

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2



(TSSM's 2015 trial exam updated for the current study design)

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: A

Explanation:

Solve the two equations on CAS.

Question 2

Answer: C

Explanation:

It is negative cubic so either C or D. Check the x-intercept.

Question 3

Answer: E

Explanation:

Define the functions on CAS and find $f(g(x))$

Question 4

Answer: D

Explanation:

$$f(x) = 2\left(\sqrt{x} + \frac{1}{2}\right)$$

$$g(x) = 2 \times \frac{1}{2}\left(\sqrt{x} + \frac{1}{2}\right)$$

Question 5

Answer: C

Explanation:

Domain: $4 - x \geq 0$ gives $x \leq 4$ and the graph is above the x-axis.

Question 6

Answer: A

Explanation:

$$Av ROC = \frac{f(8)-f(2)}{8-2}$$

Question 7

Answer: C

Explanation:

Note the shaded end-points.

Question 8

Answer: C

Explanation:

$$f(g(x)) = \frac{3}{x+5}, x \neq -2$$

Question 9

Answer: E

Explanation:

Eliminate incorrect options

Question 10

Answer: D

Explanation:

$$\text{Amp} = 2, \text{ Period} = \frac{2\pi}{\frac{1}{5}}.$$

Question 11

Answer: E

Explanation:

$$\frac{dy}{dx} \text{ at } x = 4 \text{ on CAS.}$$

Question 12

Answer: B

Explanation:

$$A_1 = A_2$$

Question 13

Answer: B

Explanation:

normalline($f(x), x = 0$) on CAS.

Question 14

Answer: C

Explanation:

$$(f(x))^2 \times (f(y))^2 = e^{2x} \times e^{2y} = e^{2x+2y} = f(2x+2y)$$

Question 15

Answer: A

Explanation:

$$\frac{1}{k} \int_0^k x^3 dx = 9 \text{ gives } k = 6^{\frac{2}{3}} \text{ on CAS.}$$

Question 16

Answer: B

Explanation:

$$\text{binompdf}\left(10, \frac{1}{5}, 6\right)$$

Question 17

Answer: C

Explanation:

$$\text{normcdf}(165, 170, 165, 7.62).$$

Question 18

Answer: A

Explanation:

$$\text{binomcdf}(6, 0.2, 5, 6) \text{ on CAS.}$$

Question 19

Answer: D

Explanation:

50th percentile means she is on average, due to the symmetry of the normal distribution

Question 20

Answer: C

Explanation:

Sketch on CAS and read the maximum value.

Question 21

Answer: C

Explanation:

$$k = 0.2, E(X) = 3.9$$

Question 22

Answer: B

Explanation:

$$\frac{\pi}{n} = 3 \text{ gives } n = \frac{\pi}{3}$$

SECTION 2: Analysis Questions

Question 1

a. $r = l \sin \alpha, h = l \cos \alpha$

A2
2 marks

b. $V = \frac{1}{3} \pi r^2 h = \frac{\pi}{3} (l \sin \alpha)^2 (l \cos \alpha) = \frac{\pi}{3} l^3 \sin^2 \alpha \cos \alpha$

M1
1 mark

c. $V'(\alpha) = \frac{\pi}{3} l^3 (\sin^2 \alpha \times -\sin \alpha + \cos \alpha \times 2 \sin \alpha \cos \alpha) = 0$

$$\sin \alpha (-\sin^2 \alpha + 2 \cos^2 \alpha) = 0$$

$$\sin \alpha = 0, \tan^2 \alpha = 2$$

$$\alpha = 0, \alpha = \pm \tan^{-1} \sqrt{2}$$

$$\alpha = \tan^{-1} \sqrt{2}, V(\alpha) = \frac{2\sqrt{3}}{27} \pi l^3$$

$$\left(\tan^{-1} \sqrt{2}, \frac{2\sqrt{3}}{27} \pi l^3 \right)$$

$$\text{Alternate form: } \left(\cos^{-1} \frac{\sqrt{3}}{3}, \frac{2\sqrt{3}}{27} \pi l^3 \right) \text{ also correct}$$

M3+A1
4 marks

d. $\alpha = \tan^{-1} \sqrt{2}$ is a point of maximum volume.

$$\text{Max volume} = \frac{2\sqrt{3}}{27} \pi \times 6^3 = 16\sqrt{3} \pi \text{ cm}^3.$$

