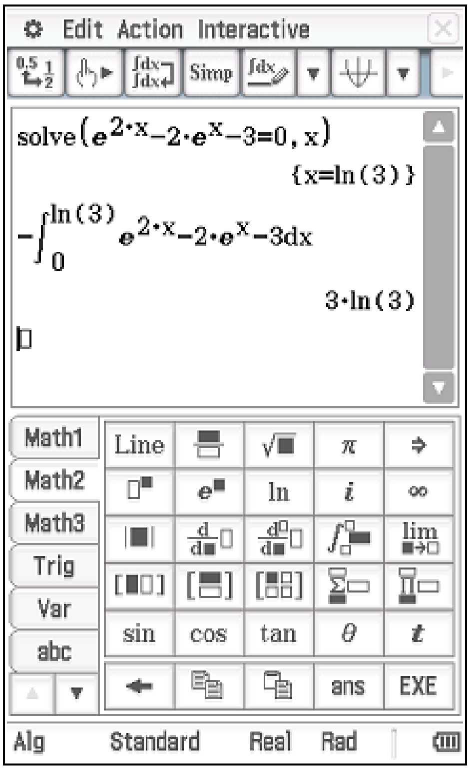
Using CAS, the area required can be seen as the shaded region (shown left).

Use the main menu to then find the exact value of the *x*-intercept (shown right).





Hence, the area is



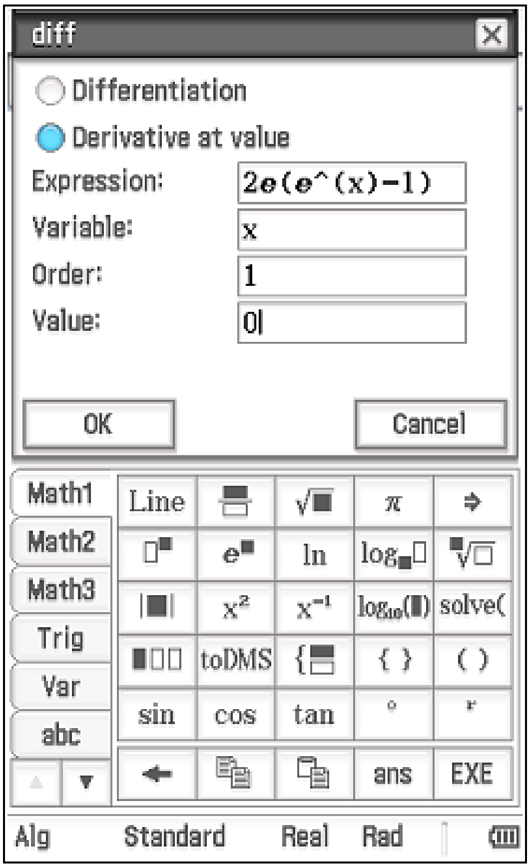
Question 19

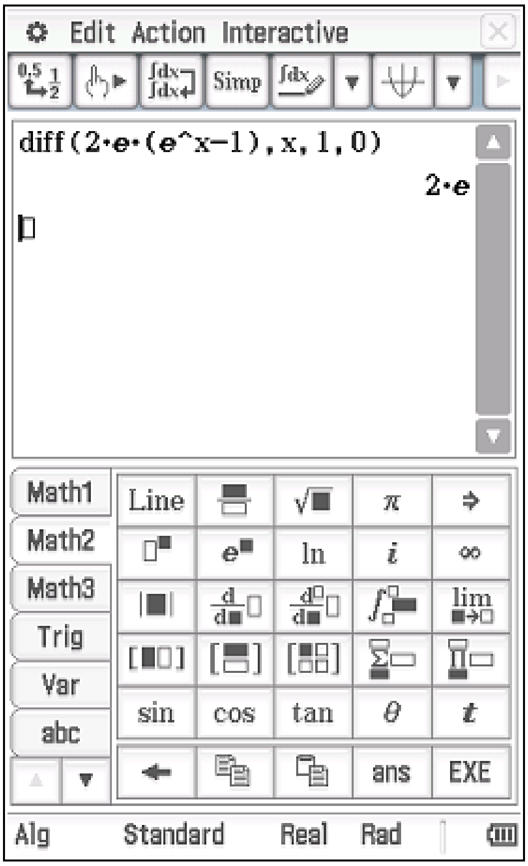
Answer: D

Explanatory notes

The rate of change means $\dfrac{dy}{dx}$.

Using CAS gives





Question 20

Answer: C

Explanatory notes

The graph drawn is the graph of the derivative; that is, the graph of the gradient function. The graph has a positive value over this domain and it does not have an x-intercept or change signs (therefore there are no turning points and stationary points of inflexion over this interval).

SECTION B

Question 1a.

Worked solution

Maximum height is 4 metres.

Mark allocation: 1 mark

- 1 answer mark

Question 1b.

Worked solution

The gradient function is given by $\frac{dy}{dx} = 2\sin\left(\frac{\pi x}{3}\right) \times \frac{\pi}{3} = \frac{2\pi}{3}\sin\left(\frac{\pi x}{3}\right)$.

The range of this function is $\left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$, so the gradient is always less than or equal to $\frac{2\pi}{3}$.

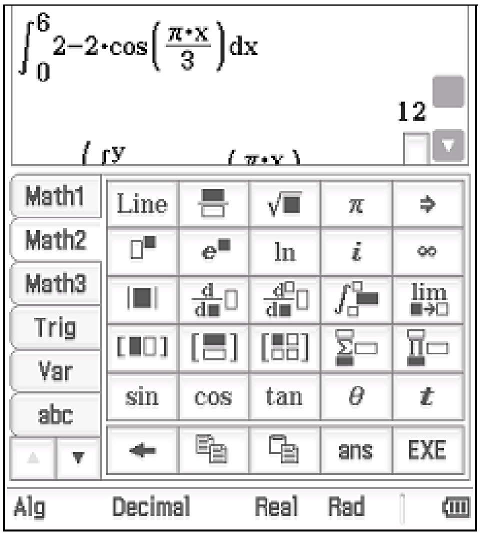
Mark allocation: 1 mark

- 1 mark for finding the range of the derivative

Question 1c.

Worked solution

Using CAS gives



So the area is 12 square metres.

