



Quality Assessment Tasks

NAME: _____

UNITS 3 & 4 Practice Examination

VCE[®] Mathematical Methods

Written examination 2

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
A	20	20	20
B	5	5	60
			80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- A question and answer book of 22 pages.
- A double sided page of formulas.
- An answer sheet for multiple-choice questions.

Instructions

- Write your name in the space provided above on this page.
- Write your name on the multiple-choice answer sheet.
- Unless otherwise indicated the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At end of Examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A – Multiple-choice questions**Instructions for Section A**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1; an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

The point $P(1,2)$ lies on the graph $y = f(x)$ of a function f . The graph is translated two units along the positive direction of the x -axis and then reflected in the x -axis.

The coordinates of the final image of P are

- A. $(-1,4)$
- B. $(-1,0)$
- C. $(-1,-2)$
- D. $(3,-2)$
- E. $(3,2)$

Question 2

The linear function $f: D \rightarrow R, f(x) = 2 - x$ has range $(-2,4]$.

The domain D of the function is

- A. $(-2,4]$
- B. $[-2,4)$
- C. $(4,-2]$
- D. $[4,-2)$
- E. $(-4,8]$

Question 3

The function with rule $f(x) = -2 \tan(\pi x)$ has period

- A. -2
- B. 1
- C. 2
- D. π^2
- E. $2\pi^2$

Question 4

If $x + a$ is a factor of $5x^3 + 3x^2 - 7ax$, where $a \in \mathbb{R} \setminus \{0\}$, then the value of a is

- A. -2
- B. $-\frac{4}{5}$
- C. $\frac{4}{5}$
- D. 1
- E. 2

Question 5

The function $g: [-a, a] \rightarrow \mathbb{R}, g(x) = \sin\left(2\left(x - \frac{\pi}{12}\right)\right)$ has an inverse function.

The maximum possible value of a is

- A. $\frac{\pi}{6}$
- B. $\frac{\pi}{3}$
- C. $\frac{5\pi}{12}$
- D. $\frac{7\pi}{12}$
- E. $\frac{13\pi}{12}$

Question 6

If $\int_2^5 f(x)dx = 4$, then $\int_2^5 (3f(x) - 2)dx$ is equal to

- A. -2
- B. 2
- C. 6
- D. 9
- E. 10

Question 7

For events A and B , $\Pr(A \cap B) = p$, $\Pr(A' \cap B) = p - \frac{2}{3}$ and $\Pr(A \cap B') = \frac{p}{2}$.

If A and B are independent, then the value of p is

- A. 0
- B. $\frac{1}{4}$
- C. $\frac{1}{2}$
- D. $\frac{2}{3}$
- E. 1

Question 8

Which one of the following functions satisfies the functional equation $f(f(x)) = x$ for every real number x ?

- A. $\sqrt{1-x}$
- B. $\sqrt{1-x^2}$
- C. $\sqrt[3]{1-x}$
- D. $\sqrt[3]{1-x^2}$
- E. $\sqrt[3]{1-x^3}$

Question 9

A bag contains three red marbles, one black and three blue marbles.

Two marbles are drawn from the bag, **without replacement**, and the results are recorded.

The probability of only one red ball may be calculated using

- A. $\frac{3}{7} \times \frac{4}{7}$
- B. $\frac{3}{7} \times \frac{4}{6}$
- C. $2 \times \frac{3}{7} \times \frac{4}{7}$
- D. $2 \times \frac{3}{7} \times \frac{4}{6}$
- E. $\frac{2!}{1!1!} \left(\frac{3}{7}\right) \left(\frac{4}{7}\right)$

Question 10

The transformation

$$T: R^2 \rightarrow R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

maps the line with equation $2x - y = 1$ onto the line with equation

- A. $y = -2 + x$
- B. $x + y = -2$
- C. $-x + y = 2$
- D. $x + y = 2$
- E. $x - y = 2$

Question 11

If the tangent to the graph of $y = e^{\frac{x}{a}}, a \neq 0$, at $x = c$ passes through the origin, then c is equal to

- A. 0
- B. $\frac{1}{a}$
- C. 1
- D. a
- E. $-\frac{1}{a}$

Question 12

The simultaneous linear equations $ax - 2y = 4 - a$ and $2x - ay = a$ have **no solution** for

- A. $a = -2$
- B. $a = 2$
- C. both $a = 2$ and $a = -2$
- D. $a \in R \setminus \{-2\}$
- E. $a \in R \setminus \{2\}$

Question 13

It is known that 13% of Australian adults prefer white chocolate to dark chocolate.

If a random sample of ten Australian adults is taken, the probability, correct to four decimal places, that more than half of them prefer **dark chocolate** is

- A. 0.9947
- B. 0.5000
- C. 0.4336
- D. 0.0260
- E. 0.0006

Question 14

In a random sample of 900 voters across the country, 52% indicated a preference to vote for the major opposition party at the next federal election.

An approximate 99.7% confidence interval for the proportion of the total voting population with a preference to vote for the major opposition party can be found by evaluating

A. $(52 - 0.15\sqrt{52 \times 48 \times 900}, 52 + 0.15\sqrt{52 \times 48 \times 900})$

B. $\left(52 - 1.96\sqrt{\frac{52 \times 48}{900}}, 52 + 1.96\sqrt{\frac{52 \times 48}{900}} \right)$

C. $\left(0.52 - 1.96\sqrt{\frac{0.52 \times 0.48}{900}}, 0.52 + 1.96\sqrt{\frac{0.52 \times 0.48}{900}} \right)$

D. $\left(52 - 3\sqrt{\frac{52 \times 48}{900}}, 52 + 3\sqrt{\frac{52 \times 48}{900}} \right)$

E. $\left(0.52 - 3\sqrt{\frac{0.52 \times 0.48}{900}}, 0.52 + 3\sqrt{\frac{0.52 \times 0.48}{900}} \right)$

Question 15

The cubic function $f(x) = ax^3 + bx^2 + cx$, has **no** stationary points when

A. $b^2 - 3ac = 0$

B. $b^2 - 3ac \neq 0$

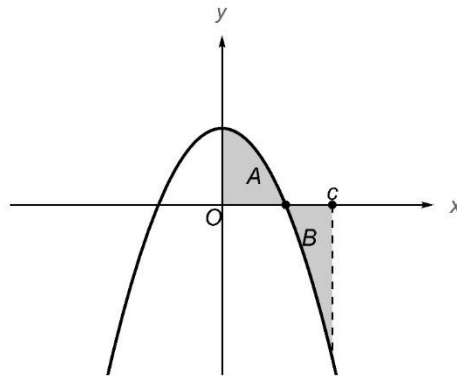
C. $c < \frac{b^2}{3a}$ and $a < 0$

D. $c < \frac{b^2}{3a}$ and $a > 0$

E. $b^2 - 3ac > 0$

Question 16

A part of the graph of $g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = 9 - x^2$ is shown below.



