

MATHEMATICAL METHODS

Units 3 & 4 – Written examination 1



(TSSM's 2011 trial exam updated for the current study design)

SOLUTIONS

Question 1

a. $f'(x) = \frac{-4(2-x)^3}{(2-x)^4} = \frac{4}{x-2}$

M1+A1

2 marks

b. $x = \log_e(2-y)^4$

$$x = 4\log_e(2-y)$$

$$\frac{x}{4} = \log_e(2-y)$$

$$e^{\frac{x}{4}} = 2-y$$

$$y = 2 - e^{\frac{x}{4}}$$

$$f^{-1}(0) = 1$$

M2+A1

3 marks

c. $g'(x) = 2xe^{\sin(x)} + x^2 \cos(x)e^{\sin(x)}$

$$g'(x) = 2 \times \frac{\pi}{2} \times e^{\sin(\frac{\pi}{2})} + \left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) e^{\sin(\frac{\pi}{2})} = \pi e$$

M1+A1

2 marks

Question 2

a. $\int \sqrt{x} - 2 \sin\left(\frac{\pi x}{4}\right) dx = \frac{2x^{\frac{3}{2}}}{3} + \frac{8}{\pi} \cos\left(\frac{\pi x}{4}\right) + c$ where c is a real constant (but not necessary)

M1+A1

2 marks

b. $\left[\frac{(2x-1)^4}{2 \times 4} \right]_0^2 = e^{\log_e m}$

$$\frac{81}{8} - \frac{1}{8} = m$$

$$m = 10$$

M1+A1

2 marks

Question 3

a. Amplitude = 3

$$\text{Period} = \frac{2\pi}{n} = 2\pi \times 5 = 10\pi$$

A2

2 marks

b. $\frac{-\cos(2x)}{\cos(2x)} = \frac{\sin(2x)}{\cos(2x)}$

$$\tan(2x) = -1, -\pi \leq x \leq \pi$$

$$\text{Reference } \angle: \frac{\pi}{4}$$

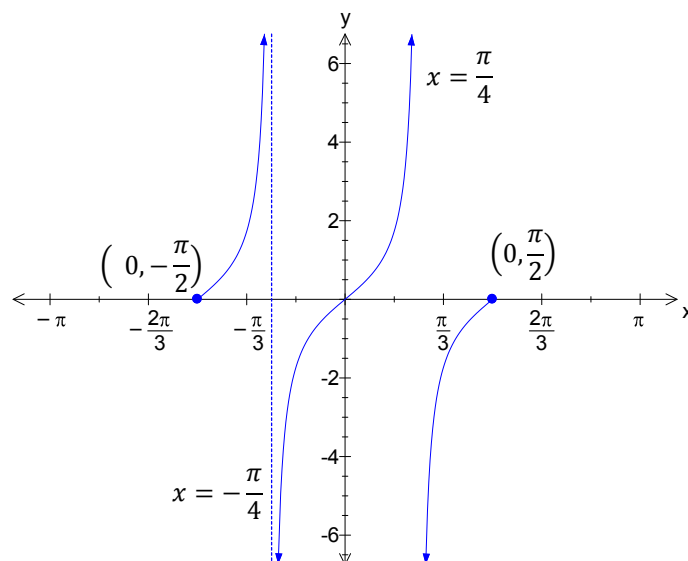
$$2x = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \text{ or } 2x = \frac{3\pi}{4} - \pi = -\frac{\pi}{4}$$

$$x = \frac{3\pi}{8} \quad x = -\frac{\pi}{8}$$

M2+A1

3 marks

c.



A2

2 marks

Question 4

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -4x + 3 \\ 2y - 1 \end{bmatrix}$$

$$x' = -4x + 3 \text{ and } y' = 2y - 1$$

$$x = \frac{3-x'}{4} \text{ and } y = \frac{y'+1}{2}$$

$$\frac{y'+1}{2} = 2 \log_e(3 - x') + 1$$

$$y' = 4 \log_e(3 - x') + 1$$

$$a = 4, b = -1, c = 3 \text{ and } d = 1$$

M2+A2

4 marks

Question 5

$$\text{Let } y = 0 \text{ for } x \text{ intercepts } \frac{1}{(x-2)^2} - 1 = 0$$

$$x = 1 \text{ or } x = 3$$

$$\text{Then find the derivative } h'(x) = \frac{-2}{(x-2)^3}$$

$$h'(1) = 2, h'(3) = -2$$

Using points (1,0) and (3,0) respectively:

We get equations $y = 2x - 2$ and $y = -2x + 6$

M2+A1

3 marks

Question 6

a. $\hat{p} = 0.18$

A1

1 mark

b. $M = \sqrt{\frac{0.18 \times 0.82}{n}}$

A1

1 mark

c. If n was halved, the value of M will increase by a factor of $\sqrt{2}$

A2

2 marks

Question 7

a. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} m \cos(x) dx = 1$

$$m[\sin(x)]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} = 1$$

$$m(-1 - 1) = 1$$

$$m = \frac{-1}{2}$$

M2+A1
3 marks

b. $Pr(X > \pi) = \int_{\pi}^{\frac{3\pi}{2}} -0.5 \cos(x) dx$

$$-0.5[\sin(x)]_{\pi}^{\frac{3\pi}{2}} = -0.5(-1) = 0.5$$

M1+A1
2 marks

Question 8

a. $Pr(X < 7) = 0.025$

$$z = \frac{x - \mu}{\sigma} = \frac{7 - 11}{2} = -2 \Rightarrow 2 \text{ standard deviations below the mean}$$

$$Pr(Z < -2) = Pr(X < 7) = 0.025$$

A1
1 mark

b. $Pr(X < 8) = Pr(X > 14) = Pr\left(Z > \frac{14 - 11}{2}\right) = Pr\left(Z > \frac{3}{2}\right), m = \frac{3}{2}$

M1+A1
2 marks

Question 9

a.
$$\frac{-2}{x + 2} - \frac{-2x + 3}{-2x - 4}$$

$$\frac{3 - 2x}{x + 2} = \frac{7}{x + 2} - 2$$

M1+A1
2 marks

b. $\int \frac{3 - 2x}{x + 2} dx = \int \frac{7}{x + 2} - 2 dx = 7 \log_e |x + 2| - 2x + c$

A1
1 mark