

Trial Examination 2016

MATHEMATICAL METHODS

Written Examination 2 - SOLUTIONS

SECTION A

1. A 2. C 3. E 4. C 5. A 6. D 7. E 8. B 9. B 10. A

11. E 12. D 13. A 14. E 15. B 16. D 17. D 18. C 19. C 20. B

SECTION A

Question 1

$$f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = -2 - 3 \cos\left(\frac{x}{4} + 1\right)$$

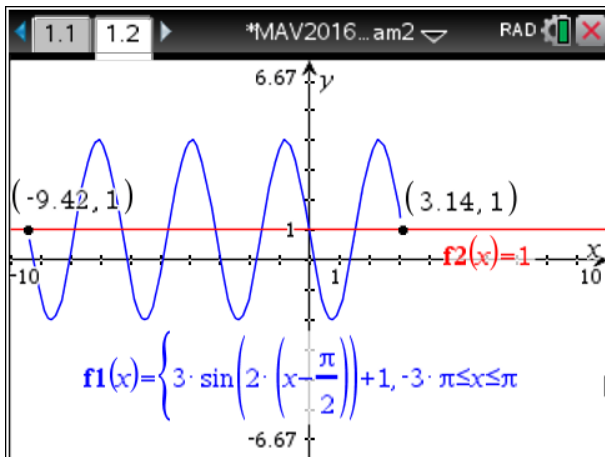
The range is $[-3 - 2, 3 - 2] = [-5, 1]$

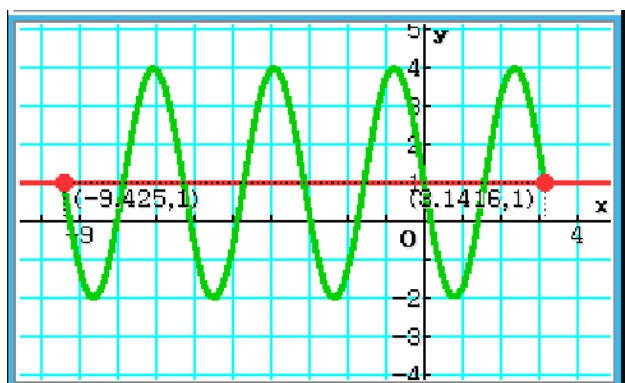
The Period is $\frac{2\pi}{\frac{1}{4}} = 8\pi$ **A**

Question 2

$$g: [-3\pi, \pi] \rightarrow \mathbb{R}, g(x) = 3 \sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 1$$

There are 9 intersections when $k = 1$. **C**





$$\text{define } f(x) = 3\sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 1$$

$$\text{solve}(f(x) = 1 \mid -3\pi \leq x \leq \pi, x)$$

$$\left\{ x = 0, x = -3\pi, x = -2\pi, x = -\pi, x = \pi, x = \frac{-5\pi}{2}, x = \frac{-3\pi}{2}, x = \right\}$$

done

Question 3

The domain of $f(g(x))$ is the same as the domain of g where $g(x) = \tan(\pi x)$.

The asymptotes of g have equations $y = \frac{1}{2} + k, k \in \mathbb{Z}$.

Hence the domain of $f(g(x))$ is $\mathbb{R} \setminus \left\{ \frac{1}{2} + k \right\}, k \in \mathbb{Z}$ **E**

Question 4

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos(x)$$

$$\sqrt{15^2 - 2^2} = \sqrt{221}$$

$$\sin\left(\frac{3\pi}{2} - x\right) = -\frac{\sqrt{221}}{15} \quad \mathbf{C}$$

Question 5

$$y = 2x^2 + 3x - 1 = 2\left(x + \frac{3}{4}\right)^2 - \frac{17}{8}$$

The graph of $y = x^2$ has been dilated by a factor of 2 from the x -axis and

Then translated $\frac{3}{4}$ units to the left and $\frac{17}{8}$ units down.

A possible rule for the transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \frac{3}{4} \\ \frac{17}{8} \end{bmatrix} \quad \mathbf{A}$$

completeSquare($2 \cdot x^2 + 3 \cdot x - 1, x$)

$$2 \cdot \left(x + \frac{3}{4}\right)^2 - \frac{17}{8}$$

Question 6

$$f(x) = x^{\frac{3}{2}}$$

$$f(x^2 y^2) = (x^2 y^2)^{\frac{3}{2}} = x^3 y^3$$

$$f(x^2) f(y^2) = (x^2)^{\frac{3}{2}} (y^2)^{\frac{3}{2}} = x^3 y^3$$

D

Define $f(x) = x^{\frac{3}{2}}$

$f(x^2 \cdot y^2) = f(x^2) \cdot f(y^2)$ true

define $f(x) = x^{\frac{3}{2}}$

done

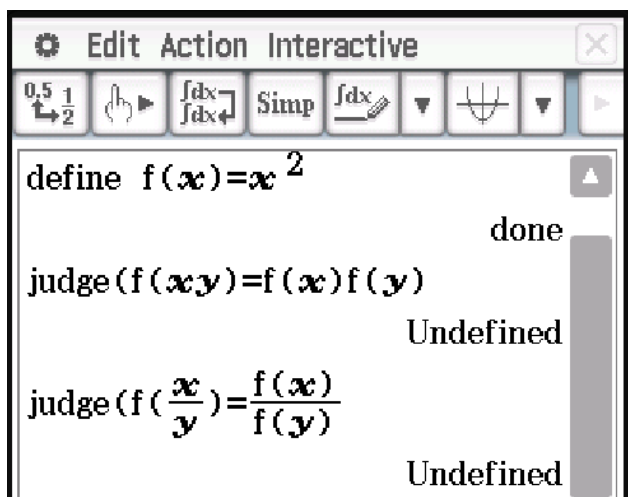
judge($f(x^2 y^2) = f(x^2) f(y^2)$)