The area of the region labelled A is the same as the area of the region labelled B .
The exact value of the positive real number c is

- A. 3
- B. $3\sqrt{3}$
- C. 6
- D. 9
- E. 18

Question 17

A cubic function has the rule $y = f(x)$. The graph of the derivative function f' crosses the x -axis at $(-2, 0)$ and $(3, 0)$. The minimum value of the derivative function is -2 .

The value of x for which the graph of $y = f(x)$ has a local maximum is

- A. -2
- B. 2
- C. -3
- D. 3
- E. $-\frac{1}{2}$

Question 18

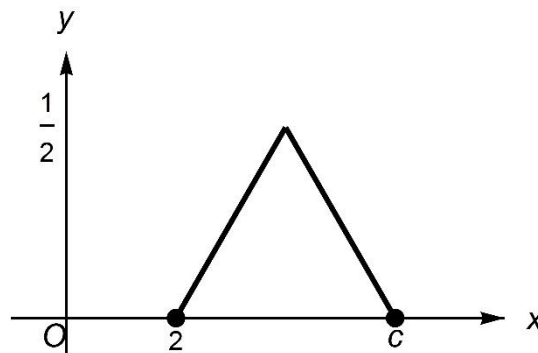
The time T days required to process insurance claims at a large insurance company is a normally distributed random variable with a mean of 90 days and a standard deviation of σ days.

Of the last 1000 claims, 110 were processed within 70 days. The value of σ is closest to

- A. 10
- B. 12
- C. 14
- D. 16
- E. 18

Question 19

The graph of a probability density function of a continuous variable, X , is shown below.

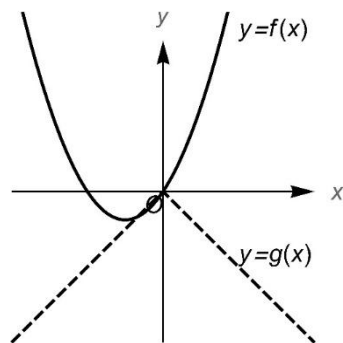


If $c > 2$, and the mode occurs at the middle of the interval $[2, c]$, then $E(X)$ is equal to

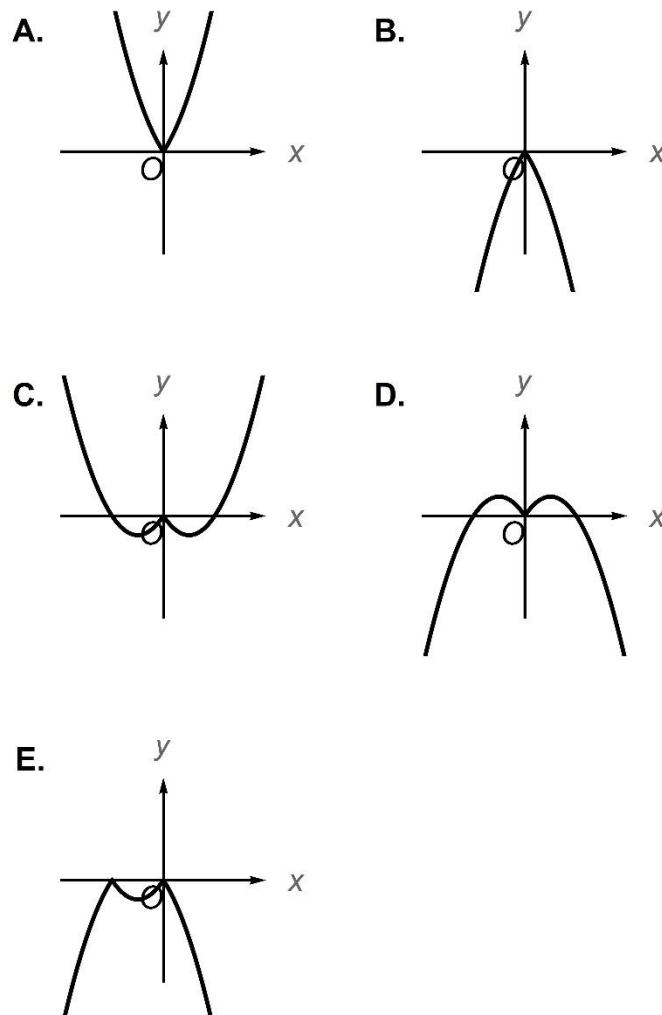
- A. 3
- B. $\frac{7}{2}$
- C. 4
- D. $\frac{9}{2}$
- E. 5

Question 20

The graphs of $y = f(x)$ and $y = g(x)$ are shown below.



The graph of $y = f(g(x))$ is best represented by



SECTION B**Instructions for Section B**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (7 marks)

The temperature, T ($^{\circ}\text{C}$), inside a climatically controlled greenhouse, t hours after midnight, is given by the rule

$$T(t) = 20 - 4 \cos\left(\frac{\pi t}{12}\right).$$

- a.** Find the period and amplitude of the function T . 2 marks

- b.** Find the maximum and minimum temperature of the greenhouse. 2 marks

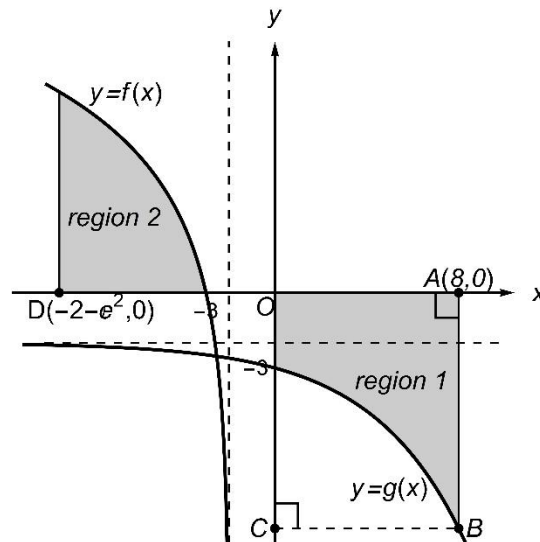
- c.** Find $T(8)$. 1 mark

- d.** Find the fraction of one period when the temperature of the greenhouse is at least as warm as $T(8)$. 2 marks

Question 2 (7 marks)

Let $f: (-\infty, -3) \rightarrow \mathbb{R}, f(x) = 4 \log_e(-x - 2)$ and $g: \mathbb{R} \rightarrow \mathbb{R}, g(x) = -e^{x/4} - 2$.

