

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 2



(TSSM's 2011 trial exam updated for the current study design)

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: E

Explanation:

If $k = 2$: $(2 - 1)x + 4y = 8$ and $3x - (-2 + 2)y = 2 + 1$

$x + 4y = 8$ and $3x = 3$

$x = 1$ and $y = \frac{7}{4}$

Therefore it will have a unique solution.

Question 2

Answer: C

Explanation:

$$f(x - y) = (x - y)^3$$

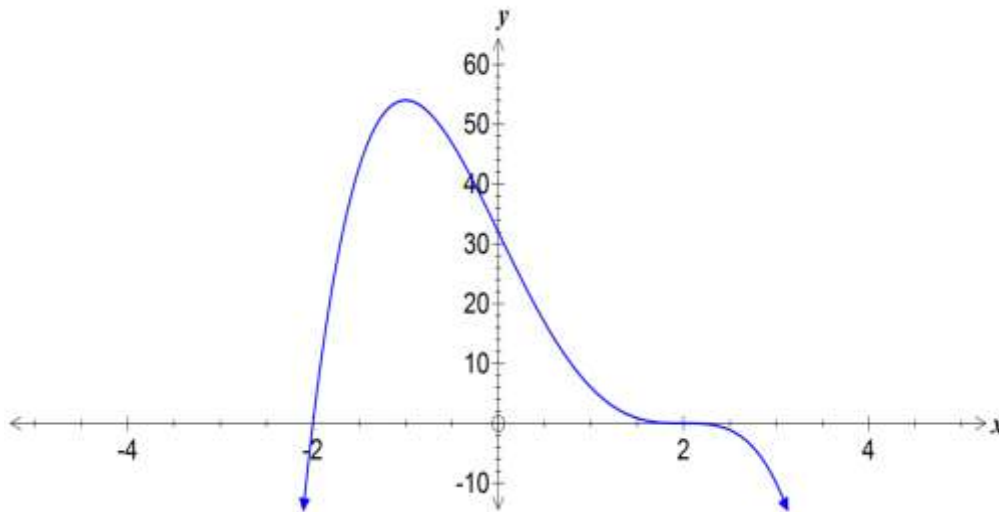
$$f(x) - f(y) = x^3 - y^3$$

$$f(x - y) \neq f(x) - f(y)$$

Question 3

Answer: B

Explanation:
Sketch graph:



Question 4

Answer: C

Explanation:

$$\begin{aligned} & \int_1^4 (2f(x) + 6) dx \\ &= 2 \int_1^4 f(x) dx + \int_1^4 6 dx \\ &= 2 \int_1^4 f(x) dx + 18 \end{aligned}$$

Question 5

Answer: B

Explanation:

$$\begin{aligned} m_{\text{tangent}} &= -3 \\ y &= -3(x - 3) + 4 - 6 = -3x + 7 \end{aligned}$$

Question 6

Answer: B

Explanation:

$$Pr(Z < -z) = Pr(Z > z) = 0.75$$

Use CAS Probability menu: inverse normal. Change the area to 0.25 because the TI-nspire calculators use $Z < c$ and not $Z > c$.

$$c = -0.6745$$

Question 7

Answer: B

Explanation:

At $x = 2$, the gradient function changes from positive to negative, therefore local max on $f(x)$

Question 8

Answer: A

Explanation:

$f \circ g$ exists if $\text{ran } g \subseteq \text{dom } f$, therefore $D = x < -1$ or $x > 3$.

(For $f \circ g$ to exist, the range of g : $(1, \infty)$)

Question 9

Answer: C

Explanation:

$$\text{Use CAS: } f(x) = \int (2 - x)(2x + 1)^2 dx = -x^4 + \frac{4}{3}x^3 + \frac{7}{2}x^2 + 2x$$

Question 10

Answer: B

Explanation:

$$\text{Solve } \int_k^{\frac{\pi}{2}} \cos^2(x) dx = 0.3 \text{ on CAS}$$

$$k = 2.63$$

Question 11

Answer: B

Explanation:

$$\int_0^6 f\left(\frac{1}{3}x\right) + 1 dx = \int_0^6 f\left(\frac{x}{3}\right) dx + \int_0^6 1 dx = 12 + [x]_0^6 = 12 + 6 = 18$$