TRUE

Using counter examples

$$f(xy) = f(x)f(y), f(-1 \times -1) = 1, f(-1) \times f(-1) \text{ has no real solution}$$

$$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}, f\left(\frac{-1}{-1}\right) = 1, \frac{f(-1)}{f(-1)} \text{ has no real solution}$$



$$f(x+y) = f(x) + f(y), \quad f(1+1) = 2^{\frac{3}{2}}, \quad f(1) + f(1) = 2$$

$$f(x-y) = f(x) - f(y), \quad f(2-1) = 1, \quad f(1) - f(1) = 0$$

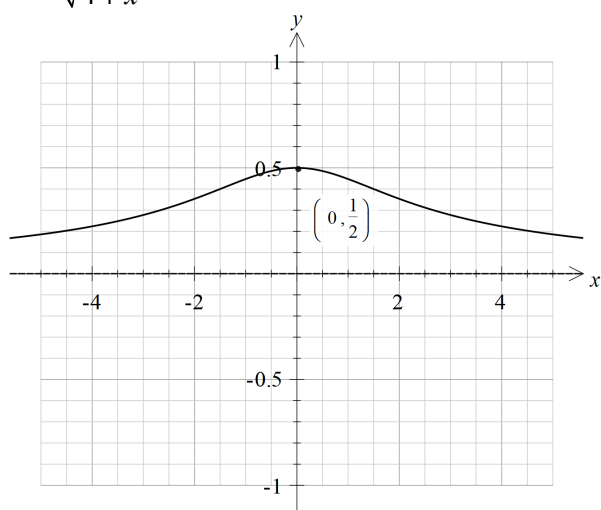
Question 7

When $x = 0$, $y = 0.5$.

Either **C** or **E**

When $x = 2$, $y \approx 0.35$

$$y = \frac{1}{\sqrt{4+x^2}} \quad \mathbf{E}$$



Question 8

$$f: (1, \infty) \rightarrow \mathbb{R}, f(x) = \frac{3}{(x-1)^2} + 2$$

Range is $(2, \infty)$ which is the domain of the inverse function.

$$\text{Let } y = \frac{3}{(x-1)^2} + 2$$

Inverse swap x and y .

$$x = \frac{3}{(y-1)^2} + 2$$

$$y = \pm \sqrt{\frac{3}{x-2}} + 1$$

$$y = \sqrt{\frac{3}{x-2}} + 1, \text{ as the domain of } f \text{ is } (1, \infty).$$

$$f^{-1}: (2, \infty) \rightarrow \mathbb{R}, f^{-1}(x) = \sqrt{\frac{3}{x-2}} + 1 \quad \mathbf{B}$$

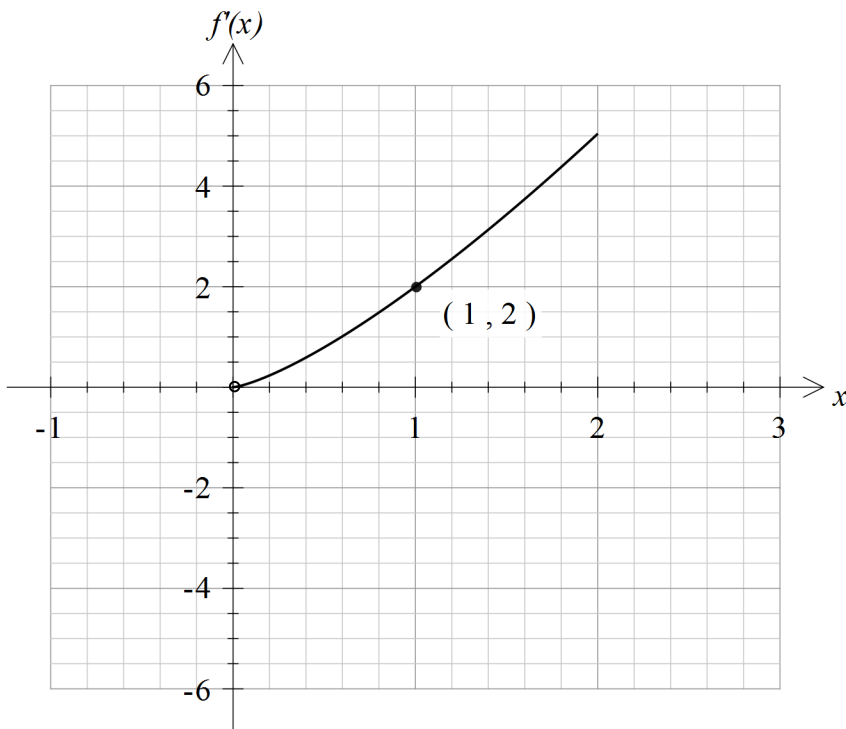
TI-84 Plus calculator screen showing the solution of the equation $x = \frac{3}{(y-1)^2} + 2$ for y . The screen displays the equation, the solutions $y = \frac{\sqrt{x-2} - \sqrt{3}}{\sqrt{x-2}}$ or $y = \frac{\sqrt{x-2} + \sqrt{3}}{\sqrt{x-2}}$, and the simplified form of the second solution: $\frac{\sqrt{3}}{\sqrt{x-2}} + 1$.