The graphs $y = f(x)$ and $y = g(x)$ are shown in the diagram below.



a. Express the rule for each composite function in its simplest form:

i. $f(g(x))$

1 mark

ii. $g(f(x))$

1 mark

b. Find the area of the rectangle $OABC$ in the figure above.

1 mark

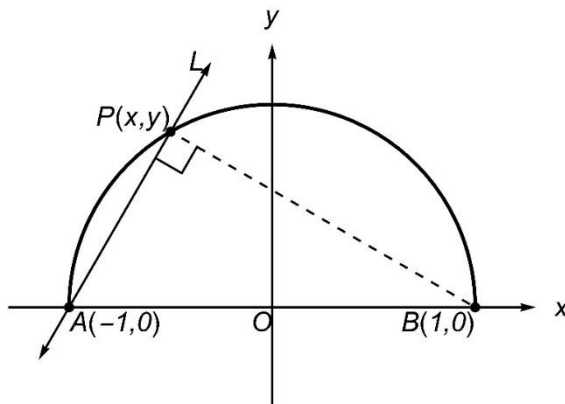
- c. Evaluate the integral $\int_0^8 g(x) dx$. 1 mark

- d. Find the area of region 1 (shaded) in the figure above. 1 mark

- e. **Hence**, find the area of region 2 (shaded) in the figure above. 2 marks

Question 3 (11 marks)

The points A, B and P are on the circumference of a semicircle of the unit radius, as shown in the figure below. The line L with gradient m , and equation $y = m(x + 1)$ passes through point A and point P .



- a. Solve the quadratic equation $x^2 + m^2(x + 1)^2 = 1$ to express x in terms of m . 1 mark

- b. Hence show that point P has coordinates $P\left(\frac{1-m^2}{1+m^2}, \frac{2m}{1+m^2}\right)$. 3 marks

- c. Show that $\triangle APB$ is a right angled triangle. 2 marks

- d. Express the area of $\triangle APB$ in terms of m .

2 marks

- e. Find $\frac{d}{dm} \left(\frac{2m}{1+m^2} \right)$ in simplest terms.

1 mark

- f. Hence, find the value of m so that the area of $\triangle APB$ is a maximum, and state the value of this maximum area. Justify your choice of m in the context of the current problem.

2 marks

Question 4 (17 marks)

Let $f: R \rightarrow R, f(x) = (x - a)(x - b)(x - c)$, where a, b and c are real constants such that $a < b < c$

- a.** State the equation of the tangent line to the graph of $y = f(x)$ at the point with x –coordinate equal to $\frac{a+b}{2}$. Express your answer in a fully factored representation.

2 marks

- b.** Find x –intercept of the tangent line from part **a**.

1 mark

- c.** State the equation of the tangent line to the graph of $y = f(x)$ at the point with x –coordinate equal to $\frac{b+c}{2}$. Express your answer in a fully factored representation.

2 marks

- d.** Find the x –intercept of the tangent line from part **c**.

1 mark

- e. Use algebra to show that $b = \frac{a+c}{2}$ whenever the tangent line in part **b** is parallel to the tangent line in part **d**. 3 marks

In the remainder of this question, $b = \frac{a+c}{2}$.

Let $g: R \rightarrow R, g(x) = x + b$.

- f. Find the **rule** for the composite function $h(x) = f(g(x))$.
Express your answer in a factored form $h(x) = x(x - u)(x + u)$ where u is a suitably defined function of the parameters a and c . 2 marks

- g. Verify that $h(x)$ is a solution of the functional equation $k(x) + k(-x) = 0$. 2 marks

- h. Find the coordinates of the point of inflection of the graph of $y = h(x)$. 3 marks

- i. **Hence**, find the coordinates of the point of inflection of the graph of $y = f(x)$ when

$$b = \frac{a+c}{2}.$$

1 mark

Question 5 (18 marks)

Recent international medical research has revealed that individuals across the world with a particular gene have a $\frac{3}{5}$ probability of developing a specific disease.

- a.** A random sample of 24 gene carriers has been collected. Let X be the random variable that represents the number of these carriers who eventually develop the disease.

- i.** Find $\Pr(X \geq 12)$ correct to four decimal places. 2 marks

- ii.** Find $\Pr(X \geq 18 | X \geq 12)$ correct to three decimal places. 3 marks

For samples of 24 gene carriers, $\hat{P}$ is the random variable of the distribution of sample proportions for the people who develop the disease.

- iii. Find the expected value and variance of $\hat{P}$. 3 marks

- iv. Find the probability that a sample proportion lies within two standard deviations of $\frac{3}{5}$.
Give your answer correct to three decimal places. Do not use a normal approximation. 3 marks

- v. Find $\Pr\left(\hat{P} \geq \frac{3}{4} \mid \hat{P} \geq \frac{3}{5}\right)$. Give your answer correct to three decimal places.
Do not use a normal approximation. 2 marks

- b.** The age (in years) when symptoms are first experienced by a sufferer of this disease is modelled by a random variable W , with a probability density function $g(w)$, given by the following hybrid function

$$g(w) = \begin{cases} 0 & w < 1 \\ -\frac{12(w-35)(w-1)}{99127} & 1 \leq w \leq 18 \\ -\frac{4(w-87)^3}{37559529} & 18 < w \leq 87 \\ 0 & w > 87 \end{cases}$$

- i.** Find $E(W)$ correct to four decimal places. 2 marks

- ii.** In a random sample of 200 disease sufferers, how many would be expected to be older than 18 years of age? Give your answer to the nearest integer. 2 marks

- c.** From a random sample of 100 disease sufferers, it was found that 40 of these individuals developed symptoms of the disease before their 18th birthday.

