

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1



(TSSM's 2012 trial exam updated for the current study design)

SOLUTIONS

Question 1

a. $f(x^4 + 2) = \log_e(x^4 + 2 + 3) = \log_e(x^4 + 5)$

M1+A1

2 marks

b. $f'(g(x)) = \frac{1}{(x^4+5)} \times 4x^3 = \frac{4x^3}{(x^4+5)}$

M1+A1

2 marks

c. $f'(g(-2)) = \frac{4(-2)^3}{((-2)^4+5)} = \frac{-32}{21}$

A1

1 mark

Question 2

a. $\left(\frac{5x^2}{2} - 10x\right)_0^a = 0$

$$\frac{5a^2}{2} - 10a = 0$$

$$5a(a - 4) = 0$$

$$a = 0 \text{ or } a = 4$$

Since $a \neq 0$, $a = 4$

M1+A1

2 marks

b. $\int_0^\pi \cos\left(\frac{x}{2}\right) dx = 2\sin\left(\frac{x}{2}\right)_0^\pi = 2$

M1+A1

2 marks

Question 3

a. $x = -4e^{\frac{y}{2}} + 1$

$$\frac{x-1}{-4} = e^{\frac{y}{2}}$$

$$\frac{y}{2} = \log_e\left(\frac{1-x}{-4}\right)$$

$$f^{-1}(x) = 2\log_e\left(\frac{1-x}{-4}\right)$$

M2+A1

3 marks

b. $1-x > 0$

Domain of $f^{-1}(x)$ is $(-\infty, 1)$

M1

1 mark

c. $2\log_e\left(\frac{1-x}{4}\right) = 0$

$$\left(\frac{1-x}{4}\right) = 1$$

$$\frac{1-x}{4} = 1$$

$$x = -3$$

M1+A1

2 marks

Question 4

a. $3\cos(2x) = -\frac{3\sqrt{3}}{2}$

$$\cos(2x) = -\frac{\sqrt{3}}{2} \text{ for } -\pi \leq 2x \leq \pi$$

$$2x = \frac{5\pi}{6}, -\frac{5\pi}{6}$$

$$x = \frac{5\pi}{12}, -\frac{5\pi}{12}$$

M1+A1

2 marks

b. $-3 \sin(2x) \times 2 = 0$

$$2x = 0, \pi, -\pi$$

$$x = 0, \frac{\pi}{2}, -\frac{\pi}{2}$$

M1+A1

2 marks

c. $(0, \frac{\pi}{2})$

M1

1 mark

Question 5

a. $Pr(\hat{p} = \frac{1}{4}) = Pr(1 \text{ blue ball})$

$$\begin{aligned} Pr(1 \text{ blue}) &= \frac{3}{10} \times \frac{7}{9} \times \frac{6}{8} \times \frac{5}{7} + \frac{7}{10} \times \frac{3}{9} \times \frac{6}{8} \times \frac{5}{7} + \frac{7}{10} \times \frac{6}{9} \times \frac{3}{8} \times \frac{5}{7} + \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{3}{7} \\ &= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} \\ &= \frac{1}{2} \end{aligned}$$

M1+A1

2 marks

b. $Pr(\hat{p} \geq \frac{1}{4}) = 1 - Pr(\hat{p} < \frac{1}{4}) = 1 - Pr(\text{no blue balls})$

$$Pr(\text{no blue}) = \frac{7}{10} \times \frac{6}{9} \times \frac{5}{8} \times \frac{4}{7} = \frac{1}{6}$$

$$Pr(\hat{p} \geq \frac{1}{4}) = 1 - \frac{1}{6} = \frac{5}{6}$$

M1+A1

2 marks

Question 6

$$f(1) = e^2$$

$$(1, e^2)$$

$$f'(x) = 2e^{2x}$$

$$f'(1) = 2e^2$$

$$y - e^2 = 2e^2(x - 1)$$

$$y = 2e^2x - e^2$$

M2+A1

3 marks

Question 7

$$\sin(2x) = \cos(2x)$$

$$\tan(2x) = 1$$

$$2x = \frac{\pi}{4}, \frac{5\pi}{4} \dots$$

$$2x = k\pi + \frac{\pi}{4}, k \in \mathbb{Z}$$

$$x = \frac{1}{2}(k\pi + \frac{\pi}{4})$$

$$x = \frac{\pi}{8}(4k + 1), k \in \mathbb{Z}$$

M2+A1
3 marks

Question 8

a. $\int_1^4 k(-x^2 + 5x - 4)dx = 1$

$$k \left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x \right)_1^4 = 1$$

which gives $k = \frac{2}{9}$

M1+A2
3 marks

b. $\Pr(X > 3) = \frac{2}{9} \int_3^4 (-x^2 + 5x - 4)dx$

$$= \frac{2}{9} \left(\frac{-64}{3} + 40 - 16 + \frac{27}{3} - \frac{45}{2} + 12 \right)$$

$$= \frac{7}{27}$$

A2
2 marks

Question 9

- a. $m = 200$ (95% means 2 standard deviations from the mean)

M1
1 mark

b. $\Pr(X < 210) = \Pr\left(Z < \frac{210-220}{10}\right)$
 $= \Pr(Z < -1) = \Pr(Z > 1) = 0.16$

M1+A1
2 marks

c. $\Pr(X > 230|X > 220) = \frac{\Pr(X > 230)}{\Pr(X > 220)} = \frac{\Pr(Z > 1)}{\Pr(Z > 0)} = \frac{0.16}{0.5} = \frac{16}{50} = \frac{8}{25} \text{ or } 0.32$

M1+A1
2 marks