

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2



(TSSM's 2013 trial exam updated for the current study design)

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: D

Explanation:

Domain of sum of functions is given by $\text{dom of } f \cap \text{dom of } g$

Question 2

Answer: B

Explanation:

Swap x and y and use CAS to make y the subject

Note that range of f^{-1} = domain of f

And domain of f^{-1} = range of f

Question 3

Answer: E

Explanation:

$$\frac{-2k}{x-1} = 3x$$

$$3x^2 - 3x + 2k = 0$$

$\Delta = 0$ (as the line is a tangent, it will cut the graph at only one point)

$$k = \frac{9}{24}$$

Question 4

Answer: B

Explanation:

$$\left| \begin{matrix} k & 2 \\ 1 & k-1 \end{matrix} \right| = 0 \text{ gives } k = -1, 2$$

$k \neq 2$, so $k = -1$

Question 5

Answer: A

Explanation:

Eliminate incorrect solutions from formula (binomial)

Question 6

Answer: D

Explanation:

$9 + 3x = 0$ gives $x = -3$ as the horizontal asymptote.

Question 7

Answer: D

Explanation:

Use CAS to define the functions and find $g(f(x))$

Note that $g(f(x))$ is defined only when Range of f is a subset of domain of g

Question 8

Answer: C

Explanation:

Solve $(x = 2 - 3\log_e(1 - y), y)$ on CAS to get $f^{-1}(x) = 1 - e^{-\frac{x}{3} + \frac{2}{3}}$

Question 9

Answer: A

Explanation:
Use chain rule.

Question 10

Answer: C

Explanation:

Only statement ii is correct

Question 11

Answer: C

Explanation:

Substitute (0,6) in $y = a \log_e(x + 4)$ to get $a = \frac{6}{\log_e(4)}$

Question 12

Answer: A

Explanation:
 $\int 2 \sin\left(\frac{5x}{2}\right) dx$ on CAS

Question 13

Answer: D

Explanation:

Period = π and it is reflected in the x -axis.

Question 14

Answer: D

Explanation:

$\Pr(X < 2.3) = \Pr(Z < -2)$ which is the same as $\Pr(Z > 2)$

Question 15

Answer: E

Explanation:

Use CAS to find the value of a , $\int_0^a 3x^2 dx = 0.125$

Question 16

Answer: E

Explanation:

$np = 20$, $np(1 - p) = 16$. Solve the two equations to get $p = 0.2$ and $n = 100$

Question 17

Answer: B

Explanation:

Solve on CAS: $\text{solve}\left(\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}, x\right) | 0 \leq x \leq 4\pi$ and then add the two solutions.

Question 18

Answer: A

Explanation:

$\text{Average ROC} = \frac{1000e - 1000}{10 - 0}$ on CAS.

Question 19

Answer: D

Explanation:

$$f'(2) = 0 \text{ implies } a = 6 \text{ and } f(2) = 5 \text{ implies } b = 21$$

Question 20

Answer: A

Explanation:

$$\begin{aligned} 2\pi r(r + h) &= 726\pi \text{ implies } h = \frac{363-r^2}{r} \\ V &= 363\pi r - \pi r^3, V' = 0 \text{ implies } r = 11 \\ \text{Max volume} &= V(11) = 2662\pi \end{aligned}$$

Question 21

Answer: C

Explanation:

$$E(X) = (0.3 + 0.4 + 1.6 + 0.8) = 3.1$$

Question 22

Answer: B

Explanation:

$$\text{Period} = \frac{2\pi}{\left(\frac{\pi}{15}\right)} = 30 \text{ seconds}$$