Question 12

Answer: C

Explanation:

$$Pr(-1 < Z < 2) = Pr(\mu - \sigma < X < \mu + 2\sigma).$$

$$\mu + 2\sigma = 4.9$$

$$\mu - \sigma = 4$$

Question 13

Answer: D

Explanation:

$$\text{Use CAS: } \frac{d}{dx}(e^{-4x} \sin(x - 2))|_{x=2}$$

Question 14

Answer: C

Explanation:

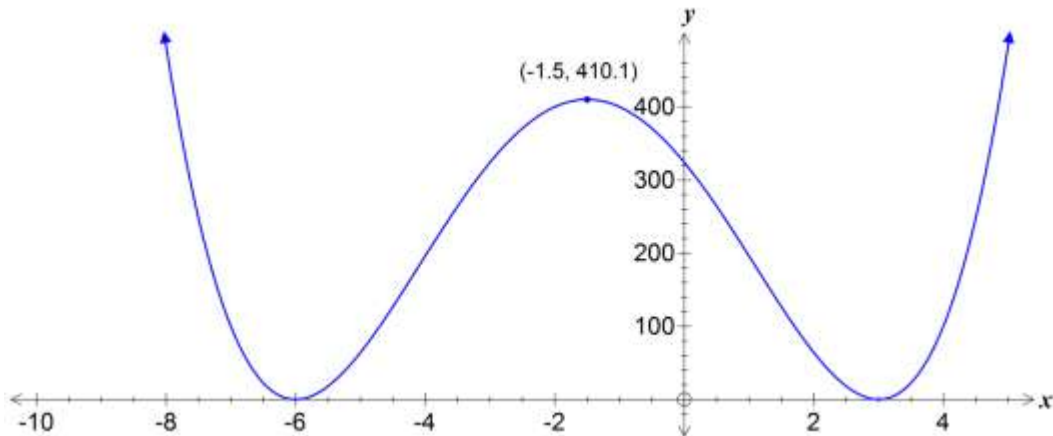
$$\text{Use CAS and solve } \cos(2x) = -\frac{\sqrt{3}}{2}, \text{ then expand to get } \pm \frac{5\pi}{12} + k\pi, \quad k \in \mathbb{Z}$$

Question 15

Answer: E

Explanation:

Use CAS to sketch graph and find local maximum, then $(-6, -1.5) \cup (3, \infty)$.



Question 16

Answer: E

Explanation:

$$y = -5e^{2(x+2)} + 2$$

$$T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

Question 17

Answer: B

Explanation:

Use CAS: Binomial pdf $(20, 0.45, 5)$, $\Pr(X = 5) = 0.0365$

Question 18

Answer: A

Explanation:

$$\text{Period} = \frac{\pi}{b} = 5\pi, \text{ asymptote } x = \frac{\pi}{2b} = \frac{5\pi}{2}$$

Question 19

Answer: C

Explanation:

$$l_1 = 3 - e^{1-1} = 3 - 1 = 2, \quad l_2 = 3 - e^{2-1} = 3 - e$$

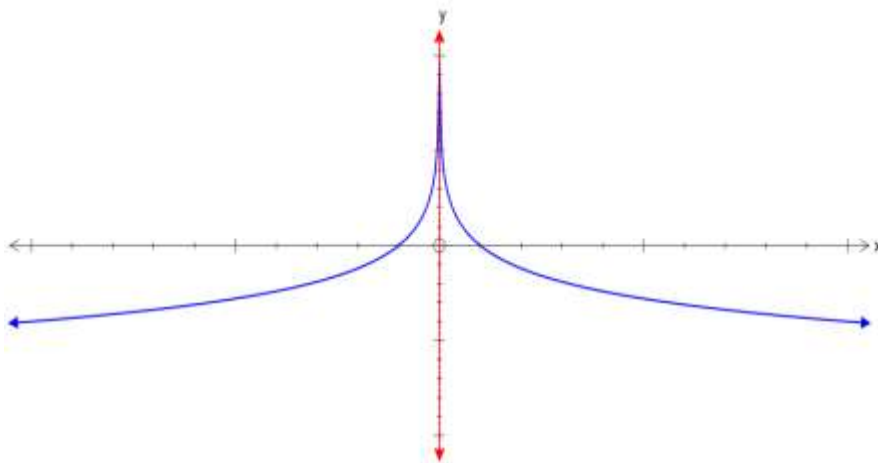
$$A = 2 + 3 - e = 5 - e$$

Question 20

Answer: D

Explanation:

Use CAS to sketch the graph.