TI-84 Plus calculator screen showing the solution of the equation $x = \frac{3}{(y-1)^2} + 2$ for y . The screen displays the equation, the solutions $y = \frac{\sqrt{x-2} - \sqrt{3}}{\sqrt{x-2}}$ or $y = \frac{\sqrt{x-2} + \sqrt{3}}{\sqrt{x-2}}$, and the simplified form of the second solution: $\frac{\sqrt{3}}{\sqrt{x-2}} + 1$.

Question 9

The point $(1, 2)$ is on the curve of f' .

Thus the gradient of the graph of f at $x = 1$ is 2.

B**Question 10**

If $f(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + h$ has a stationary point of inflection at $(-3, 0)$ and a turning point at $(2, 0)$ then $f(x) = a(x+3)^3(x-2)^2$.

$$h = a \times 3^3 \times (-2)^2 = 108a$$

$$\frac{h}{a} = 108$$

A**Question 11**

$$f(x) = -2x^{\frac{7}{2}}, \quad y = -4 \text{ to } y = -1$$

$$\text{Solve } f(x) = -4, x = 2^{\frac{2}{7}}$$

$$\text{and } f(x) = -1, x = 2^{-\frac{2}{7}}$$

Edit Action Interactive

define $f(x) = -2x^{\frac{7}{2}}$

done

solve($f(x) = -4, x$)

$$\left\{ x = 4^{\frac{1}{7}} \right\}$$

solve($f(x) = -1, x$)

$$\left\{ x = \frac{1}{4^{\frac{1}{7}}} \right\}$$

$$\begin{aligned} \text{Average rate of change} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 + 4}{2^{\frac{2}{7}} - 2^{\frac{2}{7}}} \\ &\approx -7.5 \end{aligned}$$

E

1.4 1.5 1.6 ▶ *MAV2016...am2 RAD

solve($-2 \cdot x^{\frac{7}{2}} = -4, x$) $x = 2^{\frac{2}{7}}$

solve($-2 \cdot x^{\frac{7}{2}} = -1, x$) $x = \frac{2^{\frac{2}{7}}}{2}$

$\frac{-1+4}{\frac{5}{2} - 2}$ $\frac{5}{3 \cdot 2^{\frac{2}{7}}}$

1.4 1.5 1.6 ▶ *MAV2016...am2 RAD

$\frac{-1+4}{\frac{5}{2} - 2}$ $\frac{5}{3 \cdot 2^{\frac{2}{7}}}$

$\frac{5}{2^{\frac{2}{7}} - 2}$ -7.52486

$$\frac{f\left(4^{\frac{1}{7}}\right) - f\left(\frac{1}{4^{\frac{1}{7}}}\right)}{4^{\frac{1}{7}} - \frac{1}{4^{\frac{1}{7}}}}$$

$$-7.524864066$$

Question 12

$$\log_a(b) = c \text{ and } \log_c(a) = b$$

$$\log_a(b) + \log_c(a) = b + c$$

$$\log_a(b) + \frac{\log_a(a)}{\log_a(c)} = b + c$$

$$\log_a(b) + \frac{1}{\log_a(c)} = b + c$$

$$\log_a\left(\frac{b}{c}\right) \neq b + c \quad \mathbf{D}$$

Question 13

$$f'(x) = 2x^3 - 2x$$

$$f(x) = \int (2x^3 - 2x) dx$$

$$= \frac{x^4}{2} - x^2 + c$$

$$f(2) = \frac{9}{2}$$

$$\frac{9}{2} = \frac{2^4}{2} - 2^2 + c$$

$$c = \frac{1}{2}$$

$$\text{Solve } \frac{x^4}{2} - x^2 + \frac{1}{2} = 0 \text{ for } x.$$

$$x = \pm 1$$

A

The screenshot shows a TI-84 Plus calculator interface. At the top, the window title is '*MAV2016...am2' and the mode is set to 'RAD'. The main display area shows the integral $\int (2 \cdot x^3 - 2 \cdot x) dx$ on the left and the result $\frac{x^4}{2} - x^2$ on the right. Below this, the equation $\text{solve}\left(\frac{2^4}{2} - 2^2 + c = \frac{9}{2}, c\right)$ is entered on the left, with the result $c = \frac{1}{2}$ on the right. At the bottom, the equation $\text{solve}\left(\frac{x^4}{2} - x^2 + \frac{1}{2} = 0, x\right)$ is entered on the left, with the result $x = -1 \text{ or } x = 1$ on the right.

Edit Action Interactive
 0.5 1/2 $\int \frac{dx}{dx}$ $\int \frac{dx}{dx}$ Simp $\frac{dx}{dx}$ $\int \frac{dx}{dx}$
 $\int 2 \cdot x^3 - 2 \cdot x dx$
 $\frac{x^4}{2} - x^2$
 define $f(x) = \frac{x^4}{2} - x^2 + c$
 done
 solve $\left(f(2) = \frac{9}{2}, c \right)$
 $\left\{ c = \frac{1}{2} \right\}$
 solve $\left(\frac{x^4}{2} - x^2 + \frac{1}{2} = 0, x \right)$
 $\{ x = -1, x = 1 \}$
 Alg Standard Real Rad

Question 14

$$\int_2^3 (g(x)) dx = -3$$

$$= 2 \int_3^2 (1 - 2g(x)) dx$$

$$= 2 \int_3^2 (1) dx - 4 \int_3^2 (g(x))$$

$$= 2[x]_3^2 - 4 \times 3$$

$$= -14$$

E**Question 15**

$$\text{Area} = \frac{1}{2} (e^0 + e + e^2 + e^3)$$

$$= \frac{1}{2} (1 + e + e^2 + e^3)$$

The rectangles are below the curve.

