

Victorian Certificate of Education
2016

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

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MATHEMATICAL METHODS

Written examination 1

Wednesday 2 November 2016

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
8	8	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 13 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \frac{\cos(x)}{x^2 + 2}$.

Find $\frac{dy}{dx}$.

2 marks

b. Let $f(x) = x^2 e^{5x}$.

Evaluate $f'(1)$.

2 marks

Question 2 (3 marks)

Let $f : \left(-\infty, \frac{1}{2}\right] \rightarrow \mathbb{R}$, where $f(x) = \sqrt{1-2x}$.

a. Find $f'(x)$.

1 mark

b. Find the angle θ from the positive direction of the x -axis to the tangent to the graph of f at $x = -1$, measured in the anticlockwise direction.

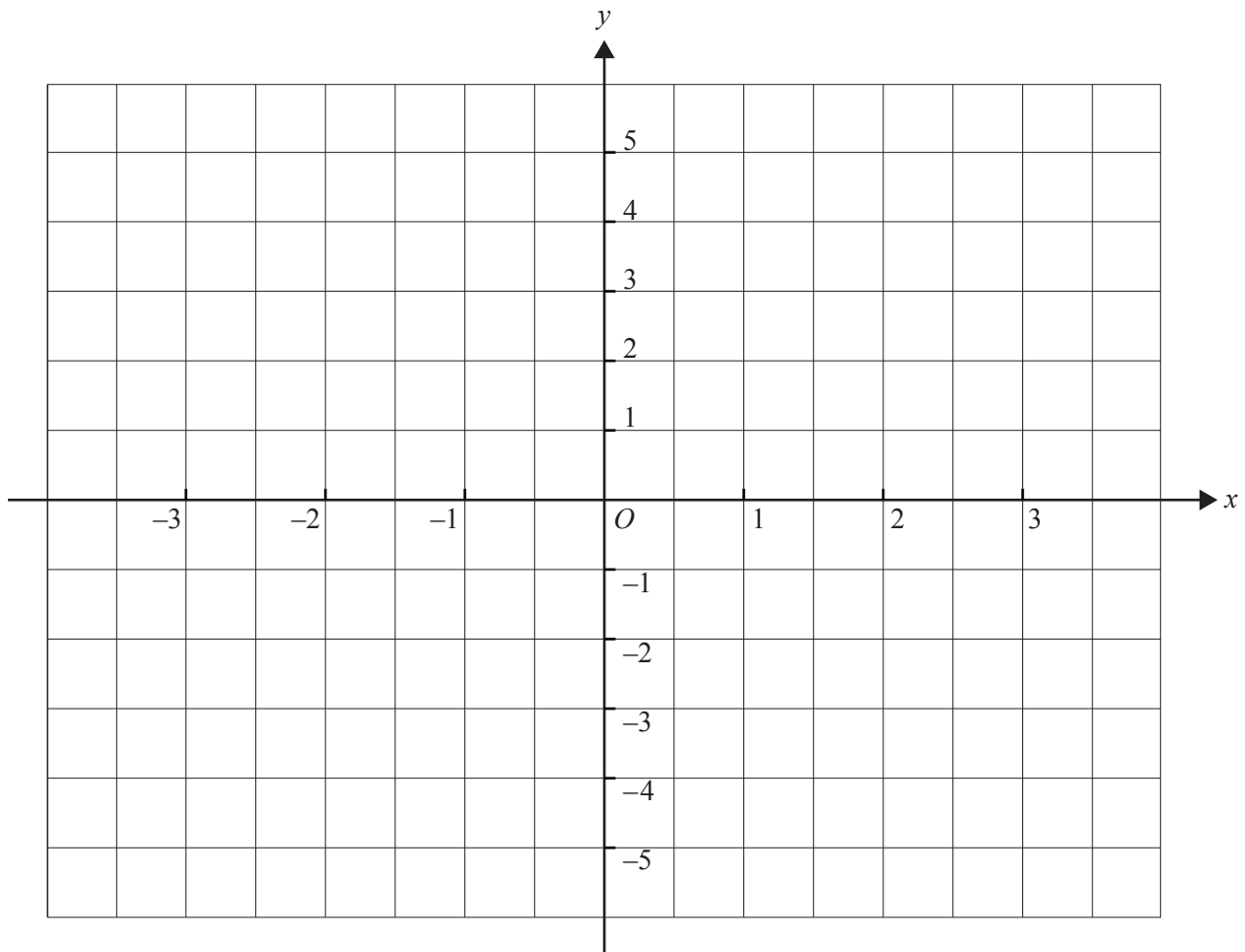
2 marks

Question 3 (5 marks)

Let $f: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R}$, where $f(x) = 2 + \frac{3}{x-1}$.