There is a vertical line at $x = 0$.

Question 21

Answer: C

Explanation:

You have to use the points of intersection and for each section it is the top graph minus the bottom graph.

Question 22

Answer: E

Explanation:

The graph has a vertical asymptote at $x = a$ and the function lies to the right of the asymptote, sketch the graph.

SECTION 2: Analysis Questions

Question 1

a. $f(x) = \frac{2}{x-3} - 4 = 2(x-3)^{-1} - 4$

Use chain rule: $m_{\text{tangent}} = f'(x) = -2(x-3)^{-2} \times 1 = \frac{-2}{(x-3)^2}$

$m_{\text{normal}} = \frac{(x-3)^2}{2}$

M1+A1

2 marks

b. $m = 2, (1, -5)$

$y - y_1 = m(x - x_1)$

$y + 5 = 2(x - 1)$

$g(x) = 2x - 7$

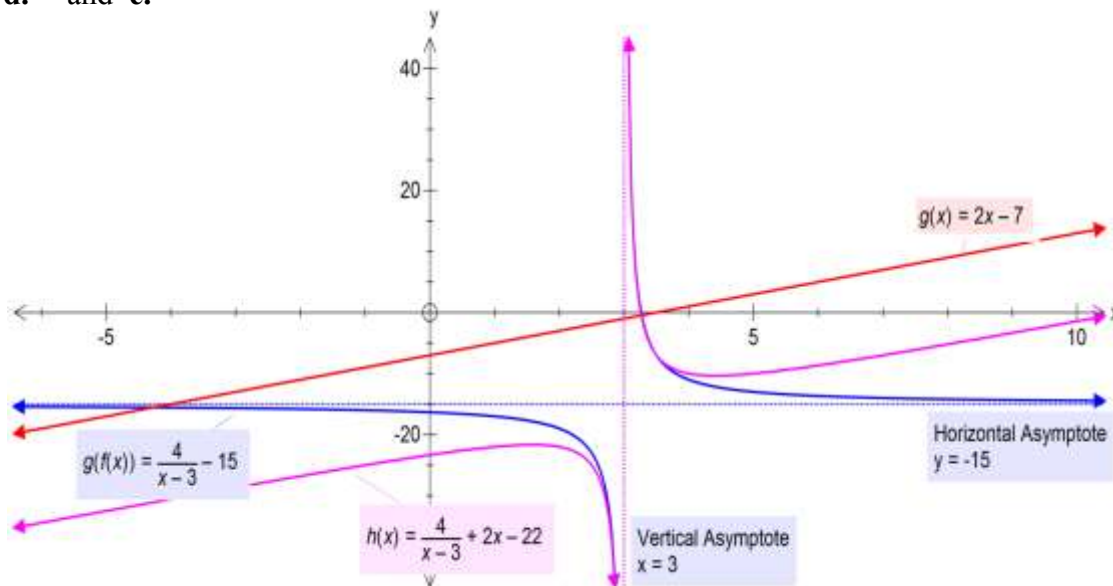
M1+A1

2 marks

c. $g(f(x)) = g\left(\frac{2}{x-3} - 4\right) = 2\left(\frac{2}{x-3} - 4\right) - 7 = \frac{4}{x-3} - 15$, the domain is the set of all values in the domain of f (the domain of the inner function). Domain: $x \in \mathbb{R} \setminus \{3\}$.
M2+A1

3 marks

d. and e.



A2

A2

2 + 2 marks

- f. Reflection in the x –axis, dilation factor of 2 away from the x –axis, dilation factor of $\frac{1}{2}$ away from the y –axis, translation of 2 units to the right and a translation of 1 unit up.

$$\text{equation: } y = -2 \left(\frac{2}{2x-4-3} - 4 \right) + 1 = 9 - \frac{4}{2x-7}$$

M2+A2

4 marks

Question 2

- a. $np = 212.5$
 $np(1 - p) = 31.875$
 Solve for n & p
 $n = 250$
 $p = 0.85$

M1+A1

2 marks

- b. *Binomial CDF* (5, 0.9, 3, 5)
 $= 0.991$

A1

1 mark

- c.
 $\Pr(\text{at least one lasts less than 200 hours}) > 0.95$
 $\Pr(\text{zero last less than 200 hours}) < 0.05$
 $\binom{n}{0} (0.1)^0 (0.9)^n < 0.05$
 $n > 28.433$
 $n = 29$

M2+A1

3 marks

- d. $\text{normcdf}(200, \infty, 196, 4.2) = 0.17$

A1

1 mark

- e. $\Pr(X > m) = 0.42$
 $\Pr(X < m) = 0.58$
 $m = 196.85$

M1+A1

2 marks

f. $Pr(X \geq 2) \geq 0.8$

$$Pr(X = 0) + Pr(X = 1) \leq 0.2$$

$$0.6^n + \binom{n}{1} \times 0.4 \times (0.6)^{n-1} \leq 0.2$$

$$n = 6.4$$

At least 7 coloured light globes needs to be sold.