Underestimate

B**Question 16**

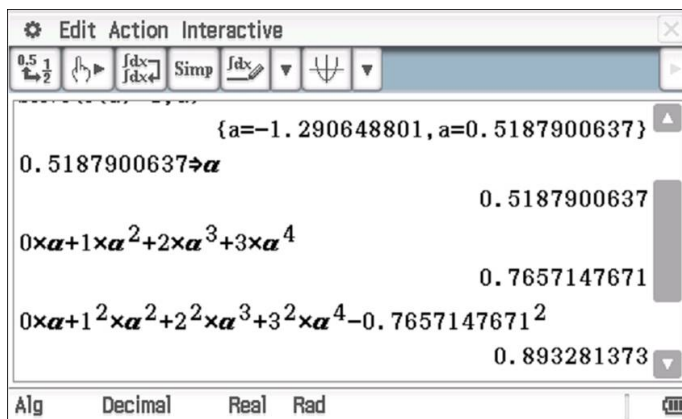
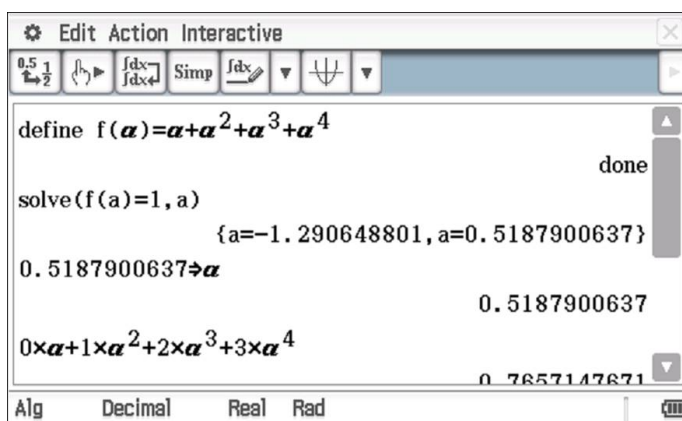
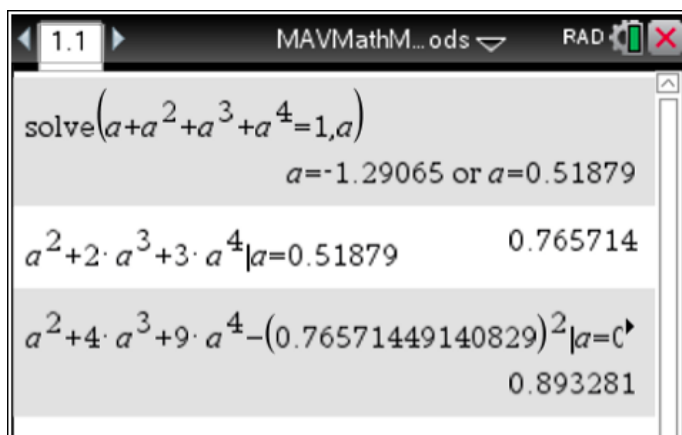
$$\text{Solve } a + a^2 + a^3 + a^4 = 1 \text{ for } a$$

$$a \approx 0.51879$$

$$E(X) = a^2 + 2 \times a^3 + 3 \times a^4 \approx 0.766$$

$$\text{Var}(X) = a^2 + 4a^3 + 9a^4 - (0.765714 \dots)^2 \approx 0.893$$

D



Question 17

	J	J'	
R	$0.3p$	$0.7p$	p
R'	$0.3 - 0.3p$	p^2	$1 - p$
	0.3	0.7	1

Independent events $\Pr(J \cap R) = \Pr(J) \times \Pr(R)$

Let $a = \Pr(J \cap R) = 0.3p$

$$0.3 + p - a + p^2 = 1$$

Solve $p^2 + 0.7p - 0.7 = 0$ for p .

$$p = \frac{\sqrt{329} - 7}{20}$$

$$a = 0.3p$$

$$a = \frac{3(\sqrt{329} - 7)}{200}$$

D

$\text{solve}\left(p^2 + \frac{7}{10} \cdot p - \frac{7}{10} = 0, p\right)$
 $p = \frac{-(\sqrt{329} + 7)}{20} \text{ or } p = \frac{\sqrt{329} - 7}{20}$
 $\text{solve}\left(p^2 + \frac{7}{10} \cdot p - \frac{7}{10} = 0, p\right)$
 $p = -1.256918 \text{ or } p = 0.5569179$

$\text{solve}(0.7 \cdot p + p^2 = 0.7, p)$
 $\left\{ p = \frac{-\sqrt{329} - 7}{20}, p = \frac{\sqrt{329} - 7}{20} \right\}$

Question 18

Solve $\int_1^a (\log_e(x)) dx = 1$ for a .

$$a = e$$

$$\text{Var}(X) = E(X^2) - (E(X))^2$$

$$\begin{aligned} \text{Var}(X) &= \int_1^e (x^2 \log_e(x)) dx - \left(\int_1^e (x \log_e(x)) dx \right)^2 \\ &= -\frac{e^4}{16} + \frac{2e^3}{9} - \frac{e^2}{8} + \frac{7}{144} \end{aligned}$$

C

TI-Nspire calculator screen showing the solution to the equation $\int_1^a \ln(x) dx = 1, a$. The variable a is set to e . The expression to be expanded is $\int_1^e x^2 \cdot \ln(x) dx - \left(\int_1^e x \cdot \ln(x) dx \right)^2$. The result is $\frac{-e^4}{16} + \frac{2 \cdot e^3}{9} - \frac{e^2}{8} + \frac{7}{144}$.