SECTION 2: Analysis Questions

Question 1

a. $f'(x) = 3ax^2 + 2bx + c$

$f'(2) = 0$ implies $12a + 4b + c = 0$

$f(2) = -1$ implies $8a + 4b + 2c - 5 = -1$

Solve the above two equations to get $a = \frac{c-4}{4}$ and $b = 3 - c$

M1+A2

3 marks

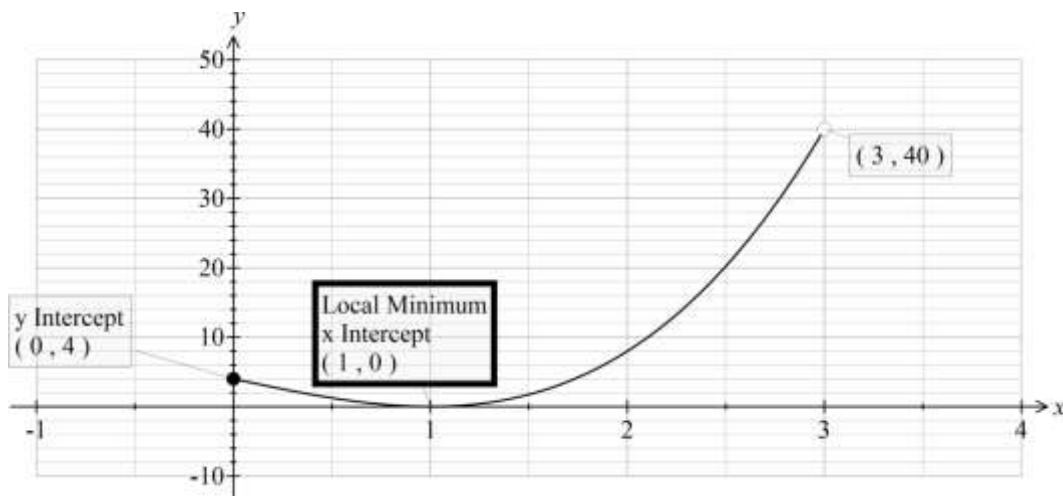
b. $f(-1) = 9$ gives $-a + b - c - 5 = 9$

Substitute a and b from part a. in the above equation to get $c = -\frac{40}{9}$ $a = \frac{-19}{9}$, $b = \frac{67}{9}$

M1+A1

2 marks

c.



1 mark for y-intercept, 1 mark for turning point, 1 mark for showing the range

3 marks

d. On CAS: $\frac{d}{dx}(g(x)) = 6x^2 - 6$

A1

1 mark

- e. The point is (2, 8)

$$m_T = g'(2) = 18$$

$$m_N = -\frac{1}{18}$$

Equation of the normal is:

$$y - 8 = -\frac{1}{18}(x - 2) \text{ or } y = -\frac{1}{18}x + \frac{73}{9}$$

M2+A1

3 marks

Question 2

a. $\binom{4}{1}(0.6)^1(0.4)^3$
 $= 0.1536$

A1

1 mark

b. $1 - \binom{4}{4}(0.4)^4$
 $= 0.9744$

M1+A1

2 marks

c. $\Pr(X < 5) = 0.8$
 $\Pr(Z < 0.8416) = 0.8$
 $\frac{5 - \bar{x}}{1.1} = 0.8416$
 $\bar{x} = 4.07$

M+A1

2 marks

d. Solve $k \int_0^4 t(4 - t)dt = 1$ on CAS to get $k = \frac{3}{32}$

M2

2 marks

e. $\Pr(X \geq 2) = \frac{3}{32} \int_2^4 t(4 - t)dt = 0.5 = \frac{1}{2}$

M1+A1

2 marks

f. Let $X \sim Bi(4, \frac{1}{2})$

$$Pr(X \leq 3) = 0.9375 \quad (\text{using } binomcdf(4, \frac{1}{2}, 0, 3))$$

M1+A1

2 marks

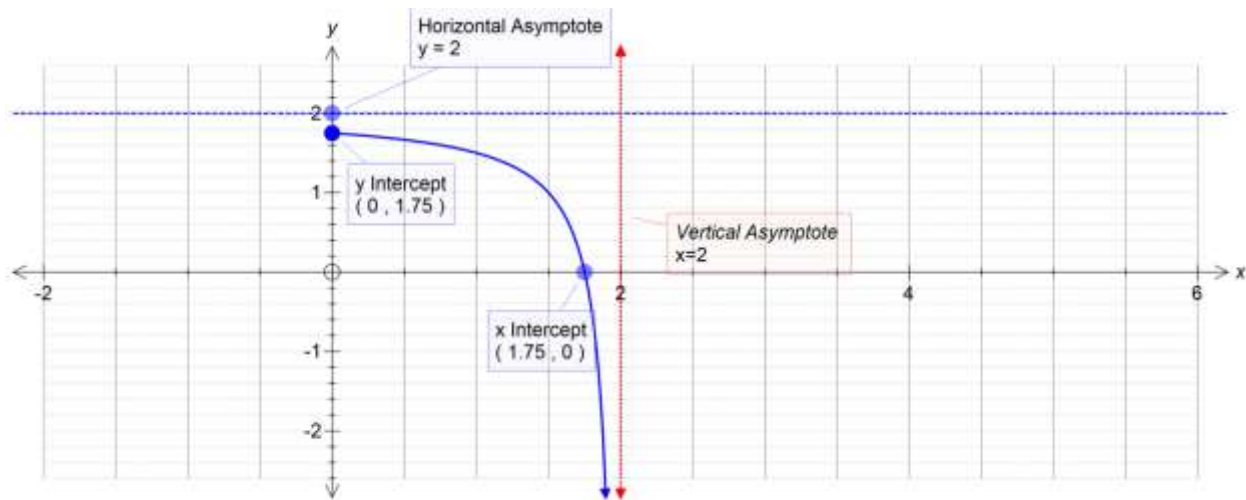
g. $Pr(X < n) = 0.15$. Solve $\frac{3}{32} \int_0^n t(4-t)dt = 0.15$ to get $n = 0.9776hr = 59 \text{ mins}$

M1+A1

2 marks

Question 3

a.