- a. Sketch the graph of f . Label the axis intercepts with their coordinates and label any asymptotes with the appropriate equation.

3 marks



- b. Find the area enclosed by the graph of f , the lines $x = 2$ and $x = 4$, and the x -axis.

2 marks

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Question 4 (3 marks)

A paddock contains 10 tagged sheep and 20 untagged sheep. Four times each day, one sheep is selected at random from the paddock, placed in an observation area and studied, and then returned to the paddock.

- a.** What is the probability that the number of tagged sheep selected on a given day is zero? 1 mark

- b.** What is the probability that at least one tagged sheep is selected on a given day? 1 mark

- c.** What is the probability that no tagged sheep are selected on each of six consecutive days?
Express your answer in the form $\left(\frac{a}{b}\right)^c$, where a , b and c are positive integers. 1 mark

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Question 5 (11 marks)

Let $f: (0, \infty) \rightarrow \mathbb{R}$, where $f(x) = \log_e(x)$ and $g: \mathbb{R} \rightarrow \mathbb{R}$, where $g(x) = x^2 + 1$.

- a. i.** Find the rule for h , where $h(x) = f(g(x))$. 1 mark

- ii.** State the domain and range of h . 2 marks

- iii.** Show that $h(x) + h(-x) = f\left((g(x))^2\right)$. 2 marks

- iv.** Find the coordinates of the stationary point of h and state its nature. 2 marks

b. Let $k: (-\infty, 0] \rightarrow \mathbb{R}$, where $k(x) = \log_e(x^2 + 1)$.

i. Find the rule for k^{-1} .

2 marks

ii. State the domain and range of k^{-1} .

2 marks

Question 6 (5 marks)

Let $f: [-\pi, \pi] \rightarrow \mathbb{R}$, where $f(x) = 2 \sin(2x) - 1$.

- a. Calculate the average rate of change of f between $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{6}$.

2 marks

- b. Calculate the average value of f over the interval $-\frac{\pi}{3} \leq x \leq \frac{\pi}{6}$.

3 marks

Question 7 (3 marks)

A company produces motors for refrigerators. There are two assembly lines, Line A and Line B. 5% of the motors assembled on Line A are faulty and 8% of the motors assembled on Line B are faulty. In one hour, 40 motors are produced from Line A and 50 motors are produced from Line B. At the end of an hour, one motor is selected at random from all the motors that have been produced during that hour.

- a. What is the probability that the selected motor is faulty? Express your answer in the form $\frac{1}{b}$, where b is a positive integer. 2 marks

- b. The selected motor is found to be faulty.

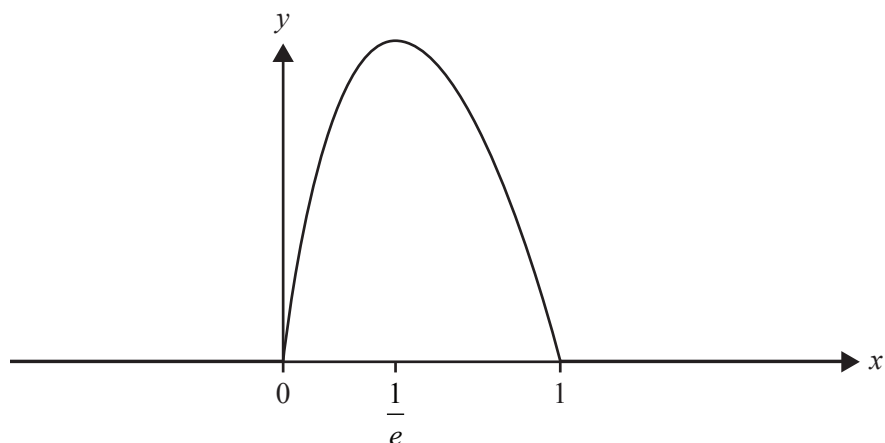
What is the probability that it was assembled on Line A? Express your answer in the form $\frac{1}{c}$, where c is a positive integer. 1 mark

Question 8 (6 marks)

Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} -4x \log_e(x) & 0 < x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

Part of the graph of f is shown below. The graph has a turning point at $x = \frac{1}{e}$.



a. Show by differentiation that

$$\frac{x^k}{k^2} (k \log_e(x) - 1)$$

is an antiderivative of $x^{k-1} \log_e(x)$, where k is a positive real number.

2 marks

- b. i.** Calculate $\Pr\left(X > \frac{1}{e}\right)$.

2 marks

- ii.** Hence, explain whether the median of X is greater than or less than $\frac{1}{e}$, given that $e > \frac{5}{2}$. 2 marks

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MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$		
$\frac{d}{dx}((ax+b)^n) = an(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$		
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$		
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$		
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$		
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$		
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$			
product rule	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$		

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{p}) = p$
standard deviation	$\text{sd}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$