

# MATHEMATICAL METHODS

## Written Examination 2



## 2016 Trial Examination

### **SOLUTIONS**

#### **Question 1**

*Answer:* D

*Explanation:*

vertical translation of 2 and horizontal translation of  $\frac{1}{2}$

#### **Question 2**

*Answer:* B

*Explanation:*

$Period = \frac{2\pi}{4} = \frac{\pi}{2}$  and  $Range = [-3 - 2, -3 + 2]$

#### **Question 3**

*Answer:* C

*Explanation:*

solve  $\left(x = \frac{1}{3}\sqrt{2-y}, y\right), \quad x \geq 0$

**Question 4**

*Answer:* E

*Explanation:*

Turning point at c so a repeated factor.

**Question 5**

*Answer:* B

*Explanation:*

Find the derivative on CAS.

**Question 6**

*Answer:* C

*Explanation:*

Equation of tangent is  $y = -6x - 7$ . Substitute  $y = 2$ .

**Question 7**

*Answer:* D

*Explanation:*

Sketch on CAS over restricted domain.

**Question 8**

*Answer:* B

*Explanation:*

$$4 + 4a + a = -1$$

**Question 9**

*Answer:* D

*Explanation:*

$$y = \frac{1}{2(x-\frac{1}{2})} \rightarrow y = \frac{4}{2(x-\frac{1}{2})} = \frac{2}{x-\frac{1}{2}} \rightarrow y = \frac{2}{x-\frac{1}{2}-\frac{1}{2}} = \frac{2}{x-1}$$

**Question 10**

*Answer:* B

*Explanation:*

$$2 \times 15 - (3 - 0) = 27$$

**Question 11**

*Answer:* A

*Explanation:*

$$\text{Binomcdf}(10, 0.3, 0, 1)$$

**Question 12**

*Answer:* E

*Explanation:*

$$1 - \frac{C(8,3)}{C(14,3)}$$

**Question 13***Answer:* D*Explanation:*

| $x$                  | 0    | 1    | 2    | 3    | 4    | 5    |
|----------------------|------|------|------|------|------|------|
| $\Pr(X = x)$         | 0.10 | 0.05 | 0.15 | 0.25 | 0.30 | 0.15 |
| $x \cdot \Pr(X = x)$ | 0    | 0.05 | 0.30 | 0.75 | 1.20 | 0.75 |

Mean = 3.05

**Question 14***Answer:* D*Explanation:*

$$\frac{1}{3} \int_1^4 \frac{1}{x^2} dx$$

**Question 15***Answer:* E*Explanation:*

$$m^2 - m - 2 \neq 0$$

**Question 16***Answer:* B*Explanation:*

$$y = 2 \cos(3x) - 1 \rightarrow y = \frac{1}{2} (2 \cos(3x) - 1) \rightarrow y = \cos\left(3x \times \frac{2}{3}\right) - \frac{1}{2} \rightarrow y = \cos(2x) + 4$$

**Question 17**

*Answer:* A

*Explanation:*

$$\sqrt{\frac{p(1-p)}{n}}, \text{ where } p = \frac{265}{300}$$

**Question 18**

*Answer:* B

*Explanation:*

$$a = 2, E(X) = \int_0^1 x^2 dx + \int_1^2 (2-x)x dx$$

**Question 19**

*Answer:* C

*Explanation:*

$$\sqrt{0.6 \times \frac{0.4}{100}} = 0.0490, \quad 0.6 + 1.96 \times 0.0490$$

**Question 20**

*Answer:* A

*Explanation:*

$$\text{Area} = 24 - \int_{-8}^1 \sqrt{-x+1} dx = 24 - \int_{-8}^1 \sqrt{-y+1} dy$$

**SECTION 2****Question 1**

a.  $(0 - 1)(0 + 0 - a) = 1 \rightarrow a = 1$

1 mark

b.  $f'(x) = 3x^2 - 2$

1 mark

c.  $y - 0 = 1(x - 1) \rightarrow y = x - 1$

2 marks

d.  $x^3 - 2x + 1 = x - 1 \rightarrow x = -2 \quad (-2, -3)$

2 marks

e.  $f(x) = 0 \rightarrow \left(\frac{-1-\sqrt{5}}{2}, 0\right)$

1 mark

f.  $Area = \int_{-2}^1 (f(x) - (x - 1))dx = \frac{27}{4}$  square units

3 marks

**Question 2**

a.  $k = 5000 \left(1 - \frac{0}{40}\right)^2 \rightarrow k = 5000$

1 mark

b.  $\frac{x}{40} + \frac{y}{5000} = 1 \rightarrow y = 5000 - 125x$

1 mark

c.  $AB = \sqrt{(0 - 5000)^2 + (40 - 0)^2} \approx 5000.16\text{m}$

2 marks

d.  $PQ = 1250 \rightarrow 1250 = 5000 \left(1 - \frac{x}{40}\right)^2 \rightarrow x = 20$   
Distance =  $20 - 4 = 16\text{m}$