M1+A1
2 marks

Question 2

a. Period = $\frac{2\pi}{\frac{\pi}{2.2}} = 4.4$ years and Amplitude = 300

A2
2 marks

b. Min = 200, Max = 800

A2
2 marks

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- c. Solve $P(t) = 800$ over $[0, 5]$
 $t = 0.7$. After 8.4 months

M1+A1
2 marks

- d. Sketch the graph on CAS and read the domain when $P < 300$
 $2.3 < t < 3.5$ and $6.7 < t < 7.9$

M1+A2
3 marks

- e. Strictly increasing for $t \in [0, 0.7] \cup [2.9, 5]$
Note that we include endpoints for strictly increasing intervals.

A3
3 marks

Question 3

- a. Sketch on CAS and read the max: $0.45 \mu\text{g/mL}$

A1
1 mark

- b. 3.5 minutes

A1
1 mark

- c. $C(10) = 0.32 \mu\text{g/mL}$

M1+A1
2 marks

- d. $\frac{C(5) - C(\frac{3}{2})}{5 - \frac{3}{2}} = 0.0115 \frac{\mu\text{g}}{\text{mL}}/\text{minute}$

M1+A1
2 marks

- e. Solve $\frac{dC}{dt} < 0$ on CAS
 $t > 3.53$ minutes

M1+A1
2 marks

f. $\frac{dC_1}{dt} = 0$ at $t = 120 \dots \dots \dots (1)$

$C_1(120) = 120 \dots \dots \dots (2)$

Solve the above equations on CAS to get $a = e$ and $b = \frac{1}{120}$

M2+A1

3 marks

Question 4

a. $f(x) = x^2 + bx + \frac{b^2}{4} + 3 - \frac{b^2}{4} = \left(x + \frac{b}{2}\right)^2 + 3 - \frac{b^2}{4}$

$\frac{b}{2} = 5$ gives $b = 10$ ($b > 0$)

M1+A1

2 marks

b. Translation of + 5 units parallel to the x – axis

Translation of + 22 units parallel to the y – axis

A2

2 marks

c. Range of $g: [0, \infty)$

Domain of $f: \mathbb{R}$

Range of g is a subset of domain of f , hence $f(g(x))$ exists.

$f(g(x)) = (x^2 + 5)^2 - 22$

Or $f(g(x)) = x^4 + 10x^2 + 3$

M1+A2

3 marks

d. $\text{tangentline}(h(x), x, k)$

$y = (4k^3 + 20k)x + (-3k^4 - 10k^2 + 3)$

M1+A1

2 marks

e. $\text{Area} = \int_0^3 ((x^2 + 5)^2 - 22) dx$

A2

2 marks

Question 5

a.

i. Let $X \sim N(7.5, 2.5^2)$

$$\Pr(X < 11) = 0.9192$$

(using CAS: $\text{normcdf}(-\infty, 11, 7.5, 2.5)$)

ii. $\Pr(5.5 < X < 10.5) = 0.6731$ (using CAS: $\text{normcdf}(5.5, 10.5, 7.5, 2.5)$)

M1+A1
2 marks

b. $\Pr(D < d) = 0.1$

$$d = 4.3 \text{ km}$$

M1+A1
2 marks

c. $n = 6, p = \Pr(X \geq 6.8) = 0.6103, r = 4$

$$\text{Let } Y \sim \text{Bi}(6, 0.6103)$$

$$\Pr(Y = 4) = 0.3160$$

(using $\text{binompdf}(6, 0.6103, 4)$)

M2+A1
3 marks

d. $\Pr(x > 5) = 0.65 \rightarrow \Pr(X < 5) = 0.35$

$$\frac{5 - 6.4}{\sigma} = -0.3853$$

$$\sigma = 3.63$$

M2+A1
3 marks

e. $\hat{p} = 0.8$ and $n = 200$

$$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.028284$$

$$(0.8 - 1.96 \times 0.028284, 0.8 + 1.96 \times 0.028284)$$

$$(0.745, 0.855)$$

M2+A1
3 marks

f. $M = 0.02, \hat{p} = 0.8$

$$0.02 = 1.96 \sqrt{\frac{0.8 \times 0.2}{n}}$$

$$n = 1536.64$$

Thus we need a sample size of 1537 people.

M1+A1
2 marks