State an approximate 95% confidence interval for the population proportion corresponding to this sample proportion. Express your values correct to three decimal places. 1 mark

Multiple Choice Answer Sheet

Student Name:

Shade the letter that corresponds to each correct answer.

Question	A	B	C	D	E
Question 1	()	()	()	()	()
Question 2	()	()	()	()	()
Question 3	()	()	()	()	()
Question 4	()	()	()	()	()
Question 5	()	()	()	()	()
Question 6	()	()	()	()	()
Question 7	()	()	()	()	()
Question 8	()	()	()	()	()
Question 9	()	()	()	()	()
Question 10	()	()	()	()	()
Question 11	()	()	()	()	()
Question 12	()	()	()	()	()
Question 13	()	()	()	()	()
Question 14	()	()	()	()	()
Question 15	()	()	()	()	()
Question 16	()	()	()	()	()
Question 17	()	()	()	()	()
Question 18	()	()	()	()	()
Question 19	()	()	()	()	()
Question 20	()	()	()	()	()

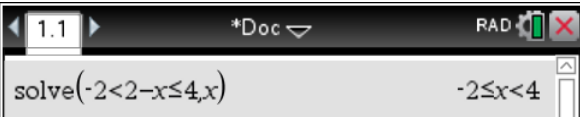
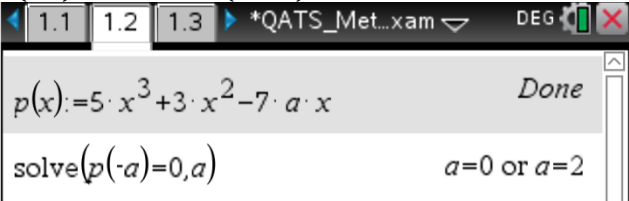
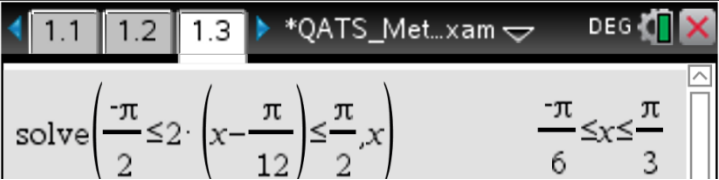
Solution Pathway

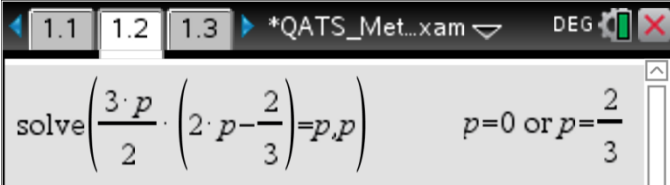
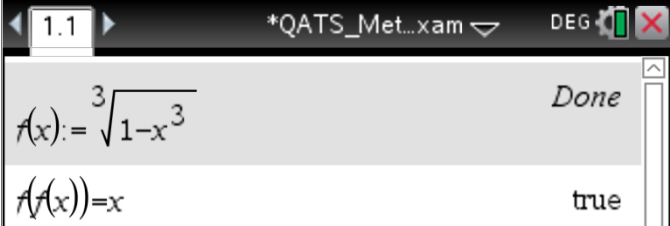
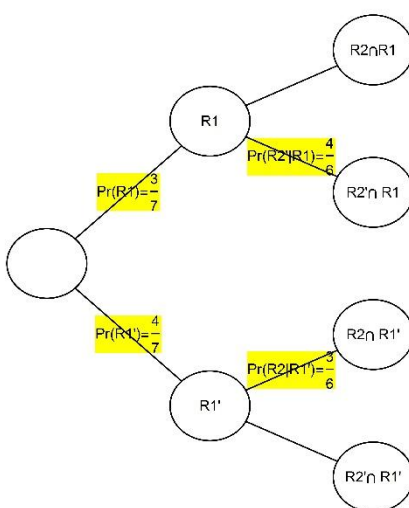
Below are sample answers. Please consider the merit of alternative responses.

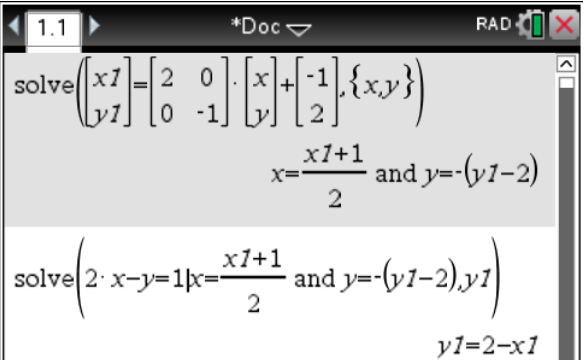
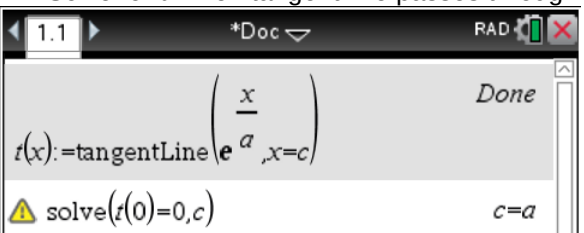
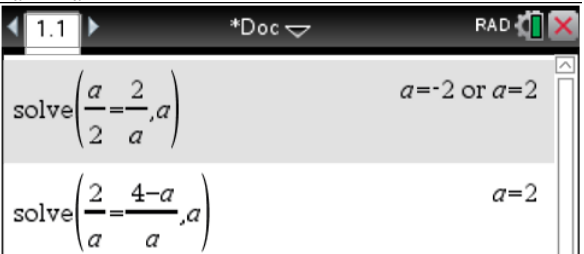
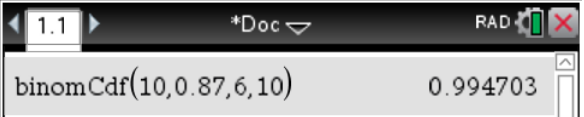
Section A: Multiple-choice answers

