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‘2016 Examination Package’ - Trial Examination 3 of 5

STUDENT NUMBER

Figures										Letter
Words										

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2

(TSSM’s *2013 trial exam updated for the current study design)*

Reading time: 15 minutes
Writing time: 2 hours

QUESTION & ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 19 pages including answer sheet for multiple-choice questions.

Instructions

- Print your name in the space provided on the top of this page and the multiple-choice answer sheet.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

SECTION 1 –Multiple-choice questions

Instructions for Section 1
Answer all questions on the answer sheet provided for multiple choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

If $f: [-2, \infty) \to R$ and $g: [-5, 5] \to R$ then the graph of $h(x)$ where $h(x) = f(x) + g(x)$ will have a domain of:

A. $[-5, -2]$

B. $[-5, \infty)$

C. $[-2, 5)$

D. $[-2, 5]$

E. $(-\infty, -2]$

Question 2

The inverse function of the function $f: [0, 5] \to R, \; f(x) = -\sqrt{25 - x^2}$ is:

A. $f^{-1}: (5, \infty) \to R, \; f^{-1}(x) = \sqrt{x^2 - 25}$

B. $f^{-1}: [-5, 0] \to R, \; f^{-1}(x) = \sqrt{25 - x^2}$

C. $f^{-1}: [0, 5] \to R, \; f^{-1}(x) = \sqrt{25 - x^2}$

D. $f^{-1}: (-5, 0] \to R, \; f^{-1}(x) = \sqrt{25 - x^2}$

E. $f^{-1}: [0, 5] \to R, \; f^{-1}(x) = -\sqrt{25 - x^2}$

F.

Question 3

The line $y = 3x$ is a tangent to the curve with equation $y = \frac{-2k}{x-1}$ when:

A. $k \in R$

B. $-\frac{9}{24} < k < \frac{9}{24}$

C. $k \leq \frac{9}{24}$

D. $k \geq \frac{9}{24}$

E. $k = \frac{9}{24}$

SECTION 1 - continued

Question 4

The value(s) of k for which the following system of equations have no solution is/are:

$$kx + 2y = 4$$
$$x + (k-1)y = 2$$

A. $k = -1, 2$

B. $k = -1$

C. $k = 2$

D. $k \neq 2$

E. $k \neq -1$

Question 5

If p is the probability of winning one game, ${}^{6}C_{5}(p)^{5}(1-p)$ is the probability of

A. exactly 1 loss out of 6 games

B. exactly 2 losses out of 6 games

C. at least 1 loss out of 6 games

D. exactly 5 losses out of 6 games

E. at least 5 losses out of 6 games

Question 6

The equations of the asymptotes of $y = -2 - \frac{1}{(9+3x)^2}$ are:

A. $x = -9,\ y = -2$

B. $x = 3,\ y = -2$

C. $x = 3,\ y = 2$

D. $x = -3,\ y = -2$

E. $x = -\frac{1}{3},\ y = -2$

SECTION 1 - continued
TURN OVER

Question 7

If $f(x) = x^2 - 1,\ x \geq 1$, and $g(x) = \sqrt{2 + 2x},\ x \geq -1$ then $g(f(x))$ is defined as:

A. $g(f(x)) = \sqrt{2}x,\ x \geq -1$

B. $g(f(x)) = -\sqrt{2}x,\ x \geq -1$

C. $g(f(x)) = -\sqrt{2}x,\ x \geq 1$

D. $g(f(x)) = \sqrt{2}x,\ x \geq 1$

E. $g(f(x)) = \sqrt{2}x,\ x \geq 0$

Question 8

The inverse of $f(x) = 2 - 3log_e(1 - x)$ is given by $f^{-1}(x) = a - e^{bx+c}$. The values of a, b and c are:

A. $a = 1,\ b = -1,\ c = 2$

B. $a = 1,\ b = \frac{2}{3},\ c = -\frac{1}{3}$

C. $a = 1,\ b = -\frac{1}{3},\ c = \frac{2}{3}$

D. $a = -1,\ b = -\frac{1}{3},\ c = \frac{2}{3}$

E. $a = 1,\ b = \frac{1}{3},\ c = -\frac{2}{3}$

Question 9

The derivative of $f(-3x^2 + 2)$ is:

A. $f'(-3x^2 + 2) \times -6x$

B. $f(-3x^2 + 2) \times -6x + (-3x^2 + 2) \times f'(-3x^2 + 2)$

C. $-6x \times f'(x)$

D. $(-6x + 2) \times f'(x)$

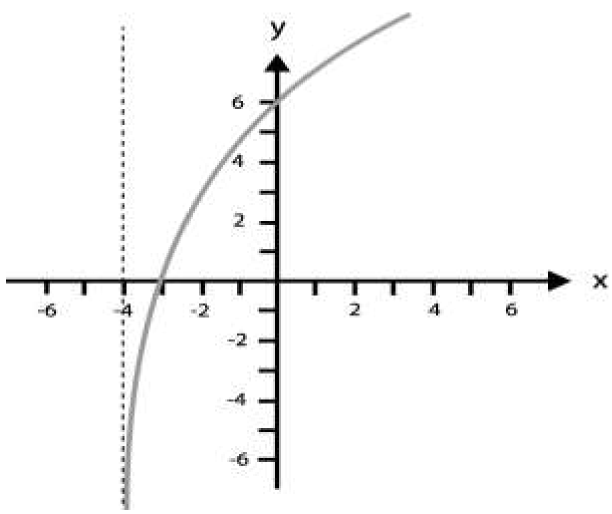
E. $f'(-3x^2 + 2)$

Question 10
Which of the following statements are true?

i. The sample size has no effect on the confidence interval
ii. The larger the sample size, the smaller the confidence interval
iii. The smaller the sample size, the smaller the confidence interval
iv. A 95% confidence interval must include the true population proportion

A. i and ii
B. i and iii
C. ii only
D. ii and iv
E. none

Question 11
Consider the graph of the function $f(x) = a\log_e(x+b)$, shown below:



A set of possible values of a and b is:

A. $a = 6, b = 4$

B. $a = 6\log_e 4$, $b = 4$

C. $a = \dfrac{6}{\log_e 4}$, $b = 4$

D. $a = \dfrac{4}{\log_e 6}$, $b = $ -4

E. $a = 6e^4$, $b = 4$

SECTION 1 - continued
TURN OVER

Question 12

If $f'(x) = 2sin\left(\frac{5x}{2}\right)$ then $f(x)$ could be:

A. $\frac{-4}{5}cos\left(\frac{5x}{2}\right)$

B. $\frac{-1}{5}cos\left(\frac{5x}{2}\right) - 9$

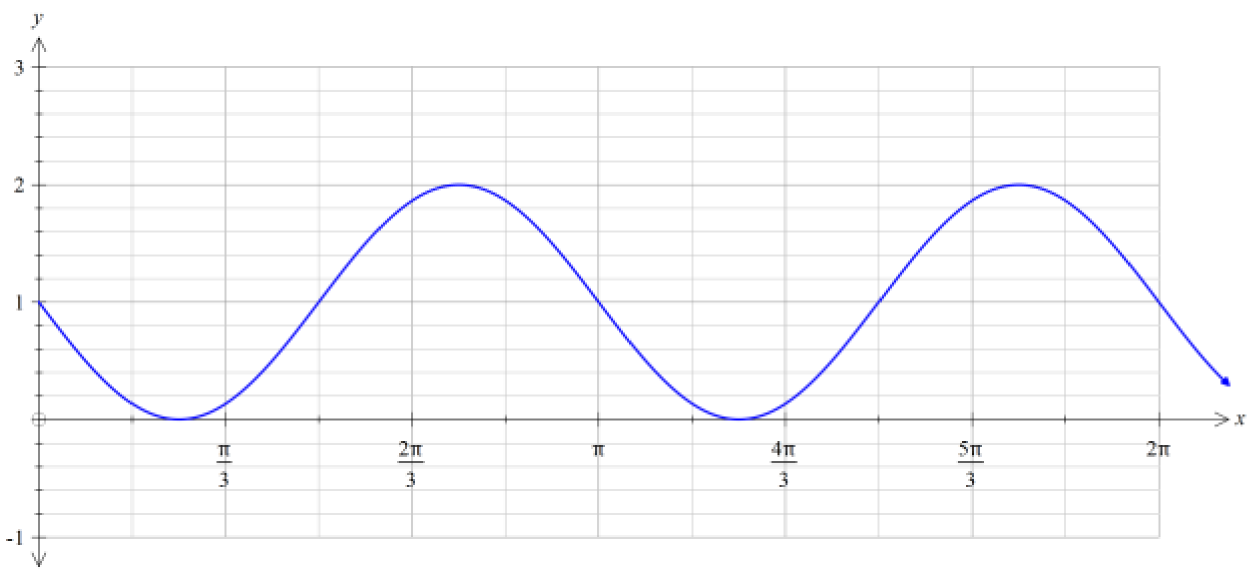
C. $\frac{4}{5}cos\left(\frac{5x}{2}\right) + 5$

