

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1



(TSSM's 2015 trial exam updated for the current study design)

SOLUTIONS

Question 1

a. $3 - 5x \geq 0$

$$x \leq \frac{3}{5}$$

$$\text{Domain: } \left(-\infty, \frac{3}{5}\right]$$

A1

1 mark

b. $f'(x) = \frac{1}{2}(3 - 5x)^{-\frac{1}{2}} \times -5$

$$f'(x) = -\frac{5}{2\sqrt{3-5x}}$$

M1+A1

2 marks

c. $f'\left(\frac{1}{5}\right) = -\frac{5}{2\sqrt{3-1}} = -\frac{5}{2\sqrt{2}} = -\frac{5\sqrt{2}}{4}$

A1

1 mark

Question 2

a. $\int \sin(3x) dx = -\frac{\cos(3x)}{3} + c$

$$0 = -\frac{1}{3} + c \text{ which gives } c = \frac{1}{3}$$

$$F(x) = -\frac{\cos(3x)}{3} + \frac{1}{3}$$

M2+A1

3 marks

b. $-\frac{\cos(3x)}{3} + \frac{1}{3} = \frac{1}{2}$
 $\cos(3x) = -\frac{1}{2}$
 $3x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$
 $x = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$

M2+A1

3 marks

Question 3

a. $y = \frac{x-2}{x+2}$
 $x = \frac{y-2}{y+2}$
 $yx + 2x = y - 2$
 $y(x - 1) = -2 - 2x$
 $y = \frac{2+2x}{1-x}$
 $f^{-1}(x) = \frac{2+2x}{1-x}$

M2+A1

3 marks

b. *Domain:* $\mathbb{R} \setminus \{1\}$
Range: $\mathbb{R} \setminus \{-2\}$

A2

2 marks

c. Using long division,

$$f^{-1}(x) = -2 + \frac{4}{1-x}$$

$$\int_0^{\frac{1}{2}} \left(-2 + \frac{4}{1-x}\right) dx = [-2x - 4\log_e(1-x)]_0^{\frac{1}{2}} = -1 - 4\ln\left(\frac{1}{2}\right) = -1 + 4\ln 2$$

M3+A1

4 marks

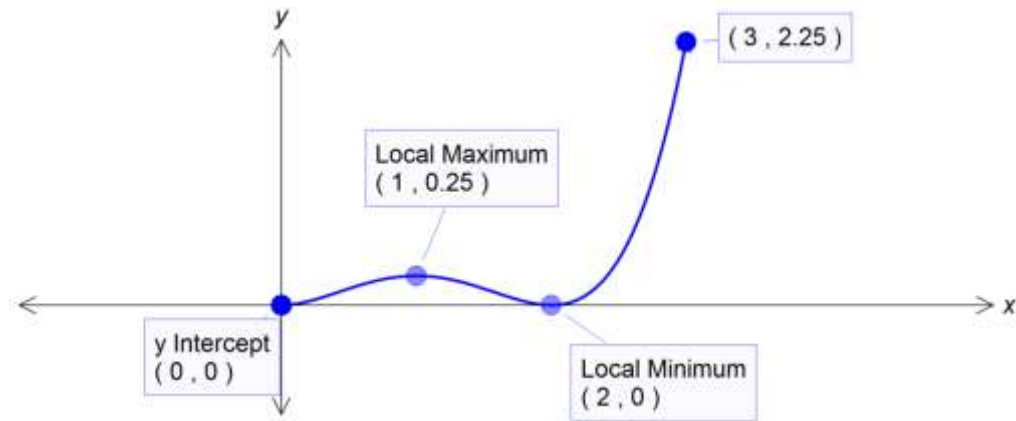
Question 4

a. $f'(x) = x^3 - 3x^2 + 2x$
 $f'(x) = 0$ gives $x(x^2 - 3x + 2) = 0$
 $x(x-2)(x-1) = 0$ gives $x = 0, 1, 2$
 $(0, 0), \left(1, \frac{1}{4}\right), (2, 0)$

M2+A1

3 marks

b.



1 for shape, 1 for stationary points, 1 for end points.

3 marks

c.
$$\text{Area} = \int_0^2 \left(\frac{1}{4}x^4 - x^3 + x^2 \right) dx = \left(\frac{x^5}{20} - \frac{x^4}{4} + \frac{x^3}{3} \right)_0^2$$

$$\text{Area} = \frac{8}{5} - 4 + \frac{8}{3} = \frac{4}{15} \text{ square units}$$

M1+A1

2 marks

Question 5

a. $4000 = 5(2 + 7^{3x})$

$$800 = 2 + 7^{3x}$$

$$798 = 7^{3x}$$

$$3x = \log_7(798)$$

$$x = \frac{1}{3} \log_7(798)$$

M1+A1

2 marks

b. $2 \times 2^{2x} + 2^x - 1 = 0$

$$2y^2 + y - 1 = 0, \text{ where } y = 2^x$$

$$(2y - 1)(y + 1) = 0$$

$$y = \frac{1}{2}, -1$$

$$2^x = \frac{1}{2}, 2^x = -1$$

$$x = -1 \text{ (} 2^x = -1 \text{ has no solution)}$$

M2+A1

3 marks

Question 6

a. $\frac{1}{5} + \frac{1}{10} + \frac{1}{3} + k = 1$
 $k = \frac{11}{30}$

A1
1 mark

b. $\Pr(X < 2) = \frac{1}{5} + \frac{1}{3} = \frac{8}{15}$

A1
1 mark

c. $\text{Mean} = \sum x\Pr(X = x) = 0 + \frac{1}{3} + \frac{1}{5} + \frac{11}{10} = \frac{49}{30}$

M1+A1
2 marks

Question 7

$$\frac{dy}{dx} = -\frac{3}{x^2}$$

$$\text{grad of tangent} = -\frac{3}{a^2}$$

$$-\frac{3}{a^2} = -9$$

$$a = \pm \frac{1}{\sqrt{3}}$$

$$a = \frac{\sqrt{3}}{3}$$

M1+A1
2 marks

Question 8

a. $\hat{p} = 0.9$

A1
1 mark

b. $M = 1.96 \sqrt{\frac{0.9 \times 0.1}{r}}$

If you double r

$$M = 1.96 \sqrt{\frac{0.9 \times 0.1}{2r}}$$

Margin of error will decrease by a factor of $\sqrt{2}$

A1
1 mark