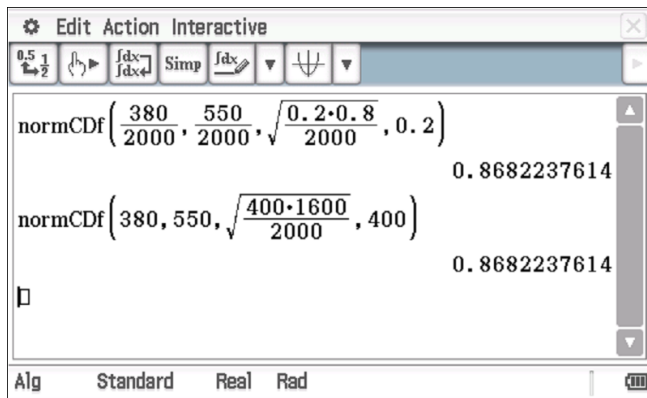
TI-Nspire calculator screen showing the same problem as above, but with the 'expand' action selected. The result is the same: $\frac{-e^4}{16} + \frac{2 \cdot e^3}{9} - \frac{e^2}{8} + \frac{7}{144}$.

Question 19

$n = 2000$ and $p = 0.2$. Since n is large, the distribution of $\hat{P}$ is approximately normal with $\mu = 0.2$ and $\sigma = \sqrt{\frac{0.2 \times 0.8}{2000}}$.

$$\Pr\left(\frac{380}{2000} < X < \frac{550}{2000}\right) \approx 0.868 \quad \mathbf{C}$$

TI-Nspire calculator screen showing the normal CDF calculation. The first line is $\text{normCdf}\left(\frac{380}{2000}, \frac{550}{2000}, 0.2, \sqrt{\frac{0.2 \cdot 0.8}{2000}}\right)$ resulting in 0.8682237. The second line is $\text{normCdf}\left(380, 550, 400, \sqrt{\frac{400 \cdot 1600}{2000}}\right)$ also resulting in 0.8682237.

**Question 20**

If 200 such samples were taken, $0.9 \times 200 = 180$ of the confidence intervals would be expected to contain p . **B**

SECTION B

EXTENDED RESPONSE QUESTIONS

Question 1 (14 marks)

a. $f(x) = ax^{\frac{2}{3}}$

Substitute the point $\left(-8, \frac{16}{3}\right)$

$$\frac{16}{3} = a(-8)^{\frac{2}{3}}$$

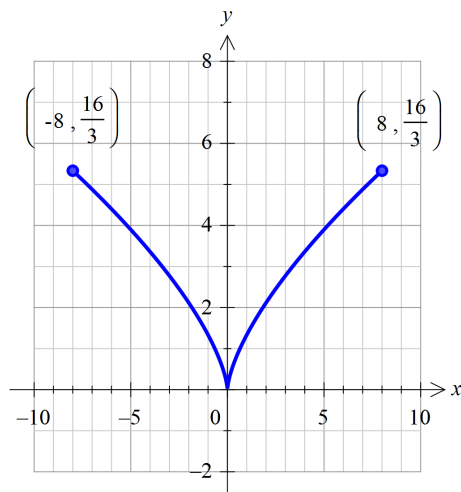
$$\frac{16}{3} = 4a$$

$$a = \frac{4}{3}$$

1M Show that

b. Shape **1A**

Endpoints **1A**



c. $f_1(x) = f(-x)$ reflects the graph over the y -axis.

As the function $f(x) = \frac{4}{3}x^{\frac{2}{3}}$ is an even function the transformation described makes no difference. **1A**

d.

- a dilation of a factor of 2 units from the x -axis gives $y = \frac{8}{3}x^{\frac{2}{3}}$
then

- a translation of 3 units up gives $f_2(x) = \frac{8}{3}x^{\frac{2}{3}} + 3$ **1A**

e.

- a reflection in the x -axis gives $y_1 = -\frac{4}{3}x^{\frac{2}{3}}$
- a reflection in the y -axis gives $y_2 = -\frac{4}{3}(-x)^{\frac{2}{3}} = -\frac{4}{3}x^{\frac{2}{3}}$
- a translation of 1 unit in the positive direction of the y -axis gives $y_3 = -\frac{4}{3}x^{\frac{2}{3}} + 1$
- a translation of 3 units in the positive direction of the x -axis gives $y_4 = -\frac{4}{3}(x-3)^{\frac{2}{3}} + 1$

The image equation is $f_3(x) = -\frac{4}{3}(x-3)^{\frac{2}{3}} + 1$ **2A**

f. $T: R^2 \rightarrow R^2, T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

gives $\begin{bmatrix} 2 & 0 \\ 0 & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2x+1 \\ -\frac{1}{3}y+2 \end{bmatrix} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

Rearrange to get

$$x = \frac{x_1 - 1}{2}$$

$$y = \frac{y_1 - 2}{-\frac{1}{3}} = -3(y_1 - 2) \quad \textbf{1M}$$

Substitute to find the image equation for $f(x) = \frac{4}{3}x^{\frac{2}{3}}$.

$$-3(y_1 - 2) = \frac{4}{3} \left(\frac{x_1 - 1}{2} \right)^{\frac{2}{3}} \quad \textbf{1M}$$