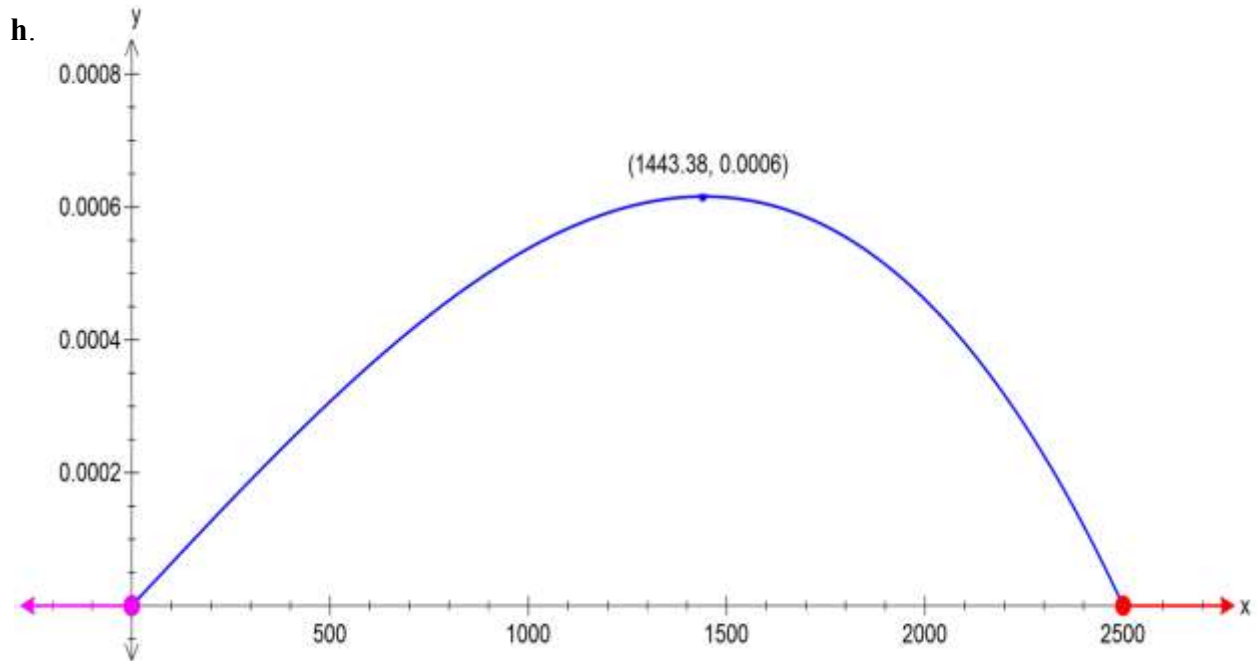
M2+A1

3 marks

g. $\int_0^{2500} \frac{1}{9.765625 \times 10^{12}} (6.25 \times 10^6 t - t^3) dt = 1$ and $f(t) \geq 0$

M1+A1

2 marks



M1+A1

2 marks

i. $\mu = \int_0^{2500} \frac{t}{9.765625 \times 10^{12}} (6.25 \times 10^6 t - t^3) dt = \frac{4000}{3}$ hours

A1

1 mark

j. $Pr\left(X > \frac{4000}{3} | X \geq 1000\right) = \frac{Pr(X > \frac{4000}{3})}{Pr(X \geq 1000)} = \frac{0.51202}{0.7056} = 0.7257$

M1+A1

2 marks

Question 3

a. average value of gradient $= \frac{1}{1.5+1.5} \int_{-1.5}^{1.5} ((x+1)^2(5-4x)) dx = 2.75$

M1+A1
2 marks

b. $f(x) = \int f'(x) dx$
 $= \int -4x^3 - 3x^2 + 6x + 5 dx$
 $= -x^4 - x^3 + 3x^2 + 5x + c$
 $f(2) = -16 - 8 + 12 + 10 + c = -2$
 $c = 0$

M1+A1
2 marks

c. $(x+1)^2(5-4x) = 0$
 $x = -1$ point of inflection – gradient is positive before and after the point.
 $x = \frac{5}{4}$ local maximum – gradient changes from positive before the point to negative after the point.