Mark allocation: 1 mark

- 1 mark for correct answer

Tip

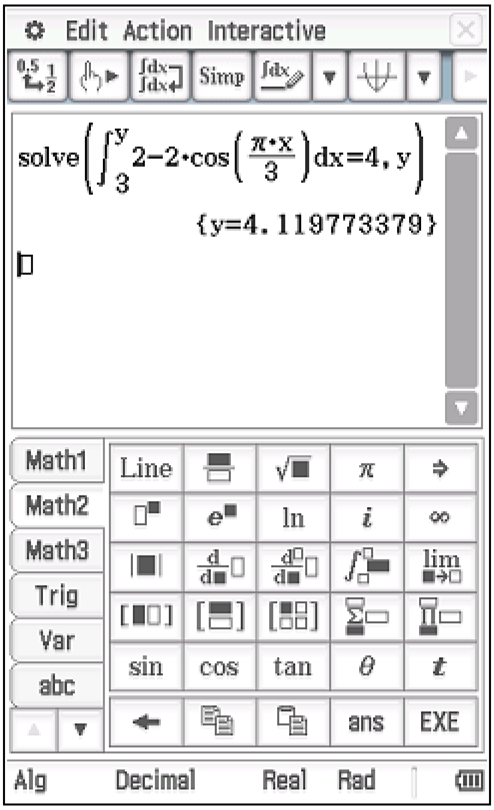
- *Be guided by the number of marks allocated for the question. In this case there is 1 mark, so there is no need to show any working; just use CAS to find the answer.*

Question 1d.

Worked solution

Two-thirds of the area is 8 and we want half of that; that is, 4 square units, to be between 3 and $3+c$.

So set up the integral and use CAS to find *c*.



Hence, $c=1.12$.

Mark allocation: 2 marks

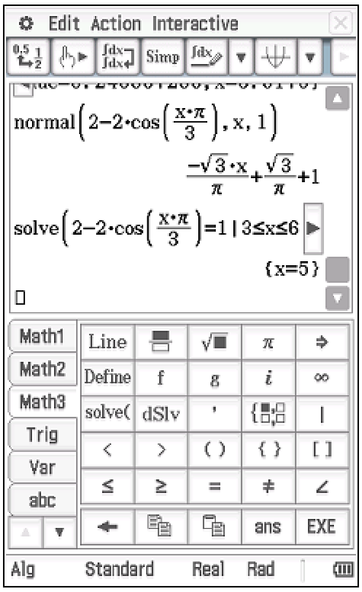
- 1 method mark for setting an integral equal to either 8 or 4
- 1 answer mark for the correct value of *c*

Question 1e.

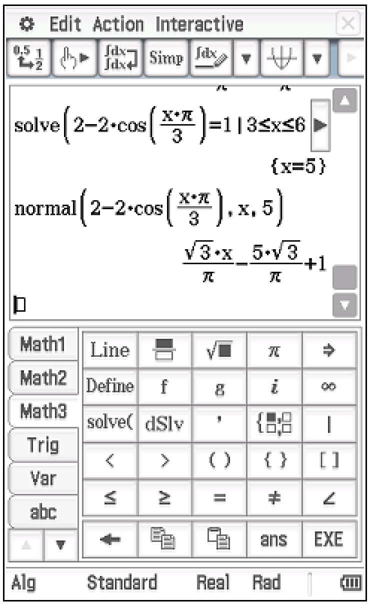
Worked solution

AB is the line drawn normal to the curve at a point when $y=1$.

First, solve $2-2\cos\left(\dfrac{\pi x}{3}\right)=1$ for $x\in[3,6]$ for *x*, to get $x=5$.



Then, use CAS to find the equation of the normal line drawn at $x=5$.



The equation of the normal is $y=\dfrac{\sqrt{3}x}{\pi}-\dfrac{5\sqrt{3}}{\pi}+1$.

Mark allocation: 2 marks

- 1 method mark for setting $2-2\cos\left(\dfrac{\pi x}{3}\right)=1$ for $x\in[3,6]$ to get $x=5$
- 1 answer mark for the equation of the normal

Question 1f.

Worked solution

$$\frac{dy}{dx} = \frac{2\pi}{3}\sin\left(\frac{\pi x}{3}\right)$$

At $x = a$, $\quad \dfrac{dy}{dx} = \dfrac{2\pi}{3}\sin\left(\dfrac{\pi a}{3}\right)$

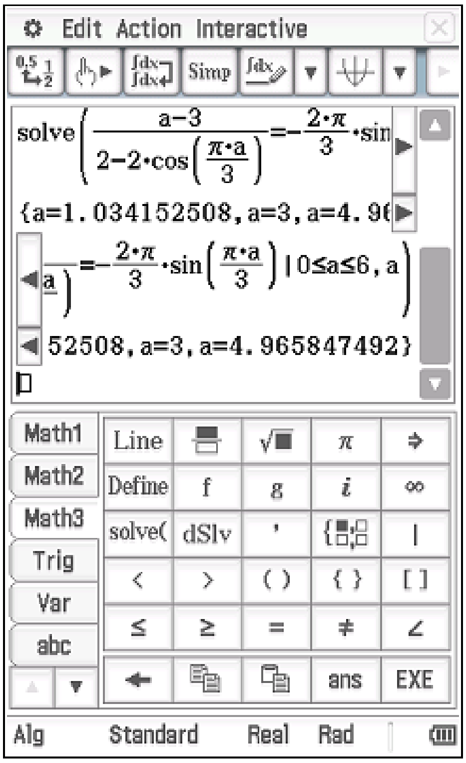
$$\Rightarrow \frac{dy}{dx}(\text{normal}) = \frac{-1}{\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)}$$

Gradient of normal line passing through $(3,0)$ and $\left(a, 2-2\cos\left(\dfrac{\pi a}{3}\right)\right)$ is $\dfrac{2-2\cos\left(\frac{\pi a}{3}\right)}{a-3}$.

So $\quad \dfrac{2-2\cos\left(\frac{\pi a}{3}\right)}{a-3} = \dfrac{-1}{\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)}$

$$\frac{a-3}{2-2\cos\left(\frac{\pi a}{3}\right)} = -\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)$$

Solving this using CAS gives $a = 4.966$.



Mark allocation: 3 marks

- 1 method mark for getting the gradient of the normal line $\frac{dy}{dx}(\text{normal}) = \dfrac{-1}{\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)}$
- 1 method mark for setting it equal to $\dfrac{-1}{f'(a)}$
- 1 answer mark for finding $a = 4.966$

Question 1g.

Worked solution

Let the longest strut be located at $x = a,\ \ y = 2 - 2\cos\left(\frac{\pi a}{3}\right)$.

$$\frac{dy}{dx}(x = a) = \frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)$$

$$\therefore m_{\text{normal}} = \frac{-1}{\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)}$$

The equation of the normal is $y - \left(2 - 2\cos\left(\frac{\pi a}{3}\right)\right) = \frac{-1}{\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)}(x - a)$.