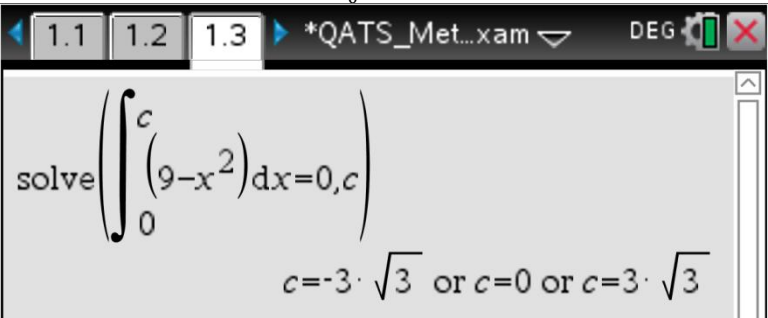
1.	D	6.	C	11.	D	16.	B
2.	B	7.	D	12.	A	17.	A
3.	B	8.	E	13.	A	18.	D
4.	E	9.	D	14.	E	19.	C
5.	A	10.	D	15.	C	20.	C

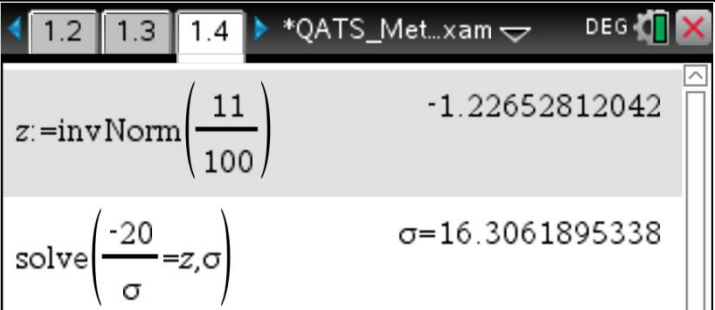
Section A: Multiple-choice solutions

MC1	$(x, y) \rightarrow (x + 2, -y) \Rightarrow (1, 2) \rightarrow (3, -2)$	Answer D
MC2	Solve $-2 < 2 - x \leq 2$ for x. 	Answer B
MC3	$f(x) = a \tan(nx)$ has period $T = \frac{\pi}{n}$ So $f(x) = -2 \tan(\pi x)$ has period $T = \frac{\pi}{\pi} = 1$	Answer B
MC4	If $(x + a)$ is a factor of $P(x)$ then by Factor Theorem $P(-a) = 0 \Rightarrow -5a^2(a - 2) = 0$  $\therefore a = 2$ since $a \neq 0$.	Answer E
MC5	$y = \sin(\theta)$ is one-one for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ Solve $-\frac{\pi}{2} \leq 2\left(x - \frac{\pi}{12}\right) \leq \frac{\pi}{2}$ for $x \Rightarrow -\frac{\pi}{6} \leq x \leq \frac{\pi}{3}$.  $\therefore a = \frac{\pi}{6}$	Answer A
MC6	$\int_2^5 (3f(x) - 2)dx = 3 \int_2^5 f(x)dx - 2 \int_2^5 dx = 3 \times 4 - 2 \times (5 - 2) = 6$	Answer C

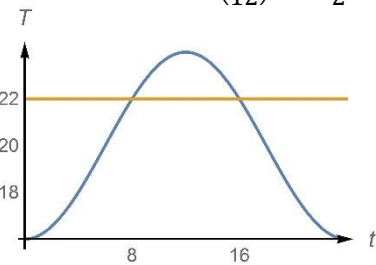
MC7	Using a 2-way table <table><tr><td></td><td>A</td><td>A'</td><td></td></tr><tr><td>B</td><td>p</td><td>$p - \frac{2}{3}$</td><td>$2p - \frac{2}{3}$</td></tr><tr><td>B'</td><td>$\frac{p}{2}$</td><td></td><td></td></tr><tr><td></td><td>$\frac{3p}{2}$</td><td></td><td>1</td></tr></table> $\Pr(A \cap B) = \Pr(A) \times \Pr(B) \Rightarrow \frac{3p}{2} \left(2p - \frac{2}{3} \right) = p$  $\therefore p = \frac{2}{3}$ only since $p \geq \frac{2}{3}$.		A	A'		B	p	$p - \frac{2}{3}$	$2p - \frac{2}{3}$	B'	$\frac{p}{2}$				$\frac{3p}{2}$		1	Answer D
	A	A'																
B	p	$p - \frac{2}{3}$	$2p - \frac{2}{3}$															
B'	$\frac{p}{2}$																	
	$\frac{3p}{2}$		1															
MC8	$f(x) = \sqrt[3]{1-x^3}$ $f(f(x)) = \sqrt[3]{1-(\sqrt[3]{1-x^3})^3} = \sqrt[3]{x^3} = x$ 	Answer E																
MC9		Answer D																

	From tree diagram $\Pr((R_1 \cap R'_2) \cup (R'_1 \cap R_2)) = \frac{3}{7} \times \frac{4}{6} + \frac{4}{7} \times \frac{3}{6} = 2 \times \frac{3}{7} \times \frac{4}{6}$	
MC10	1. Find inverse transformation by solving the matrix equation for old x and old y in terms of new x and new y. 2. Then substitute into original equation $2x - y = 1$. 3. Finally, solve for new y in terms of new x to find the image of the original equation. 	Answer D
MC11	1. Find the equation of the tangent line at the general value $x = c$ 2. Solve for c when tangent line passes through $(0,0)$. 	Answer D
MC12	For no solutions, the lines must be parallel, so they must have same slope. Same slope $\Rightarrow \frac{a}{2} = \frac{-2}{-a}$. For infinite solutions, system must be consistent with the same y -int $\Rightarrow \frac{-2}{-a} = \frac{4-a}{a}$.  So, when $a = 2$ the system of equations is consistent, with all points on either line a solution. $\therefore a = -2$ for an inconsistent system with no solutions.	Answer A
MC13	$X \sim Bi\left(n = 10, p = \frac{13}{100}\right)$ $\Pr(X \geq 6) \approx 0.9947$ 	Answer A