M2+A2
4 marks

d. Area $= - \int_{-1}^0 f(x) dx + \int_0^{1.91964} f(x) dx - \int_{1.91964}^2 f(x) dx = 9.21$.
 (separate integral using the x –ints).

M1+A1
2 marks

Question 4

a.

$$f(x) = \begin{cases} 2x^2 - 3x - 2, & x \leq 0 \cup x \geq \frac{3}{2} \\ -2x^2 + 3x - 2, & 0 < x < \frac{3}{2} \end{cases}$$

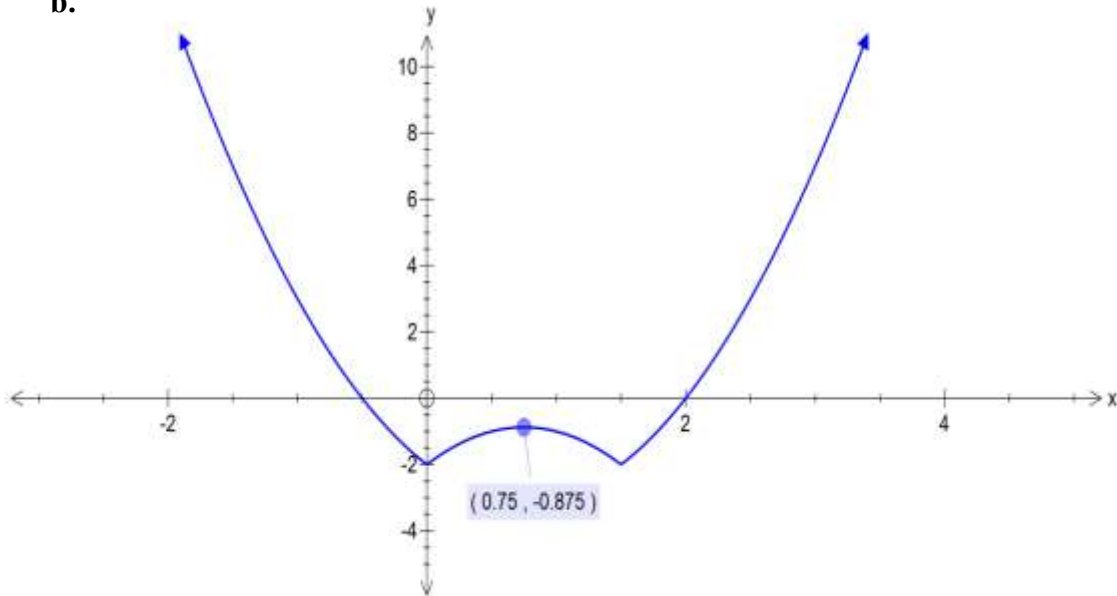
$$2x^2 - 3x - 2 = 0 \quad \text{or} \quad -(2x^2 - 3x) - 2 = 0 \Rightarrow \text{no solutions.}$$

$$(x-2)(2x+1) = 0$$

$$x = 2 \text{ or } x = -\frac{1}{2}$$

M1+A2
3 marks

b.



A2
2 marks

Question 5

a. $MS = x$

$$\triangle ANC \sim \triangle AMS$$

$$\frac{AN}{NC} = \frac{AM}{MS}$$

$$\frac{200}{100} = \frac{AM}{x}$$

$$AM = 2x \text{ mm}$$

$$\text{Area} = 2x(200 - 2x) = 400x - 4x^2 \text{ mm}^2$$

--- OR ---

Let N be the origin (0,0) then A is the point (0,200), C is the point (100,0)

And line AC has equation $y = 200 - 2x$.

Then point S has co-ordinates $(x, 200 - x)$

And the required area is $2x \times (200 - x) = 400x - 4x^2 \text{ mm}^2$

M2+A2
4 marks

b. $x \in (0, 100)$

$$\frac{dA}{dx} = 400 - 8x = 0$$

$$x = 50 \text{ mm}$$

M1+A1
2 marks

c. $A = 100 \times 100 = 10\,000 \text{ mm}^2$

A1
1 mark