To find the x-intercept, let $y = 0$.

$$\Rightarrow -\left(2 - 2\cos\left(\frac{\pi a}{3}\right)\right) = \frac{-1}{\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)}(x - a)$$

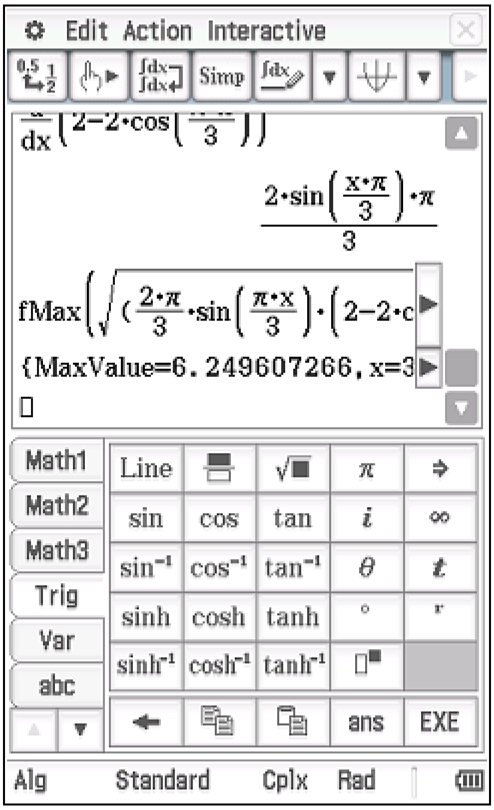
$$\Rightarrow x_{\text{int}} = a + \frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)\left(2 - 2\cos\left(\frac{\pi a}{3}\right)\right)$$

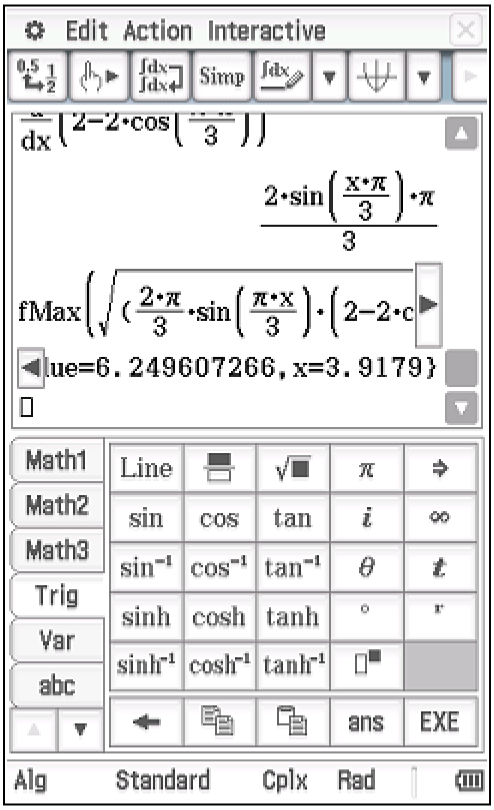
Strut extends from $\left(x_{\text{int}}, 0\right)$ to $\left(a, 2 - 2\cos\left(\frac{\pi a}{3}\right)\right)$.

Length of strut is $d = \sqrt{\left(x_{\text{int}} - a\right)^2 + \left(2 - 2\cos\left(\frac{\pi a}{3}\right)\right)^2}$.

$$d = \sqrt{\left[\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)\left(2 - 2\cos\left(\frac{\pi a}{3}\right)\right)\right]^2 + \left(2 - 2\cos\left(\frac{\pi a}{3}\right)\right)^2}$$

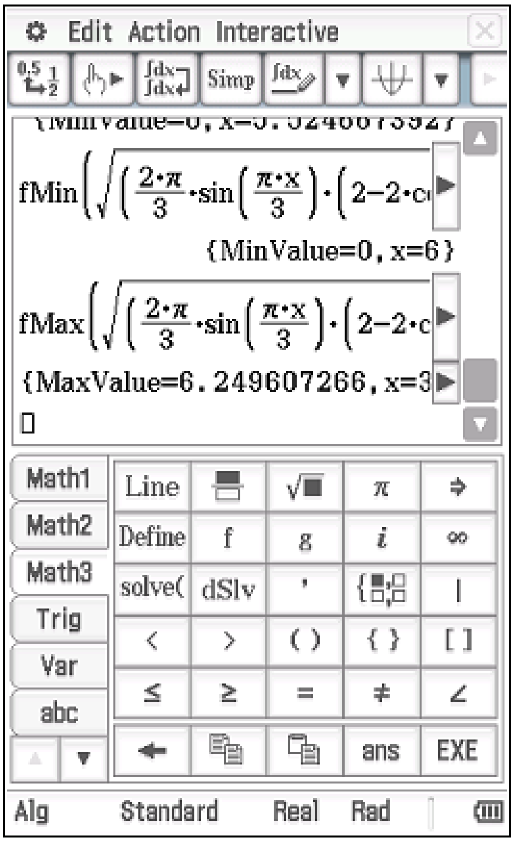
Using CAS to find the maximum value of this function gives