Image equation is

$$y_1 = -\frac{4}{9} \left(\frac{x_1 - 1}{2} \right)^{\frac{2}{3}} + 2$$

Giving $y = 2 - \frac{4}{9} \left(\frac{x-1}{2} \right)^{\frac{2}{3}}$ **1A**

g. $y = 2 - \frac{4}{9} \left(\frac{x-1}{2} \right)^{\frac{2}{3}}$ gives a dilation of a factor of 2 from y -axis and a translation of 1 unit in the positive

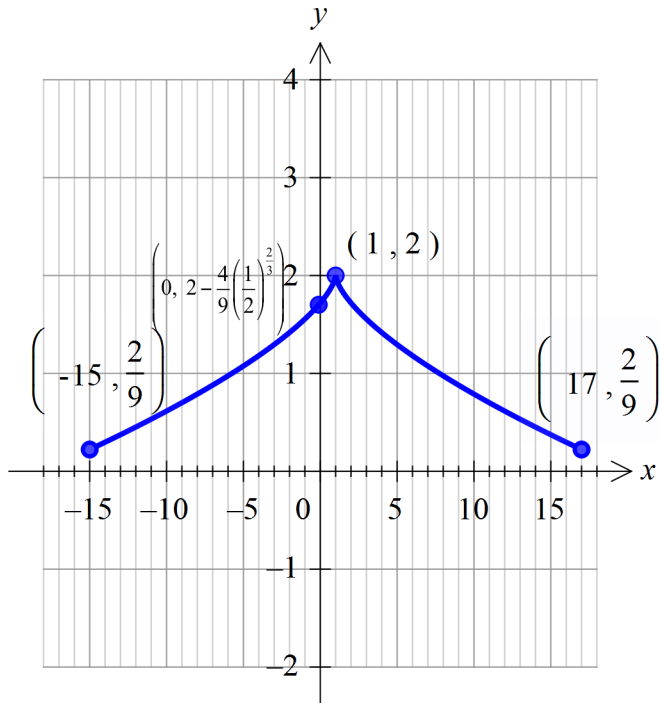
direction of the x -axis from the original function $f(x) = \frac{4}{3}x^{\frac{2}{3}}$ where the domain was $[-8, 8]$.

A dilation of a factor of 2 from y -axis gives the domain $[-16, 16]$.

Then a translation of 1 unit in the positive direction of the x -axis gives a domain $[-15, 17]$. **1A**

h. Shape 1A**Endpoints 1A****Coordinates of Cusp and**

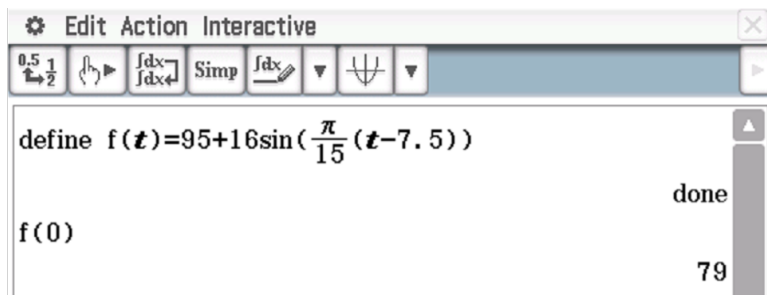
$$y\text{-intercept } \left(0, 2 - \frac{4}{9} \left(\frac{1}{2} \right)^{\frac{2}{3}} \right) \quad \mathbf{1A}$$



Question 2 (15 marks)

$$\begin{aligned}
 \text{a. } P(0) &= 95 + 16 \sin\left(\frac{\pi}{15}(0 - 7.5)\right) \\
 &= 95 - 16 \sin\left(\frac{\pi}{2}\right) \\
 &= 95 - 16 \\
 &= 79
 \end{aligned}$$

79 beats per minute

1M Show that

$$\begin{aligned}
 \text{b. Period} &= \frac{2\pi}{\pi/15} = 30 && \text{1A} \\
 \text{Amplitude} &= 16 && \text{1A}
 \end{aligned}$$

$$\text{c. } P(t) = 95 + 16 \sin\left(\frac{\pi}{15}(t - 7.5)\right)$$

$$P'(t) = 16 \times \frac{\pi}{15} \cos\left(\frac{\pi}{15}(t - 7.5)\right) \quad \text{1A}$$

$$\begin{aligned}
 P'(t) &= \frac{16\pi}{15} \cos\left(\frac{\pi t}{15} - \frac{\pi}{2}\right) \\
 &= \frac{16\pi}{15} \cos\left(\frac{\pi}{2} - \frac{\pi t}{15}\right)
 \end{aligned}$$

$$\text{Giving } P'(t) = \frac{16\pi}{15} \sin\left(\frac{\pi t}{15}\right). \quad \text{1M Show that}$$

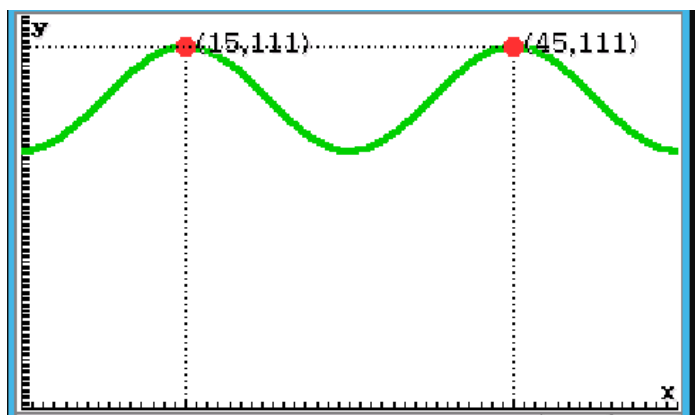
$$\text{d. Solve } \frac{16\pi}{15} \sin\left(\frac{\pi t}{15}\right) = 0 \text{ for } t \in [0, 60] \text{ 2 cycles} \quad \text{1M}$$

$$\frac{16\pi}{15} \sin\left(\frac{\pi t}{15}\right) = 0$$

$$\sin\left(\frac{\pi t}{15}\right) = 0$$

$$\frac{\pi t}{15} = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$t = 0, 15, 30, 45, 60$$