MC14	If $1 - \alpha = 0.997 = \Pr\left(-z_{1-\frac{\alpha}{2}} < Z < z_{1-\frac{\alpha}{2}}\right) \Rightarrow z_{1-\frac{\alpha}{2}} = 3$ Sub $\hat{p} = 0.52, z_{1-\alpha/2} = 3, n = 900$ into $(1 - \alpha)\% \text{ C.I.} \approx \left(\hat{p} - z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}, \hat{p} + z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \right)$ Gives $99.7\% \text{ C.I.} \approx \left(0.52 - 3 \sqrt{\frac{0.52 \times 0.48}{900}}, 0.52 + 3 \sqrt{\frac{0.52 \times 0.48}{900}} \right)$	Answer E
MC15	For stationary points $f'(x) = 3ax^2 + 2bx + c = 0$ If no stationary points, $\Delta \equiv (2b)^2 - 4(3a)c < 0$ $3ac > b^2$ $\therefore c < \frac{b^2}{3a} \text{ and } a < 0$	Answer C
MC16	When $x = c$ the signed area $\int_0^c (9 - x^2) dx = 0$  $c = 3\sqrt{3}$	Answer B
MC17	The derivative is a concave up parabola with zeros at $x = -2$ and $x = 3$. The sign diagram of the derivative is  By the first derivative test, $x = -2$ is a local maximum.	Answer A
MC18	$T \sim N(\mu = 90, \sigma)$ $\Pr(T \leq 70) \approx \frac{110}{1000}$ $\Rightarrow \Pr\left(Z \leq \frac{70 - 90}{\sigma}\right) \approx \frac{11}{100}$ $\Pr(Z \leq -1.2265) \approx \frac{11}{100}$ Upon solving $\frac{70-90}{\sigma} = -1.2265$ for σ gives $\sigma \approx 16$	Answer D

		
MC19	Mode in middle of interval $[2, c] \Rightarrow$ pdf symmetric about $x = \frac{c+2}{2}$ $\therefore \text{mean} = \text{median} = \text{mode} = \frac{c+2}{2}$ To find c, express total probability in terms of c. That is, $\frac{1}{2}(c-2) \times \frac{1}{2} = 1 \Rightarrow c = 6$. $\therefore E(X) = \frac{c+2}{2} = 4$	Answer C
MC20	$g(x) = \begin{cases} -x & x \geq 0 \\ x & x < 0 \end{cases}$ $\Rightarrow f(g(x)) = \begin{cases} f(-x) & x \geq 0 \\ f(x) & x < 0 \end{cases}$ $\therefore$ the left side of the graph is reflected about the y – axis	Answer C

Section B: Extended Answer Solutions**Question 1 (7 marks)**

(a)	Amplitude = 4 Period = $\frac{2\pi}{\left(\frac{\pi}{12}\right)} = 24$	(A1)
(b)	$\cos\left(\frac{\pi t}{12}\right) = -1 \Rightarrow T = 24$ is max $\cos\left(\frac{\pi t}{12}\right) = 1 \Rightarrow T = 16$ is min	(A1)
(c)	$T(8) = 20 - 4 \cos\left(\frac{\pi}{12} \times 8\right) = 22$	(A1)
(d)	$T(t) \geq T(8) \Rightarrow \cos\left(\frac{\pi t}{12}\right) \leq -\frac{1}{2}$  $\therefore T(t) \geq T(8)$ for $\frac{1}{3}$ of a period.	(M1)
		(A1)

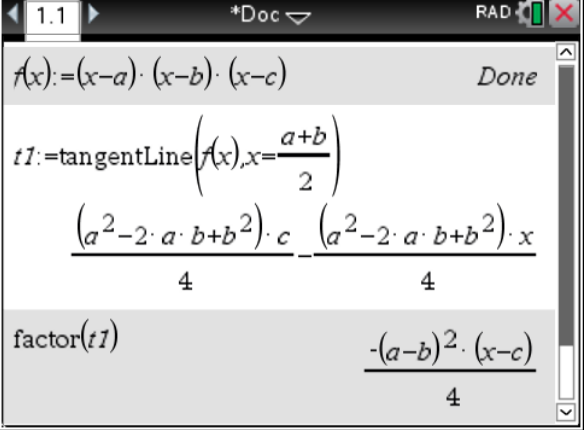
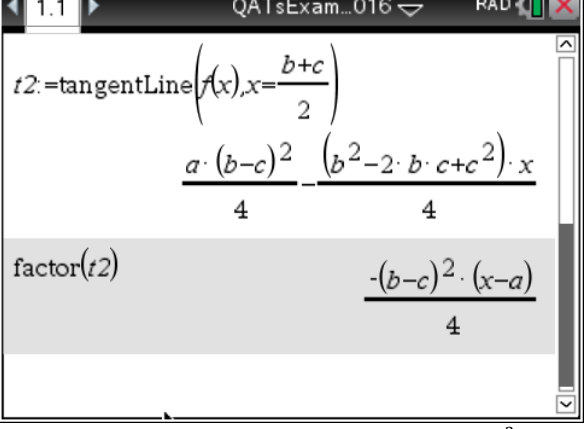
Question 2 (7 marks)

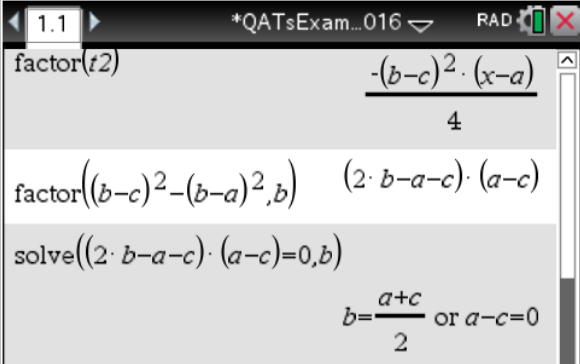
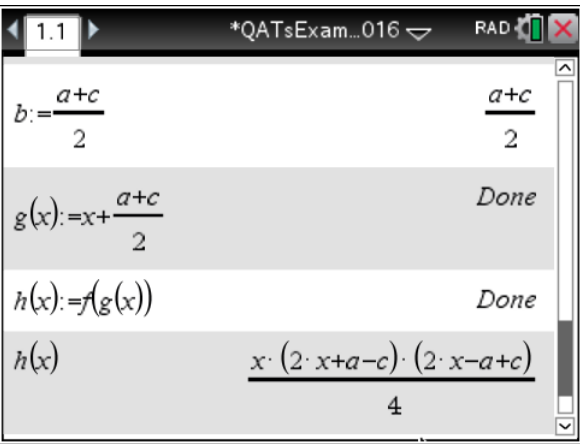
(a)i	$f(g(x)) = x$	(A1)
(a)ii	$g(f(x)) = x$	(A1)
(b)	$\text{area } OABC = 8 \times (0 - g(8)) = 8(2 + e^2)$	(A1)
(c)	$\int_0^8 g(x) dx = -4(3 + e^2)$	(A1)
(d)	$\text{area region 1} = - \int_0^8 g(x) dx = 4(3 + e^2)$	(A1)
(e)	$\begin{aligned} \text{area region 2} &= \text{area } OABC - \text{area region 1} \\ &= 4(1 + e^2) \end{aligned}$	(M1) (A1)