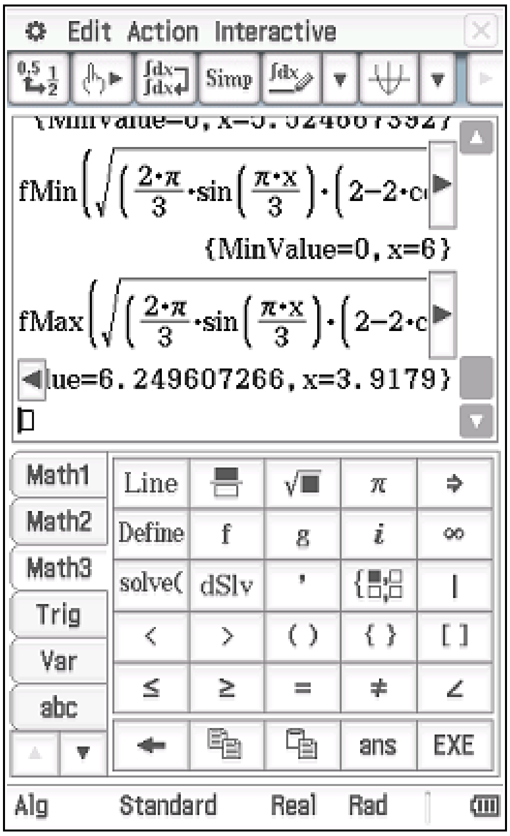


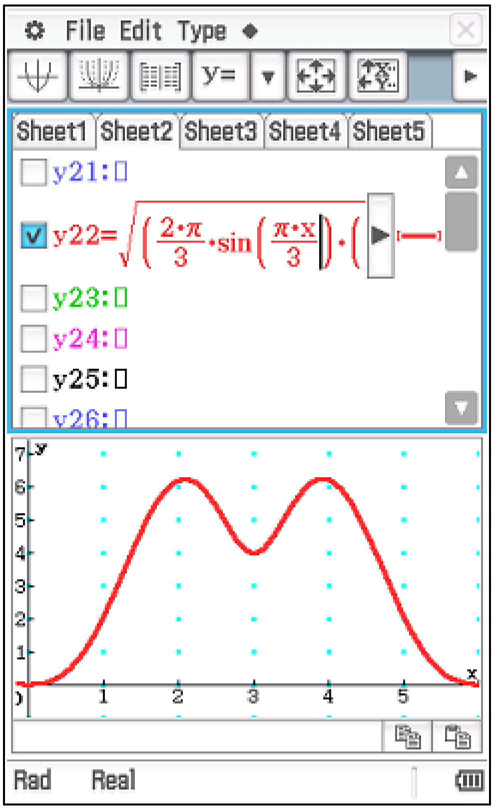


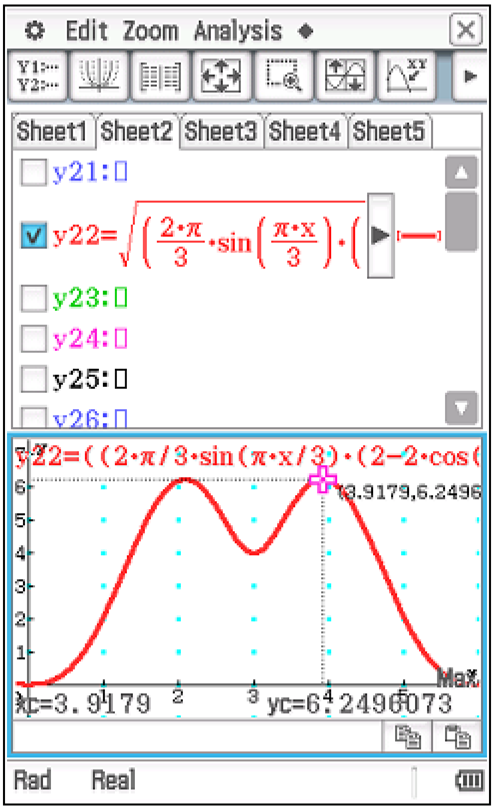
$d_{\max} = 6.2496$ at $a = 3.9179$, so $x = 3.9179$.

So the longest strut is 6.2496 metres in length and is positioned at $(3.9179, 3.1450)$.









Mark allocation: 3 marks

- 1 method mark for finding the equation of the normal line at $x = a$,

  $$y - \left(2 - 2\cos\left(\frac{\pi a}{3}\right)\right) = \frac{-1}{\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)}(x - a)$$

- 1 method mark for finding the function that gives the length of any strut drawn normal to the curve at $x = a$, $d = \sqrt{\left[\frac{2\pi}{3}\sin\left(\frac{\pi a}{3}\right)\left(2 - 2\cos\left(\frac{\pi a}{3}\right)\right)\right]^2 + \left(2 - 2\cos\left(\frac{\pi a}{3}\right)\right)^2}$

- 1 answer mark for maximum length of 6.2496 metres at $(3.9179,\ 3.1450)$

Question 2a.i.

Worked solution

$$E(X) = 120$$

$$E(X) = \int_0^a x.\frac{2x}{a^2}\,dx$$

$$= \int_0^a \frac{2x^2}{a^2}\,dx$$

$$= \frac{1}{a^2}\left[\frac{2x^3}{3}\right]_0^a$$

$$= \frac{1}{a^2}.\frac{2a^3}{3} = \frac{2a}{3}$$

$$\Rightarrow \frac{2a}{3} = 120,\ \ a = 180$$

Mark allocation: 2 marks

- 1 method mark for setting up the integral for $E(X)$
- 1 answer mark for the correct antiderivative and evaluation leading to $a = 180$

Tip

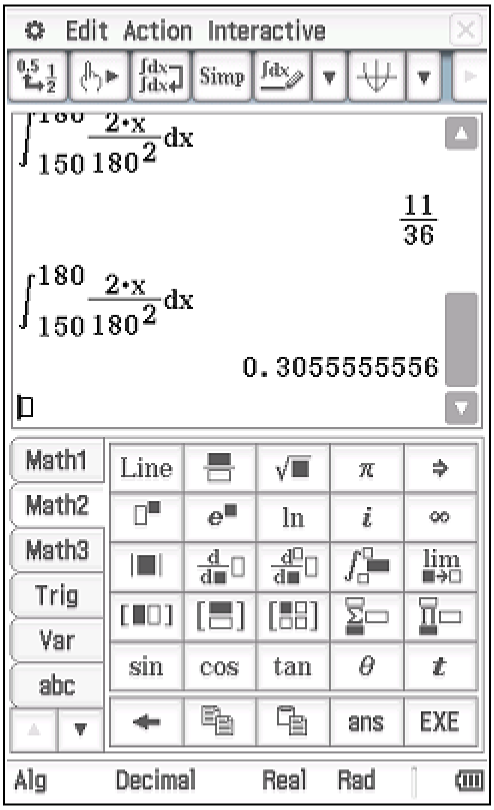
- *This is a ‘show that’ question, so appropriate working must be shown.*

Question 2a.ii.

Worked solution

$$\Pr(X > 150) = \int_{150}^{180} \frac{2x}{180^2}\, dx$$

Using CAS gives $\Pr(X > 150) = \dfrac{11}{36}$.



Mark allocation: 2 marks

- 1 method mark for setting up the integral
- 1 answer mark for $\dfrac{11}{36}$

Tip

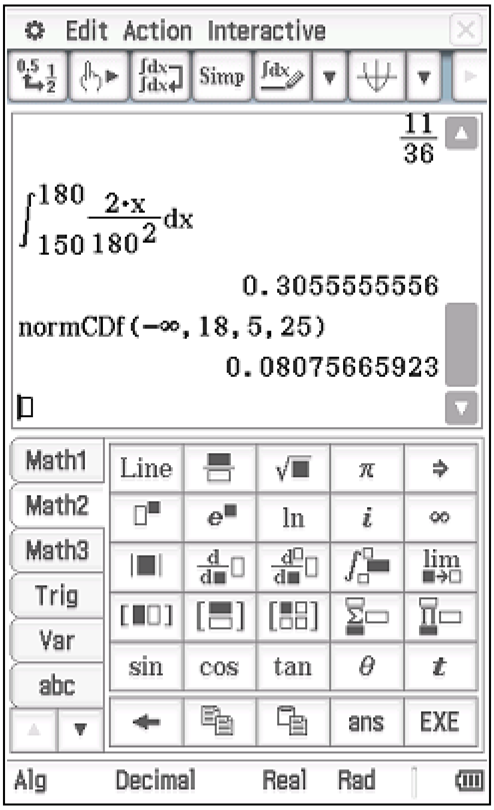
- *This question is worth 2 marks, so you must also provide a solution step.*

Question 2b.