1 mark for asymptotes, 1 mark for intercepts, 1 mark for drawing in the required domain

3 marks

b. $a = \frac{7}{4}$

A1

1 mark

c. $f'(x) = -\frac{1}{2(x-2)^2}$
Range is $(-\infty, -\frac{1}{8})$

A1+A1

2 marks

d. $\int_0^b \left(\frac{1}{2x-4} + 2 \right) dx = \frac{1}{2} \log_e \left(\frac{|b-2|}{2} \right) + 2b$

M1+A1
2 marks

e. Find the area using $\int_c^d f(y) dy$
 $\int_{-3}^0 \frac{4y-7}{2(y-2)} dy = -\frac{1}{2} \log_e \left(\frac{5}{2} \right) + 6$

M2+A1
3 marks

f. $0 = \log_e(3 - a)$ which gives $a = 2$

A1
1 mark

g. $A \left(1, \frac{3}{2} \right)$ and $B(4, \log_e 2)$

Distance between A and B = $\sqrt{(4-1)^2 + \left(\ln 2 - \frac{3}{2} \right)^2} = 3.1066 \text{ units}$

M2+A1
3 marks

Question 4

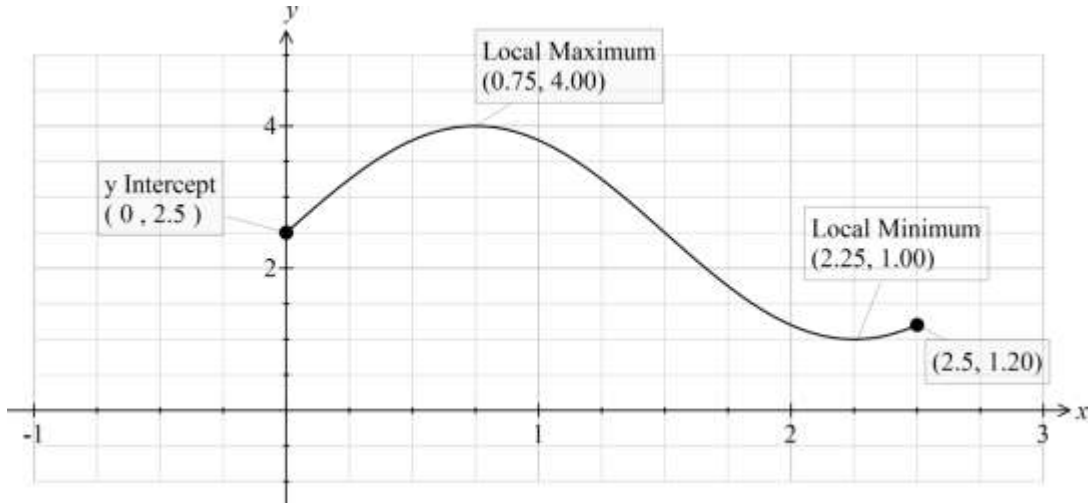
a. $B - A = 1$ and $B + A = 4$

Solve for A and B to get $A = \frac{3}{2}$ and $B = \frac{5}{2}$

$h(0.75) = 4$ gives $n = \frac{2}{3}$

A3
3 marks

b.



A3

3 marks

c. Solve $h'(x) = \pi \cos\left(\frac{2\pi}{3}x\right)$

Greatest value of $h'(x) = \pi$

Solve $\pi = \pi \cos\left(\frac{2\pi}{3}x\right)$ to get $x = 0$

M2+A1

3 marks

Question 5

a. $27000 = 2(x^2 + 2xh)$

$$h = \frac{13500 - x^2}{2x}$$

M1+A1

2 marks

b. $x > 0$, $13500 - x^2 > 0$ which gives $x < 30\sqrt{15}$

$$0 < x < 30\sqrt{15}$$

M1+A1

2 marks

c. $V = x^2 \left(\frac{13500 - x^2}{2x} \right)$

$$V = 6750x - \frac{x^3}{2}$$

$$V'(x) = 6750 - \frac{3x^2}{2}, \quad V'(x) = 0 \text{ implies } x = 30\sqrt{5}$$

Substitute $x = 30\sqrt{5}$ in part a. to get $h = 30\sqrt{5} \text{ c}$

M2+A1

3 marks

d. Substituting for x in the volume equation in part c.

$$V(30\sqrt{5}) = 135000\sqrt{5} \text{ cm}^3$$

A2

2 marks