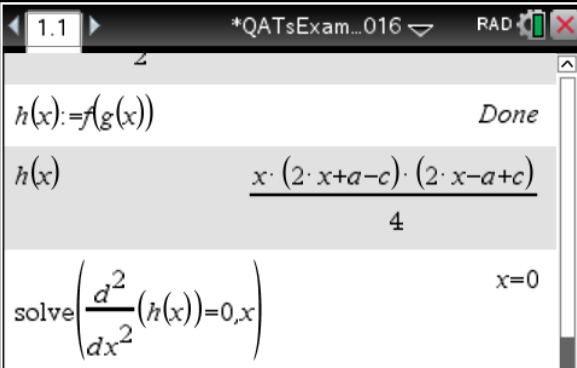
Question 3 (11 marks)

(a)	$x = -1 \text{ or } x = \frac{1 - m^2}{1 + m^2}$	(A1)
(b)	Substitute $y = m(x + 1)$ in $x^2 + y^2 = 1$ gives $x^2 + m^2(x + 1)^2 = 1$. Substitute $x = \frac{1 - m^2}{1 + m^2}$ in $y = m(x + 1)$ gives $y = m \left(1 + \frac{1 - m^2}{1 + m^2} \right) = \frac{2m}{1 + m^2}$	(M1) (M1) (M1)
(c)	$\text{slope } PB = \frac{0 - \frac{2m}{1 + m^2}}{1 - \frac{1 - m^2}{1 + m^2}} = -\frac{1}{m}$ $\text{slope } AP \times \text{slope } PB = -1 \Rightarrow \angle APB = \frac{\pi}{2}$	(M1) (M1)
(d)	$\begin{aligned} \text{area of } \triangle APB &= \frac{1}{2} \times AB \times y - \text{coordinate at } P \\ &= \frac{2m}{1 + m^2} \end{aligned}$	(M1) (A1)
(e)	$\frac{d}{dm} \left(\frac{2m}{1 + m^2} \right) = \frac{2(1 - m^2)}{(1 + m^2)^2}$	(A1)
(f)	$\frac{d}{dm} \left(\frac{2m}{1 + m^2} \right) = 0 \Rightarrow m = 1 \text{ since } m > 0$ $\therefore \text{max area of } \triangle APB = 1$	(A1) (A1)

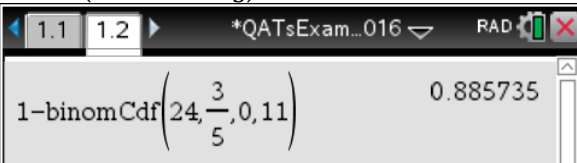
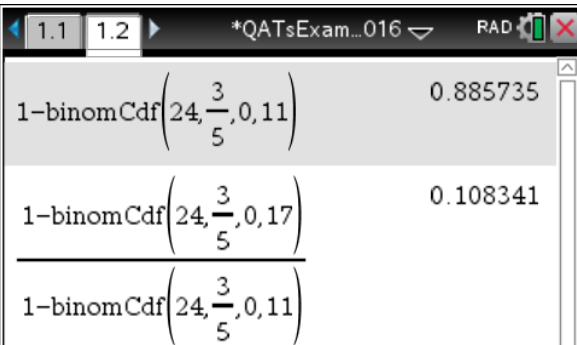
Question 4 (17 marks)

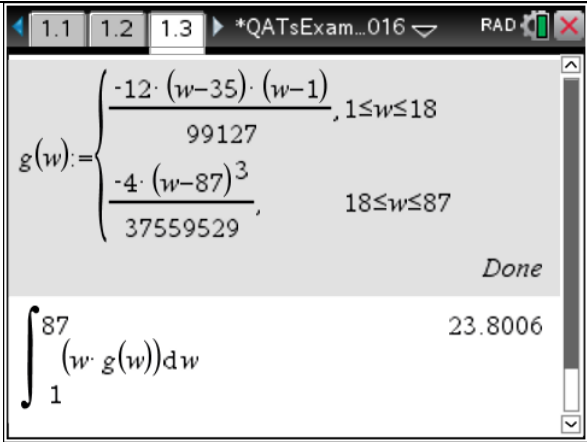
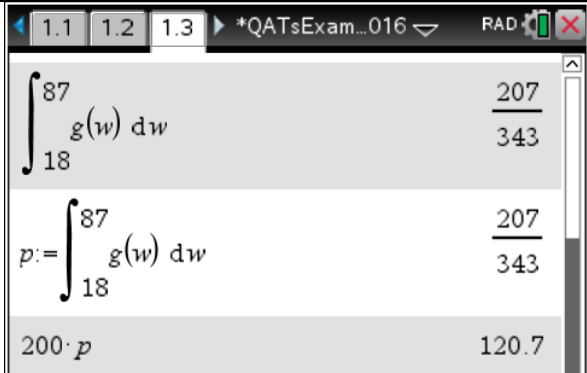
(a)	 Tangent line at $x = \frac{a+b}{2}$ has equation $y = \frac{(a^2 - 2ab + b^2)c}{4} - \frac{a^2 - 2ab + b^2}{4}x$ Factoring then gives $y = \frac{(a-b)^2(c-x)}{4}$.	(A1) (or equivalent representation) (A1) fully factored
(b)	At the $x - int, y = 0 \Rightarrow \frac{(a-b)^2(c-x)}{2} = 0$ Solving for x then gives $x = c$.	(A1)
(c)	 Tangent line at $x = \frac{b+c}{2}$ has equation $y = \frac{(b^2 - 2bc + c^2)a}{4} - \frac{(b^2 - 2bc + c^2)}{4}x$ Factoring then gives $y = \frac{(b-c)^2(a-x)}{4}$.	(A1) (or equivalent representation) (A1) fully factored
(d)	At the $x - int, y = 0 \Rightarrow \frac{(b-c)^2(a-x)}{2} = 0$ Solving for x then gives $x = a$.	(A1)
(e)	If tangent at $x = \frac{a+b}{2}$ is $\parallel$ to tangent at $x = \frac{b+c}{2}$ then they have same slope $\Rightarrow -\frac{(b-a)^2}{2} = -\frac{(b-c)^2}{2}$	(M1) or equivalent equation

