

2016 Trial Examination

STUDENT NUMBER

Figures

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Words

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Letter

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MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes

Writing time: 60 minutes

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape, notes of any kind, or a calculator of any kind.

Materials supplied

- Question and answer book of 12 pages.

Instructions

- Print your **name** in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Let $y = \frac{1}{3}(2x - 1)^6$.

Find $\frac{dy}{dx}$.

1 mark

b. Let $f(x) = \frac{\sin(x)}{x^2}$

i. Find $f'(x)$

2 marks

ii. Find $f'(\pi)$

1 mark

TURN OVER

Question 2 (4 marks)

- a. Let $f'(x) = \frac{2x-x^4}{x^2}$, for $x \neq 0$
Given that $f(1) = -2$, find $f(x)$.

3 marks

- b. Hence, find $f(e^2)$

1 mark

Question 3 (4 marks)

Consider the function $f: [0, \infty) \rightarrow R$, $f(x) = \frac{1}{2}(x-1)(k+x)^2$, where $k \in R$

The graph of $y = f(x)$ passes through the point $(2, 0)$.

- a.** Show that $k = -2$.

1 mark

The rule of the function $f(x)$ can be written as $f(x) = \frac{1}{2}(x^3 - 5x^2 + 8x - 4)$

- b.** Find the x -coordinates of the stationary points of the function.

2 marks

- c.** Find the range of the function.

1 mark

TURN OVER

Question 4 (5 marks)

Solve the following equations for x .

a. $\log_2(x) - \log_2(\sqrt{x} - 1) = 2$

3 marks

b. $e^{-2x} - 2e^{-x} = 0$

2 marks

Question 5 (3 marks)

Let the random variable X be normally distributed with mean 139.5 and standard deviation 6.2
Let Z be the standard normal random variable, such that $Z \sim N(0, 1)$.

- a. Find $\Pr(X > 151.9)$.

1 mark

- b. Find b such that $\Pr(X < b) = \Pr(Z > 1)$

2 marks

Question 6 (4 marks)

For events P and Q , $\Pr(P \cap Q) = 0.1$, $\Pr(P) = 0.3$

- a. Find $\Pr(Q)$ if P and Q are independent events.

1 mark

TURN OVER

- b.** Find $\Pr(P|Q')$, if $\Pr(Q) = 0.2$

2 marks

- c.** Explain why P and Q cannot be mutually exclusive events.

1 mark

Question 7 (3 marks)

The tangent to the curve $y = \frac{a}{x} - 1$ at the point $x = 4$ passes through the point $(3, -2)$. Find the value of a .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

TURN OVER

Question 8 (2 marks)

A transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$, is defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

Find the image of the function with the rule $f(x) = -\sqrt{x-1}$ under the above transformation.

2 marks

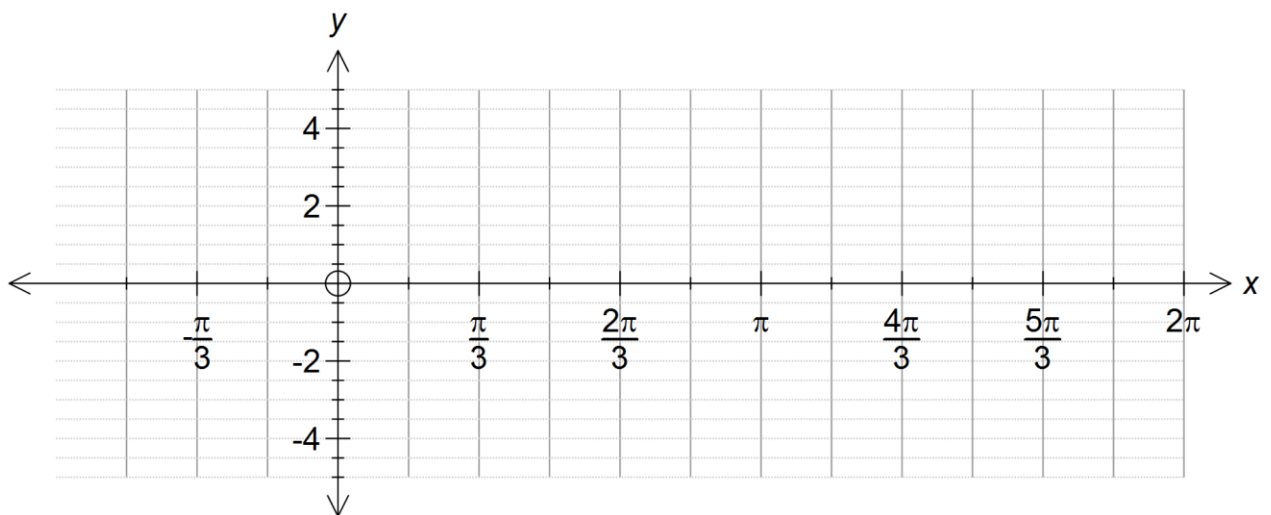
Question 9 (5 marks)

Consider the function $f: (0, 2\pi) \rightarrow \mathbb{R}$, $f(x) = 2 \sin(x) + 1$

- a.** Solve the equation $f(x) = 0$.

2 marks

- b.** Sketch the graph of $y = f(x)$ on the axes below, labelling end-points, axes intercepts and turning points.



3 marks

TURN OVER

Question 10 (6 marks)

A new packet of chips is being tested in the market. It is estimated that 60% of consumers will like the new packet of chips. A sample of 96 people tasted the new packet of chips.

- a. Find the mean of the sample proportion.

1 mark

- b. Find the standard deviation of the sample proportion.

2 marks

- c. Use $\Pr(Z \geq 2.2) = 0.0139$ to find the probability that at least 71% of consumers will indicate they like the new packet of chips.

3 marks

END OF QUESTION AND ANSWER BOOK