2 marks

e.  $Area = \int_4^{20} 5000 \left(1 - \frac{x}{40}\right)^2 dx \approx 40266.67m^2$

2 marks

f.  $Distance = \sqrt{\left(5000 \left(1 - \frac{x}{40}\right)^2 - 0\right)^2 + (x - 0)^2}$

Solve(derivative of distance = 0) on CAS to get  $x = 38.7435$

Point on rope is (38.74, 4.93)

3 marks

### Question 3

a.  $A(0) = 0, A(2\pi) = 0$

1 mark

b.  $A'(x) = \sin(x) e^x (2 \cos(x) + \sin(x))$

$\sin(x) = 0 \rightarrow x = 0, \pi, 2\pi$

$2\cos(x) = -\sin(x) \rightarrow x = \tan^{-1}\frac{1}{2} + \frac{\pi}{2}, \tan^{-1}\frac{1}{2} + \frac{3\pi}{2}$

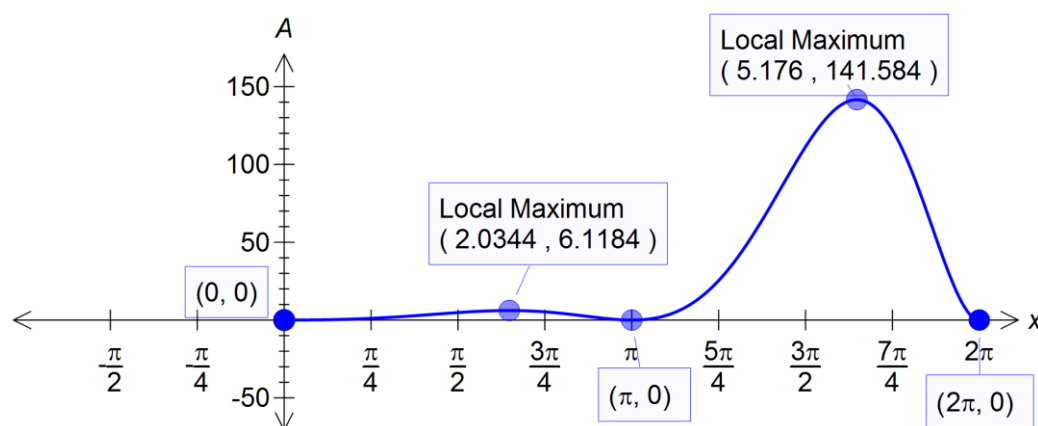
Absolute maximum at  $x = \tan^{-1}\frac{1}{2} + \frac{3\pi}{2}$

3 marks

c.  $\frac{A\left(\frac{4\pi}{3}\right) - A(\pi)}{\frac{4\pi}{3} - \pi} = \frac{9e^{\frac{4\pi}{3}}}{4\pi}$

2 marks

d.



3 marks

e. *Reflection in the  $x - axis$*

1 mark

f. Domain =  $[-2\pi, 0]$  , Range =  $[0, 141.58]$

2 marks

#### Question 4

a.  $k \int_1^3 (4x - x^2 - 3) dx = 1$

$$k \left( 2x^2 - \frac{x^3}{3} - 3x \right) \Big|_1^3 = 1$$

$$k \left( 18 - 9 - 9 - 2 + \frac{1}{3} + 3 \right) = 1 \rightarrow k = \frac{3}{4}$$

2 marks

b.  $\Pr(X > 2.5) = \int_{\frac{5}{2}}^3 \frac{3}{4} (x - 1)(3 - x) dx = \frac{5}{32}$

2 marks

c.  $\int_1^m \frac{3}{4} (x - 1)(3 - x) dx = \frac{1}{2} \rightarrow m = 0.27, 2, 3.72$

*Median = 2 kg*

2 marks

d.  $p = \int_1^{1.5} \frac{3}{4} (x - 1)(3 - x) dx = 0.15625$

$\Pr(\text{at least one small bag}) = 1 - \Pr(\text{no small bag})$

$$= 1 - C(20, 0)(0.15625)^0(1 - 0.15625)^{20} = 0.9666$$

3 marks

e.  $\text{invnorm}(0.1, 0, 1) = \frac{5 - 5.1}{\sigma}$

$$-1.28155 = -\frac{0.1}{\sigma}$$

$$\sigma = 0.0780$$

2 marks



- f.  $\text{normcdf}(5.3, \infty, 5.1, 0.0780) = 0.005172$   
Percentage is 0.5%