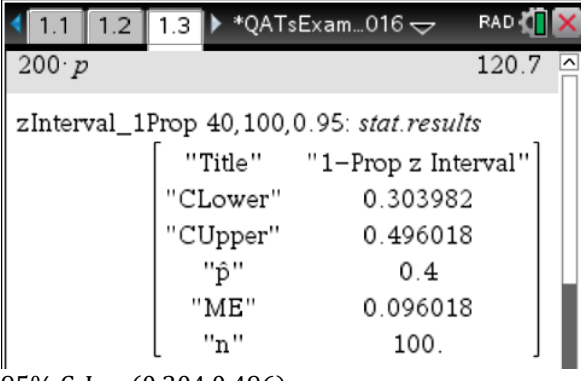
	 Cancelling and then factorising by the difference of perfect squares identity $(2b - a - c)(a - c) = 0$ By the Null Factor Law $b = \frac{a+c}{2}$ since $a - c \neq 0$.	(M1) (M1)
(f)	 $h(x) = \frac{x(2x + a - c)(2x - a + c)}{4}$ $h(x) = x(x - \frac{a-c}{2})(x + \frac{a-c}{2})$ (or $h(x) = x(x + \frac{c-a}{2})(x - \frac{c-a}{2})$)	(A1) or any equivalent equation (A1) accept either
(g)	$ \begin{aligned} h(-x) &= -x(-x - \frac{a-c}{2})(-x + \frac{a-c}{2}) \\ &= -x(x + \frac{a-c}{2})(x - \frac{a-c}{2}) \\ &= -h(x) \end{aligned} $	(M1) substitute for $-x$ (M1) demonstrate $h(x)$ is odd
(h)	A necessary and sufficient condition for the graph of the graph $y = h(x)$ to have a point of inflection at $x = v$ is $h''(v) = 0$.	(M1) attempt solve $h''(v) = 0$

	 P. O. I. = (0,0)	(A1)
(i)	The graph $y = f(x)$ is the image of the graph $y = h(x)$ after the transformation $(x, y) \rightarrow \left(x + \frac{a+c}{2}, y\right)$. P. O. I. = $\left(\frac{a+c}{2}, 0\right)$	(M1) uses transformation to motivate answer (A1)

Question 5 (18 marks)

a(i)	$X \sim \text{Bi}\left(n = 24, p = \frac{3}{5}\right)$  $\Pr(X \geq 12) = 1 - \Pr(X \leq 11) = 0.8857$	(M1) (A1)
a(ii)	$\Pr(X \geq 18 X \geq 12) = \frac{\Pr(X \geq 18 \cap X \geq 12)}{\Pr(X \geq 12)}$ $= \frac{\Pr(X \geq 18)}{\Pr(X \geq 12)}$ ≈ 0.108 	(M1) (M1) (A1)

a(iii)	$\text{mean of } \hat{p} = p = \frac{3}{5}$ $\text{variance of } \hat{p} = \frac{p(1-p)}{n}$ $= \frac{1}{100}$	(A1) (M1) (A1)
a(iv)	$\Pr\left(\frac{3}{5} - 2 \times \frac{1}{10} \leq \frac{X}{24} \leq \frac{3}{5} + 2 \times \frac{1}{10}\right)$ $= \Pr(10 \leq X \leq 19)$ ≈ 0.965	(M1) (M1) (A1)
a(v)	$\Pr\left(\hat{p} \geq \frac{3}{4} \mid \hat{p} \geq \frac{3}{5}\right) = \frac{\Pr(X \geq 18)}{\Pr(X \geq 15)}$ ≈ 0.196	(M1) (A1)
b(i)	 $g(w) = \begin{cases} \frac{-12 \cdot (w-35) \cdot (w-1)}{99127}, & 1 \leq w \leq 18 \\ \frac{-4 \cdot (w-87)^3}{37559529}, & 18 \leq w \leq 87 \end{cases}$ $\int_1^{87} (w \cdot g(w)) dw = 23.8006$ $E(W) = \int_1^{87} w g(w) dw$ ≈ 23.8006	(M1) (A1)
b(ii)	 $\int_{18}^{87} g(w) dw = \frac{207}{343}$ $p = \int_{18}^{87} g(w) dw = \frac{207}{343}$ $200 \cdot p = 120.7$ $\Pr(W > 18) = \int_{18}^{87} g(w) dw = \frac{207}{343}$ $\text{Expected Value} = 200 \times \frac{207}{343} \approx 121$	(A1) (A1)

C	 200 · p 120.7 zInterval_1Prop 40,100,0.95: <i>stat.results</i> <table><tr><td>"Title"</td><td>"1-Prop z Interval"</td></tr><tr><td>"CLower"</td><td>0.303982</td></tr><tr><td>"CUpper"</td><td>0.496018</td></tr><tr><td>"$\hat{p}$"</td><td>0.4</td></tr><tr><td>"ME"</td><td>0.096018</td></tr><tr><td>"n"</td><td>100.</td></tr></table> 95% C.I. = (0.304,0.496)	"Title"	"1-Prop z Interval"	"CLower"	0.303982	"CUpper"	0.496018	" $\hat{p}$ "	0.4	"ME"	0.096018	"n"	100.	(A1)
"Title"	"1-Prop z Interval"													
"CLower"	0.303982													
"CUpper"	0.496018													
" $\hat{p}$ "	0.4													
"ME"	0.096018													
"n"	100.													

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a + b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of a triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}((ax + b)^n) = an(ax + b)^{n-1}$	$\int (ax + b)^n dx = \frac{1}{a(n+1)}(ax + b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	
product rule $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u \frac{dv}{dx} - v \frac{du}{dx}}{v^2}$
chain rule $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(x)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum xp(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} xf(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{p}) = p$
standard deviation	$\text{sd}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$