Worked solution

$X_{\text{RF}} \sim N(\mu = 25,\ \sigma = 5)$

Using CAS gives $\Pr(X_{\text{RF}} < 18) = 0.0808.$



Mark allocation: 2 marks

- 1 method mark for $X_{\text{RF}} \sim N(\mu = 25,\ \sigma = 5)$
- 1 answer mark for 0.0808

Tip

- *Be careful to define the distribution using a new variable. Something like X_{RF} is suitable.*

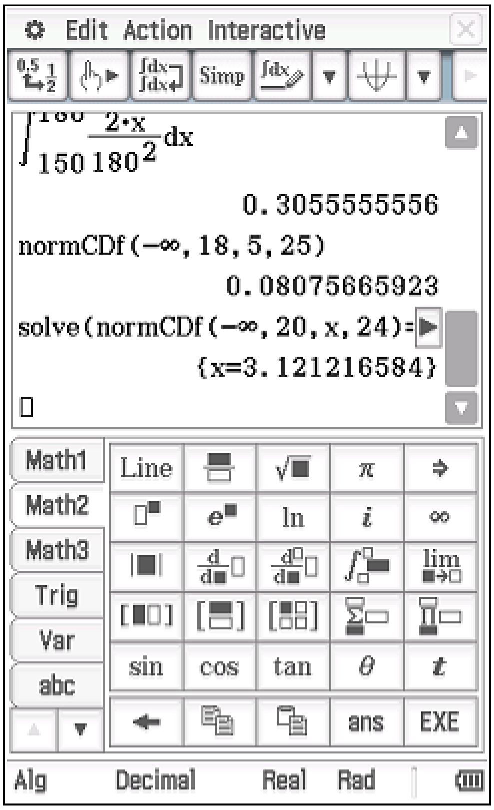
Question 2c.

Worked solution

Using symmetry, the mean is 24 mm.

$X_{JJ} \sim N(\mu = 24, \sigma = ?)$, where $\Pr(X_{JJ} < 20) = 0.1$.

Using CAS gives $\sigma = 3.12$.



Mark allocation: 3 marks

- 1 answer mark for finding the mean
- 1 method mark for setting up an integral to find the standard deviation
- 1 answer mark for the correct standard deviation

Tip

- *Be careful to use a different notation for defining the distribution.*

Question 2d.i.

Worked solution

$$\Pr(\text{JJ} \mid \text{length} < 18) = \frac{\Pr(X_{\text{JJ}} < 18)}{\Pr(\text{JJ}) \times \Pr(X_{\text{JJ}} < 18) + \Pr(\text{RF}) \times \Pr(X_{\text{RF}} < 18)}$$
$$= \frac{0.7 \times 0.027235}{0.7 \times 0.027235 + 0.3 \times 0.0808}$$
$$= 0.440$$

Mark allocation: 3 marks

- 1 method mark for recognising conditional probability or for using a tree diagram and for finding $\Pr(X_{\text{JJ}} < 18) = 0.7 \times 0.027235$
- 1 method mark for having a denominator that involves 0.7 multiplied by one probability and 0.3 multiplied by another
- 1 answer mark for 0.440

Question 2d.ii.

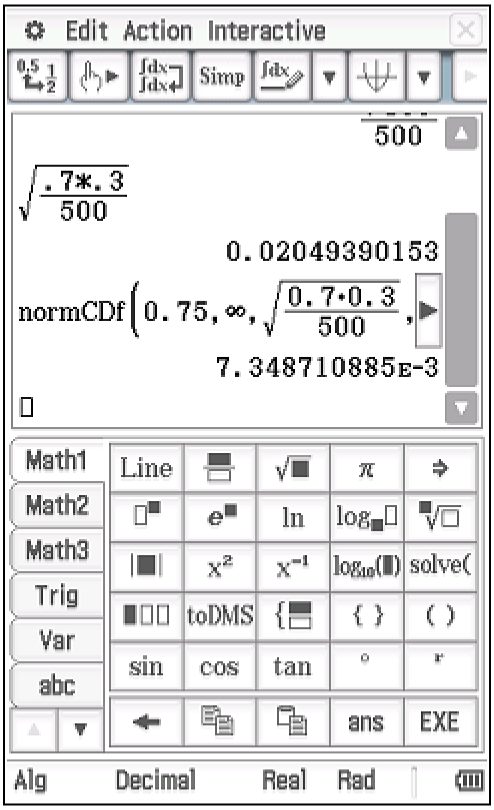
Worked solution

$p = 0.7,\ \ n = 500$

$E(\hat{p}) = p = 0.7,\ \ \sigma = \sqrt{\dfrac{p(1-p)}{n}} = \sqrt{\dfrac{0.7 \times 0.3}{500}}$

$\hat{P} \sim N(\mu = 0.7,\ \sigma = 0.0205)$

$\Pr(\hat{P} > 0.75) = 0.0073$



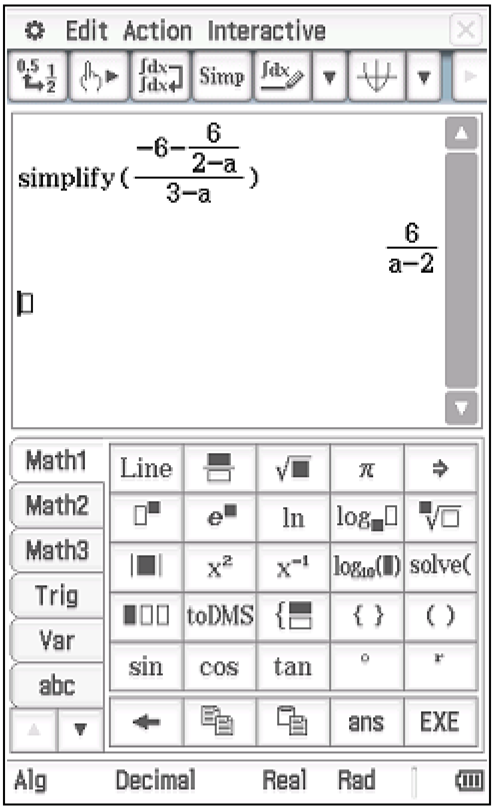
Mark allocation: 2 marks

- 1 mark for identifying parameters $\mu = 0.7,\ \sigma = 0.0205$
- 1 mark for answer $\Pr(\hat{P} > 0.75) = 0.0073$ (although if the rounded value for σ is used this will give $\Pr(\hat{P} > 0.75) = 0.0074,$ so accept either 0.0073 or 0.0074)

Question 3a.i.

Worked solution

Using CAS, the gradient of PA is $\dfrac{6}{a-2}$.



Mark allocation: 1 mark

- 1 mark for answer in simplified form

Question 3a.ii.

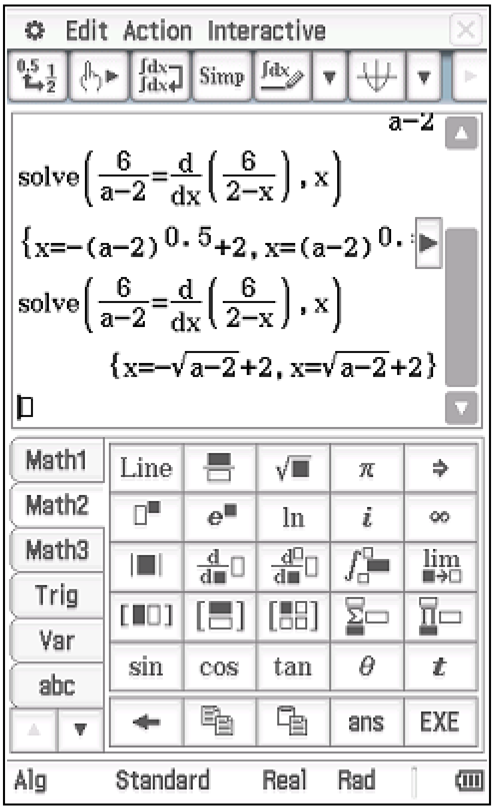
Worked solution

We need to solve $f'(x) = \dfrac{6}{a-2}$ for x.