D. $\frac{1}{5}cos\left(\frac{5x}{2}\right) + 7$

E. $\frac{4}{5}sin\left(\frac{5x}{2}\right) + 5$

Question 13

A possible equation for the curve below is:



A. $y = -1 + \sin(x)$

B. $y = 1 + \sin(x)$

C. $y = \sin(2x - 1)$

D. $y = 1 - \sin(2x)$

E. $y = 1 + \sin(2x)$

SECTION 1 - continued

Question 14

A random variable X is normally distributed with mean 4.7 and standard deviation 1.2. If Z is the standard normal variable, then $\Pr(X < 2.3)$ is:

A. $\Pr(Z < 2)$

B. $\Pr(Z < 1)$

C. $\Pr(-2 < Z < 2)$

D. $\Pr(Z > 2)$

E. $1 - \Pr(Z > 2)$

Question 15

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$

The value of a such that $\Pr(X < a) = 0.125$ is:

A. 0.021

B. 0.121

C. 0.204

D. 0.347

E. 0.500

Question 16

$X \sim Bi(n, p)$ is a binomial random variable with mean 20 and standard deviation 4. The values of n and p respectively are:

A. 80 and 0.2

B. 80 and 0.8

C. 25 and 0.8

D. 16 and 0.2

E. 100 and 0.2

SECTION 1 - continued
TURN OVER

Question 17

The sum of the solutions of $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}\ for\ 0 \leq x \leq 4\pi$ is:

A. 8π

B. 4π

C. $\frac{11\pi}{3}$

D. $\frac{\pi}{3}$

E. $\frac{\pi}{6}$

Question 18

The number of bacteria, *N*, in a colony varies with time according to the rule $N = N_0e^{0.1t}$, where *t* is the time measured in days, and $t \geq 0$. If initially there were 1000 bacteria, then the average rate of change in the number of bacteria over the first 10 days is closest to:

A. 172

B. 183

C. 272

D. 1718

E. 2718

Question 19

If the function $f(x) = x^3 - 2ax + b$ has (2, 5) as a stationary point, the values of *a* and *b* respectively are:

A. $a = 6\ \ and\ \ b = -21$

B. $a = 0\ \ and\ \ b = 21$

C. $a = -6\ \ and\ \ b = -27$

D. $a = 6\ \ and\ \ b = 21$

E. $a = 3\ \ and\ \ b = -10$

Question 20

A soup can is in the shape of a closed cylinder with a surface area of $726\pi\ cm^2$. The maximum volume of soup that can be contained in the can is:

A. 2662π cm^3

B. 1252π cm^3

C. 1452π cm^3

D. 1752π cm^3

E. 3502π cm^3

Question 21

For the following discrete probability distribution, the value of E(X) is:

x	1	2	4	8
Pr(x)	0.3	0.2	0.4	0.1

A. 3.1

B. 3

C. 3.5

D. 3.75

E. 1

Question 22

The height (h) above the ground of a carriage on a Ferris wheel is given by

$h = 15 + 13.5\cos\left(\frac{\pi}{15}t\right)$, where t is the time in seconds since the carriage was at the top of its path.

The time for the wheel to complete one revolution is:

A. 28.5 *seconds*

B. 30 *seconds*

C. 50 *seconds*

D. 53.5 *seconds*

E. 60 *seconds*

END OF SECTION 1

TURN OVER

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Where an instruction to **use calculus** is stated for a question, you must show an appropriate derivative or anti-derivative.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

Consider the function $f: R \to R, f(x) = ax^3 + bx^2 + cx - 5$.

The graph of $f(x)$ has a turning point at (2, -1).

a. Find the values of a and b in terms of c.

3 marks

The remainder when $f(x)$ is divided by $x + 1$ is 9.

b. Find the values of a, b and c.

2 marks

SECTION 2 – Question 1 – continued

Consider the function $g: [0,3) \to R, g(x) = 2(1-x)^2(x+2)$.

c. Sketch the graph of $g(x)$ clearly labelling all axes intercepts and turning point(s).

3 marks

d. Find the rule for $g'(x)$.

1 mark

e. Find the equation of the normal to the curve $g(x)$ at $x = 2$.

3 marks

Total 12 marks

SECTION 2 - continued

TURN OVER

Question 2

Rachel likes to fish each weekend. The probability that she fishes at location A is 0.6.

If Rachel fishes for the next 4 weekends:

a. Find the probability that exactly one of them is at location A.

1 mark

b. Find the probability that at least 1 of them is at location A

2 marks

The number of fish Rachel catches is normally distributed with a standard deviation of 1.1. The probability that she catches less than 5 fish is 0.8.

c. Find the mean number of fish that Rachel catches answering correct to 2 decimal places.