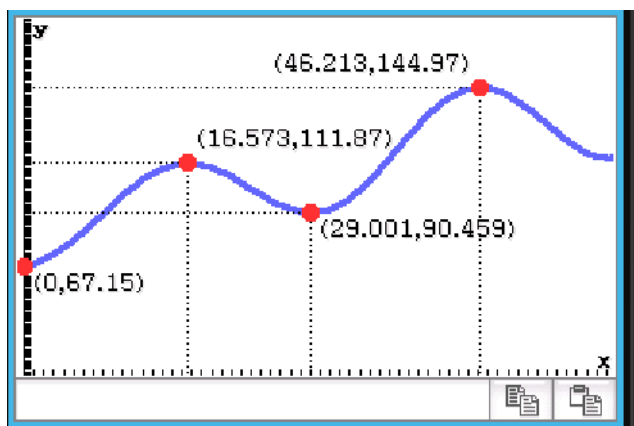
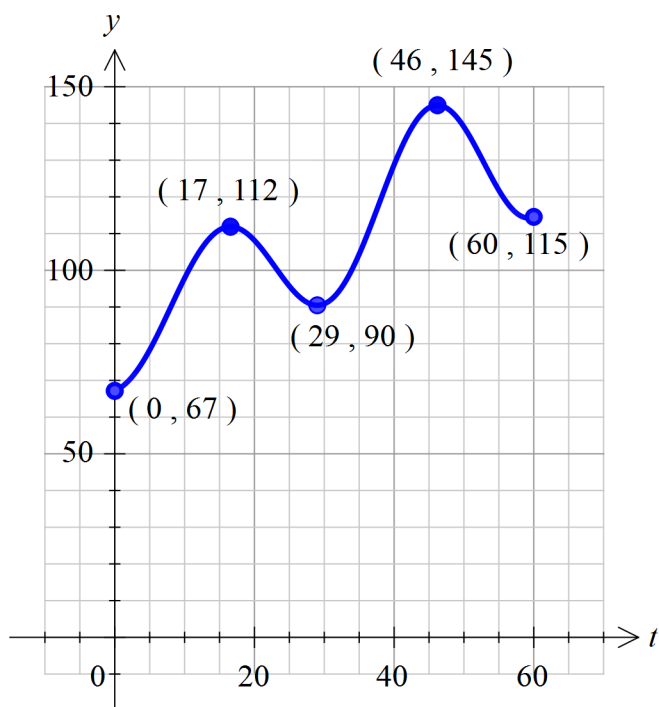
From the graph the local max occurs at $t = 15$ and $t = 45$

e. $P_N : [0, 60] \rightarrow \mathbb{R}, P_N(t) = 0.01(t + 85) \times P(t)$

Shape **1A**

Turning points **1A**

Endpoints **1A**



f. Maximum heart rate ≈ 144.97 beats /minute

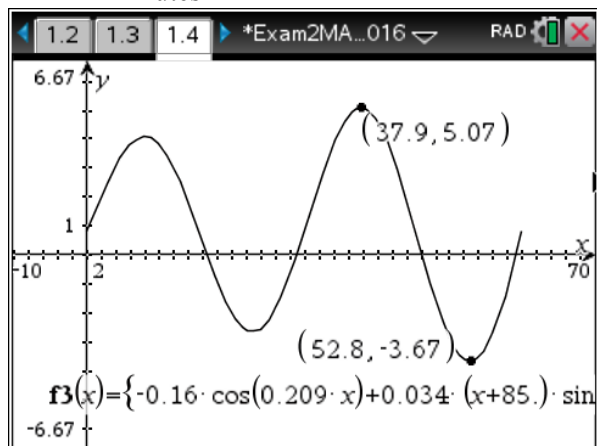
Maximum heart rate rounded to the nearest whole number = 145 beats /minute **1A**

g. Find when $P'_N(t)$ has its greatest magnitude

1M

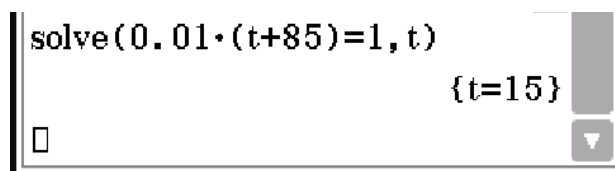
$t = 37.9$ minutes

1A



h. Solve the equation $0.01(t+85) = 1$ gives $t = 15$

1A

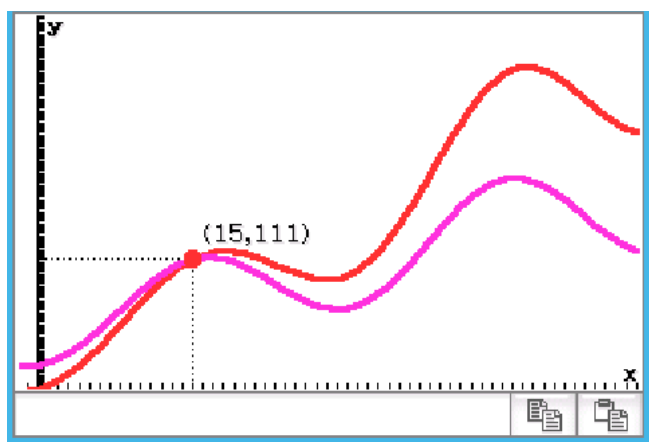


Solve $95 + 16 \sin\left(\frac{\pi}{15}(t-7.5)\right) = 0.01(t+85) \times P(t)$

$$95 + 16 \sin\left(\frac{\pi}{15}(t-7.5)\right) = 0.01(t+85) \times \left(95 + 16 \sin\left(\frac{\pi}{15}(t-7.5)\right)\right)$$