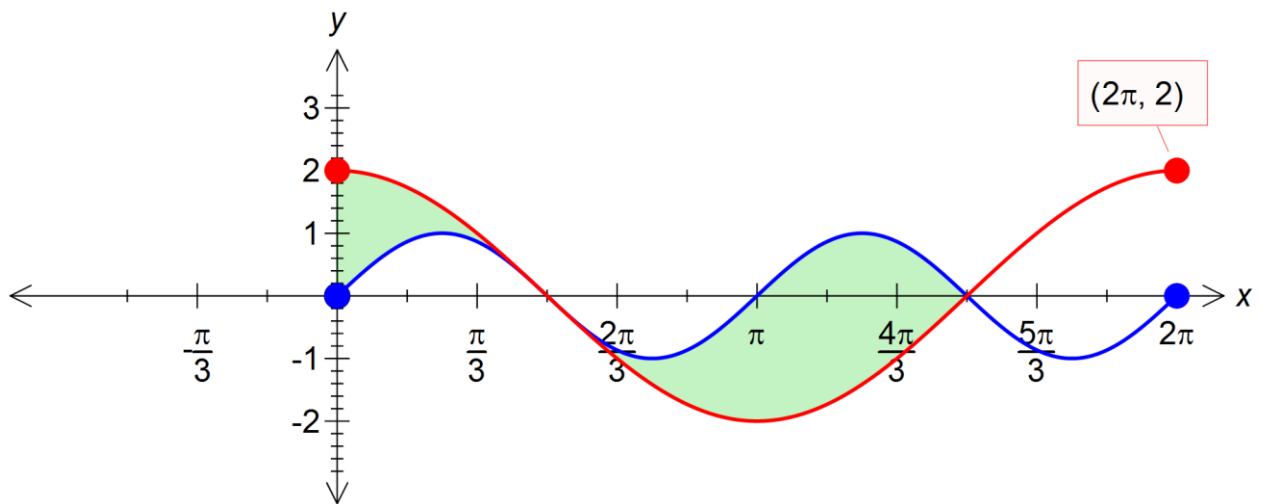
2 marks

- g.  $SD(\hat{p}) = \sqrt{0.1 \times \frac{0.9}{200}} = 0.0212$   
 $\Pr\left(Z < \frac{0.05 - 0.1}{0.0212}\right) = \Pr(Z < -2.3585) = 0.0092$

3 marks

## Question 5

a.



1 mark

- b.  $\int_0^{\frac{\pi}{2}} (2 \cos(x) - \sin(2x)) dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (\sin(2x) - 2 \cos(x)) dx$

3 marks

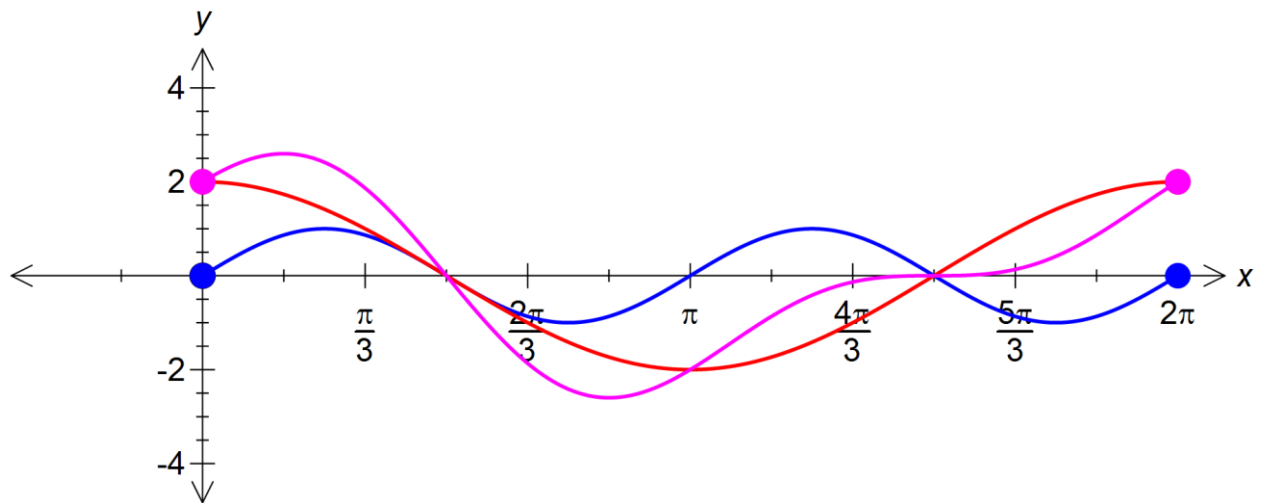
- c. Dilations of 2 units from the y-axis, dilation of 2 units from the x-axis, translation of  $\pi/2$  units in the negative direction of x-axis.

OR

Dilations of 2 units from the y-axis, dilation of 2 units from the x-axis, reflection in the y-axis and translation of  $\pi/2$  units in the positive direction of x-axis.

3 marks

d.



2 marks

- e. Solve  $\int_0^{\pi/2} (2 \cos(x) - a \sin(x)) dx = 4$  for  $a$   
 $a = -2$

2 marks