Using CAS, this gives

$x = \pm\sqrt{a-2} + 2$

Since $x > 2$, $x = \sqrt{a-2} + 2$.



Mark allocation: 2 marks

- 1 method mark for solving $f'(x) = \dfrac{6}{a-2}$ for x
- 1 answer mark for correct answer (the negative answer must be discarded)

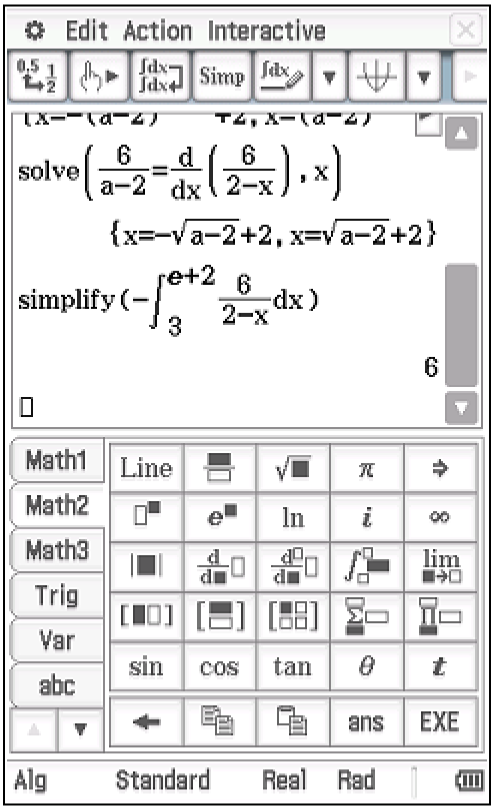
Tip

- *Always consider the domain when validating solutions.*

Question 3b.i.

Worked solution

Set up and evaluate using CAS to give an answer of 6.



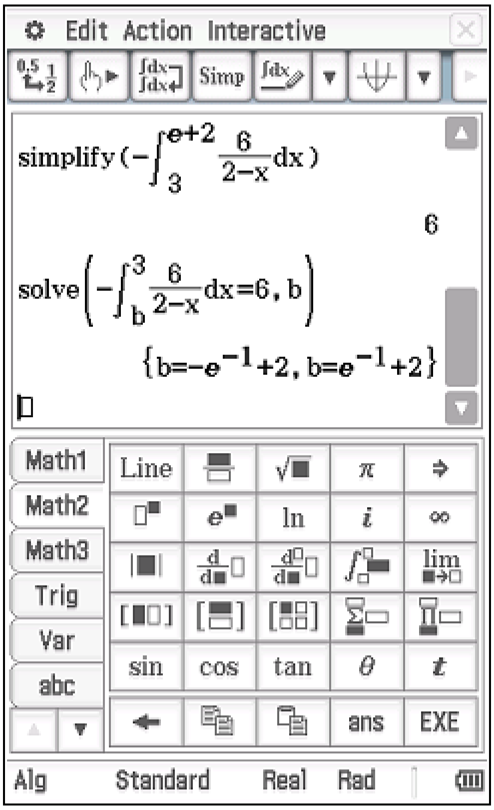
Mark allocation: 1 mark

- 1 mark for correct answer

Question 3b.ii.

Worked solution

Set up and solve using CAS. Note that $b > 2$, so $b = e^{-1} + 2$.



Mark allocation: 2 marks

- 1 method mark for $b = \pm e^{-1} + 2$
- 1 mark for answer $b = e^{-1} + 2$ (the negative answer must be discarded)

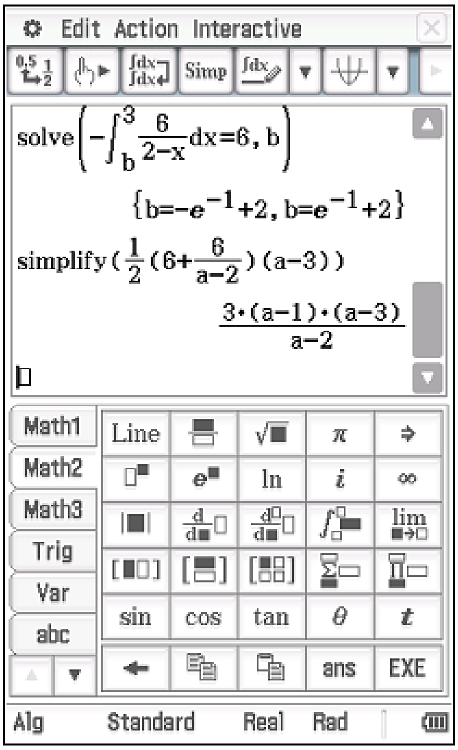
Question 3c.i.

Worked solution

Shape of the region is a trapezium with vertical sides of length 6 and $\dfrac{6}{a-2}$,

and a width of $a - 3$.

Using the formula for the area of a trapezium gives $A = \dfrac{1}{2}(a+b)h = \dfrac{3(a-1)(a-3)}{(a-2)}$.



Mark allocation: 2 marks

- 1 method mark for recognising the trapezium and finding the lengths 6 and $\dfrac{6}{a-2}$
- 1 answer mark for correct answer

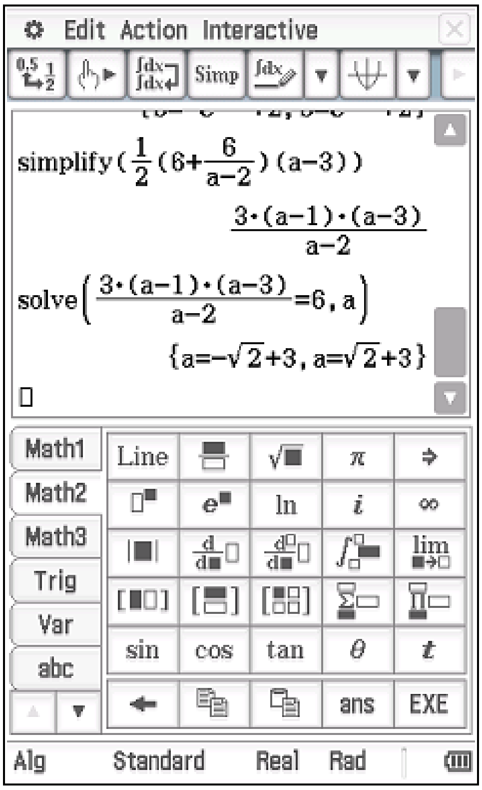
Question 3c.ii.