2 marks

SECTION 2 – Question 2 – continued

The time, t, in hours that Rachel spends fishing each weekend is a random variable with probability density function defined as follows:

$$f(t) = \begin{cases} kt(4-t), & 0 \le t \le 4 \\ 0, & elsewhere \end{cases}$$

d. Show that the value of k is $\frac{3}{32}$.

2 marks

e. Find the exact probability that Rachel spends at least 2 hours fishing on the shore.

2 marks

f. What is the probability, correct to four decimal places that Rachel spends at least 2 hours fishing on the shore at most three of the four weekends in December?

2 marks

SECTION 2 – Question 2 – continued
TURN OVER

Rachel goes fishing on each weekend for two months and spends less than n minutes on 15% of weekends.

g. Find the value of n, to the nearest minute.

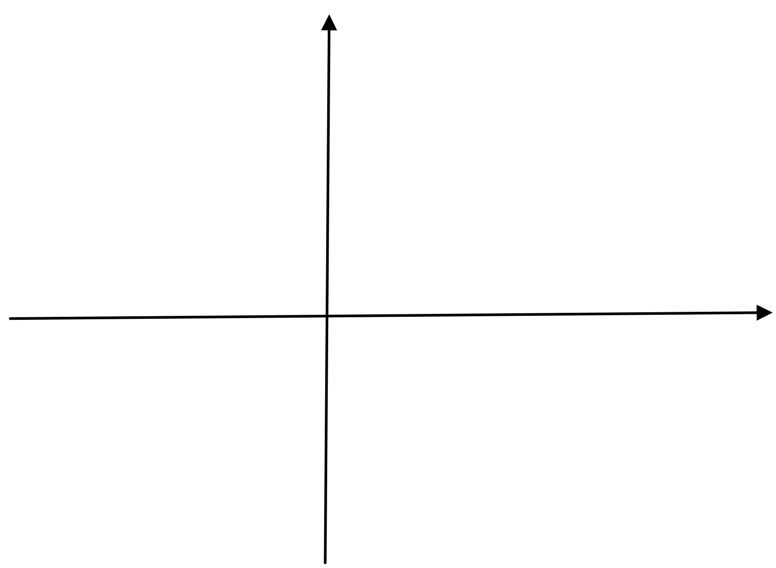
2 marks

Total 13 marks

Question 3

A plan for a mine site follows $f: [0, 2) \to R,\ f(x) = \frac{1}{2x-4} + 2$.

a. Sketch the graph of $f(x)$ on the axes below. Label all axes intercepts with their coordinates and label each of the asymptotes with its equation.



3 marks

SECTION 2 – Question 3 – continued

b. If $f(x)$ cuts the x-axis at $x = a$, find the value of a.

1 mark

c. State $f'(x)$, and the range of $f'(x)$.

2 marks

d. Evaluate $\int_0^b f(x)dx$ in terms of b.

2 marks

e. Find the area bounded by the curve $f(x)$ between the lines $y = 0$ and $y = -3$.

3 marks

SECTION 2 – Question 3 – continued
TURN OVER

An engineer decides that there is another better mine site which follows the following rule

$$h(x) = log_e(x - a), \qquad where\ x > a$$

f. If $h(x)$ crosses the x-axis at the point (3, 0), find the value of a.

1 mark

g. A bridge is constructed from $A(1, f(1))$ to $B(4, h(4))$. Find the shortest distance between A and B, correct to four decimal places.

3 marks

Total 15 marks

Question 4

A sine curve in the form $h(x) = Asin(n\pi x) + B$ is used to model a mountain profile.
The maximum and minimum points, S and T, on the function $h(x)$ occur at (0.75, 4) and (2.25, 1) respectively.

a. Find the values of A, n and B.

3 marks

SECTION 2 – Question 4 – continued

b. Sketch the graph of $h(x)$ over the domain [0, 2.5]. Label the axes intercepts and turning points correct to two decimal places.

3 marks

c. Write down the rate of change of $h(x)$ and hence find where this rate of change is greatest.

3 marks

Total 9 marks

SECTION 2 - continued

TURN OVER

Question 5

A closed box is to be constructed with a square base of side x cm and a surface area of 27 000cm^2.

a. Find the height, h of the box in terms of x.

2 marks

b. Write down the domain for x.

2 marks

c. What are the dimensions of the box for maximum volume?

3 marks

d. Find the maximum volume of the box.

2 marks

Total 9 marks

END OF QUESTION AND ANSWER BOOK

MULTIPLE CHOICE ANSWER SHEET

Student Name:___________________________________

Circle the letter that corresponds to each correct answer.

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E