Worked solution

Set $\dfrac{3(a-1)(a-3)}{(a-2)}=6.$

Using CAS to solve for *a* gives $a=\pm\sqrt{2}+3.$

Since $a>3$, $a=\sqrt{2}+3.$



Mark allocation: 2 marks

- 1 method mark for setting $\dfrac{3(a-1)(a-3)}{(a-2)}=6$
- 1 answer mark for $a=\sqrt{2}+3$

Tip

- *This question is worth two marks, so a working step must be shown. It is not sufficient to just give an answer.*

Question 3c.iii.

Worked solution

The area bounded by the curve $y = \dfrac{6}{2-x}$, the x-axis and the lines $x = 3$ and $x = a$ is less than the area of the trapezium, so

$$-\int_3^a \frac{6}{2-x}\,dx < \text{area of trapezium}$$

For $a = \sqrt{2} + 3$

$$\Rightarrow -\int_3^{\sqrt{2}+3} \frac{6}{2-x}\,dx < 6$$

Now $-\displaystyle\int_3^{e+2} \frac{6}{2-x}\,dx = 6$ from **part b.i.**, so

$$\Rightarrow -\int_3^{\sqrt{2}+3} \frac{6}{2-x}\,dx < -\int_3^{e+2} \frac{6}{2-x}\,dx$$

$$\Rightarrow \sqrt{2} + 3 < e + 2$$

$$\Rightarrow \sqrt{2} + 1 < e$$

Mark allocation: 2 marks

- 1 method mark for using $-\displaystyle\int_3^{e+2} \frac{6}{2-x}\,dx = 6$ from **part b.i.**
- 1 answer mark for a clear and logical argument

Tip

- *This question is a 'hence' question, so it relies on using arguments and results established in earlier parts of the question. It is not sufficient to simply calculate a decimal equivalent for* $\sqrt{2}+1$ *and show that* $e > \sqrt{2}+1$.

Question 4a.

Worked solution

Using the product rule gives

$$\begin{aligned} f'(x) &= 3x(x-4)^2 + (x-4)^3 \\ &= (x-4)^2(3x+x-4) \\ &= (x-4)^2(4x-4) \\ &= 4(x-1)(x-4)^2 \end{aligned}$$

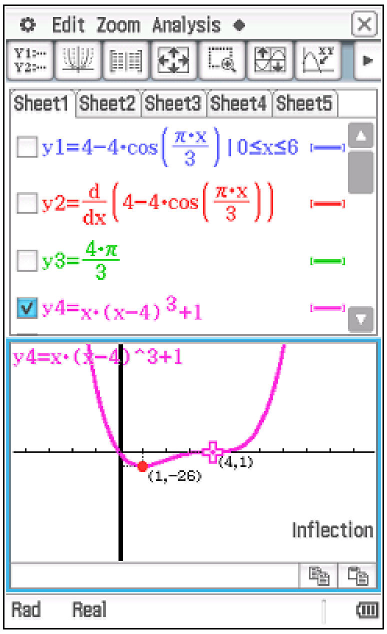
So $a = 4$.

Mark allocation: 2 marks

- 1 mark for evidence of product rule
- 1 mark for answer

Question 4b.

Worked solution



Stationary points: Let $f'(x)=0 \Rightarrow 4(x-4)^2(x-1)=0$

$$\Rightarrow x=4,\ x=1$$

At $x=1,\ y=-26$ and at $x=4,\ y=1$.

So $u=1$ and $v=1$.

Mark allocation: 2 marks

- 1 mark for correct value for u
- 1 mark for correct value for v

Question 4c.

Worked solution

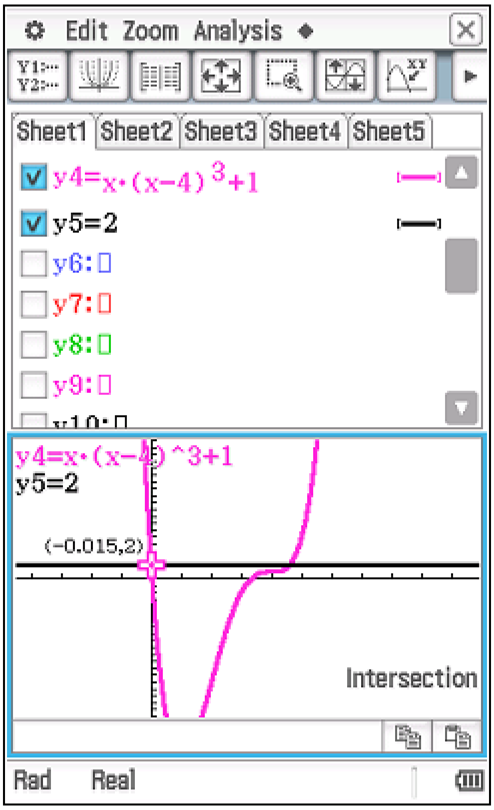
Looking at the graph of $f(x)$ gives the region of the graph from the *x*-intercept at $x = 0.016$ to the minimum turning point at $x = 1$. So the region is $[0.016, 1]$.

Mark allocation: 2 marks

- 1 mark for correct *x*-intercept
- 1 mark for correct range

Question 4d.

Worked solution



Looking at the screen, the graph must be moved to the right at least 0.015 units, so $k > 0.015$.

Mark allocation: 1 mark

- 1 mark for answer $k > 0.015$

Tip

- *Sometimes questions are completed more easily using a graph.*

Question 4e.

Worked solution

The graph of $y = f(x) - 1$ has x-intercepts at $x = 0$ and $x = 4,$ so the area can be calculated as

$$\begin{aligned}\text{Area} &= \left|\int_0^4 x^4 - 12x^3 + 48x^2 - 64x \; dx\right| \\ &= \left|\left[\frac{x^5}{5} - \frac{12x^4}{4} + \frac{48x^3}{3} - \frac{64x^2}{2}\right]_0^4\right| \\ &= \left|-51.2 - 0\right| \\ &= 51.2 \text{ square units}\end{aligned}$$

Mark allocation: 3 marks

- 1 mark for setting up the integral with correct intercepts
- 1 mark for correct antiderivative
- 1 mark for answer

Tip

- *Be careful here as the area cannot be negative – it is best to use absolute value signs around the calculation.*

Question 4f.i.

Worked solution

Dilation of factor of $\frac{1}{2}$ in x-direction (or from the y-axis)

Translation of 1 unit down

Mark allocation: 2 marks

- 1 mark for dilation specified correctly
- 1 mark for translation specified correctly

Question 4f.ii.

Worked solution

$(0,0)$ and $(2,0)$ by looking at the x-intercepts of $y = f(x) - 1$ and halving

Mark allocation: 1 mark

- 1 mark for both coordinates correct

Tip

- *Note that the question is a 'hence' question, so it needs to be obvious that the answer has come from working with the previous answer.*

Question 4f.iii.

Worked solution

$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \text{previous area} \\ &= \frac{1}{2} \times 51.2 = 25.6 \text{ square units}\end{aligned}$$

Mark allocation: 1 mark

- 1 mark for answer (showing appropriate working)

Note: Must have evidence of halving previous answer. If area is calculated using an integral, then zero marks allocated.

Question 5a.i.

Worked solution

The sum of the edges is equal to E cm; therefore, $6x+3y=E$, so $y=\frac{E}{3}-2x$.

Using Pythagoras, or otherwise, it can be shown that the vertical height of the triangle is

$$\sqrt{x^2-\left(\frac{x}{2}\right)^2}=\sqrt{\frac{3x^2}{4}}=\frac{\sqrt{3}x}{2}$$

The area of the triangle end is $\frac{1}{2}bh=\frac{1}{2}x\frac{\sqrt{3}x}{2}=\frac{\sqrt{3}x^2}{4}$.

The volume of the prism is

$$A_{\text{triangle}}\times y=\frac{\sqrt{3}x^2}{4}\left(\frac{E}{3}-2x\right)=\frac{\sqrt{3}Ex^2}{12}-\frac{\sqrt{3}x^3}{2},$$ as required.

Mark allocation: 3 marks

- 1 method mark for finding $y=\frac{E}{3}-2x$
- 1 method mark for finding the vertical height of the triangle
- 1 answer mark for finding area of triangle, leading to volume of prism

Question 5a.ii.

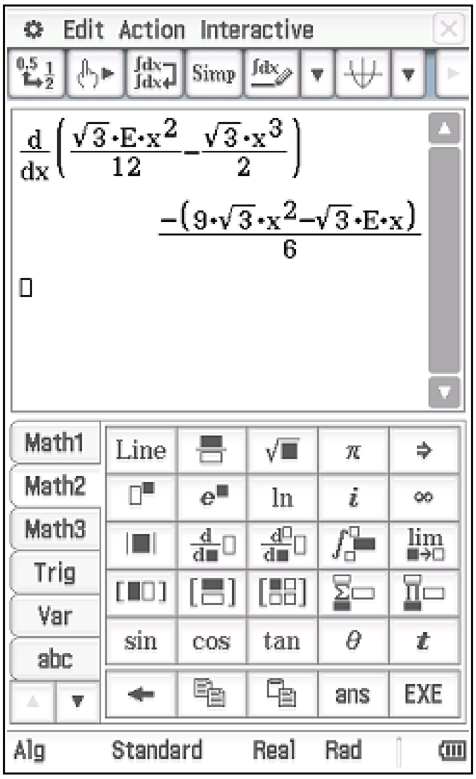
Worked solution

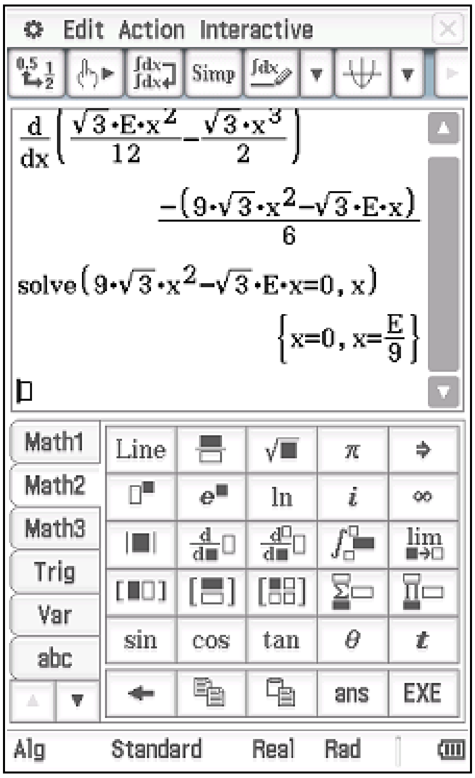
Maximum volume occurs when $\dfrac{dV}{dx} = 0.$

Using CAS gives $\dfrac{dV}{dx} = \dfrac{-\left(9\sqrt{3}\,x^2 - \sqrt{3}\,E\,x\right)}{6}$.

Set $\dfrac{-\left(9\sqrt{3}\,x^2 - \sqrt{3}\,E\,x\right)}{6} = 0$

Using CAS gives





So the maximum volume occurs $x = \dfrac{E}{9}$ cm.

Mark allocation: 2 marks

- 1 mark for setting $\dfrac{dV}{dx} = \dfrac{-\left(9\sqrt{3}\,x^2 - \sqrt{3}\,E\,x\right)}{6}$ equal to zero
- 1 mark for answer $x = \dfrac{E}{9}$

Tip

- *For questions worth more than 1 mark, appropriate working must be shown.*

Question 5b.

Worked solution

Surface area of the prism is

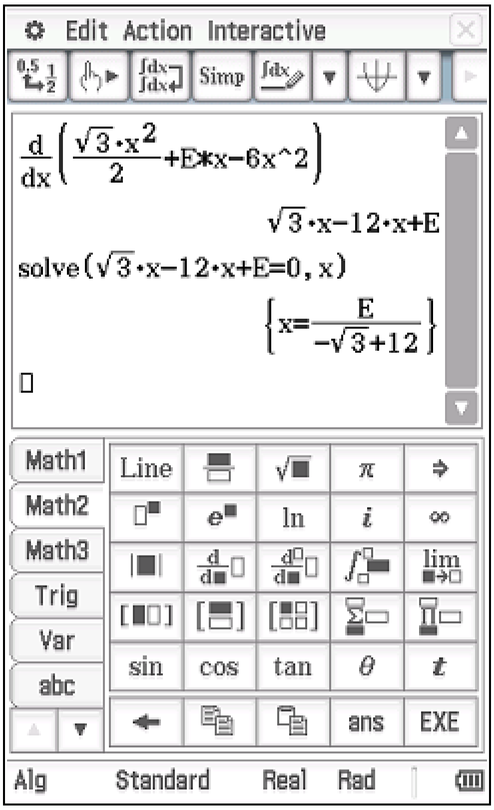
$$2 \times \text{Area}_{\text{triangle}} + 3 \times \text{Area}_{\text{rectangle}}$$

$$= \frac{\sqrt{3}x^2}{2} + 3xy$$

$$= \frac{\sqrt{3}\,x^2}{2} + 3x\left(\frac{E}{3} - 2x\right)$$

$$= \frac{\sqrt{3}\,x^2}{2} + E\,x - 6x^2$$

Using CAS gives



So, the maximum surface area occurs when $x = \dfrac{E}{12 - \sqrt{3}}$ cm.

Mark allocation: 2 marks

- 1 method mark for finding prism’s surface area
- 1 answer mark for $x = \dfrac{E}{12 - \sqrt{3}}$

END OF WORKED SOLUTIONS