Gives the equation $1 = 0.01(t+85)$ where previously found solution $t = 15$

So Sarah's pulse equals Stephen's pulse at $t = 15$.



Looking at the graph

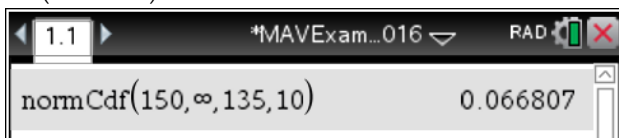
Sarah's pulse can be measured to be lower than her twin brother Stephen's

for $t \in (15, 60]$

1A

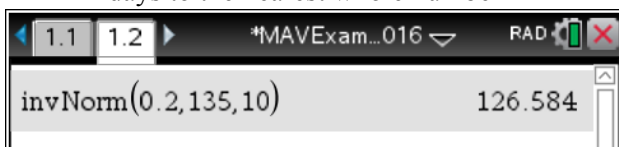
Question 3 (17 marks)**a.** $X : N(135, 100)$

$$\Pr(X > 150) = 0.0668... = 7\% \text{ to the nearest whole percent.} \quad \mathbf{1A}$$

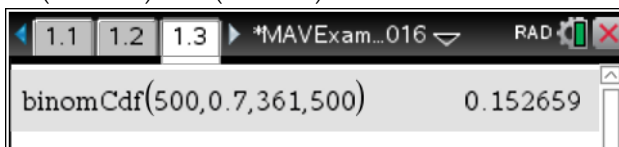
**b.** $\Pr(X > x) = 0.8$

$$\Pr(X < x) = 0.2$$

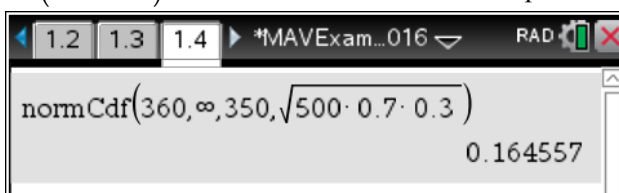
$$x = 127 \text{ days to the nearest whole number} \quad \mathbf{1A}$$

**c.** $Y : \text{Bi}(500, 0.7)$

$$\Pr(Y > 360) = \Pr(Y \geq 361) = 0.1527 \text{ correct to 4 decimal places.} \quad \mathbf{1A}$$

**d.** $W : N(350, 500 \times 0.7 \times 0.3)$

$$\Pr(W > 360) = 0.1646 \text{ correct to 4 decimal places.} \quad \mathbf{1M}$$

**e.** $\mu = 0.7$

$$\sigma = \sqrt{\frac{0.7 \times 0.3}{500}} = \frac{\sqrt{105}}{500}$$

$$\left(0.7 - 2 \times \frac{\sqrt{105}}{500}, 0.7 + 2 \times \frac{\sqrt{105}}{500} \right) \quad \mathbf{1M}$$

$$\Pr(0.6590... \leq \hat{P} \leq 0.7409...) = \Pr(329.5... \leq X \leq 370.4...) \quad \mathbf{1M}$$

$$= \Pr(330 \leq X \leq 370)$$

$$= 0.9547 \text{ correct to four decimal places.} \quad \mathbf{1A}$$

TI-84 Plus calculator screen showing calculations for a binomial distribution with $n=500$ and $p=0.7$. The screen displays the following results:

$0.7 - 2 \cdot \sqrt{\frac{0.7 \cdot 0.3}{500}}$	0.659012
$0.7 + 2 \cdot \sqrt{\frac{0.7 \cdot 0.3}{500}}$	0.740988
$0.65901219693616 \cdot 500$	329.506
$0.74098780306384 \cdot 500$	370.494
$\text{binomCdf}(500, 0.7, 330, 370)$	0.954713

f. (0.7649, 0.8351)

1A

TI-84 Plus calculator screen showing the results of a 1-Prop z Interval for $n=500$ and $p=0.8$. The screen displays the following results:

zInterval_1Prop 400,500,0.95: stat.results	
"Title"	"1-Prop z Interval"
"CLower"	0.764939
"CUpper"	0.835061
"p"	0.8
"ME"	0.035061
"n"	500.

g.

Proportion of yellow balls $\hat{p}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$\Pr(\hat{P} = \hat{p})$	$\frac{1}{22}$	$\frac{10}{33}$	$\frac{5}{11}$	$\frac{2}{11}$	$\frac{1}{66}$

Without replacement

$$\frac{6}{11} \times \frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{22}$$

1A

$$4 \times \frac{5}{11} \times \frac{6}{10} \times \frac{5}{9} \times \frac{4}{8} = \frac{10}{33}$$

1A

$\frac{6}{11} \cdot \frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8}$	$\frac{1}{22}$	$\frac{5}{11} \cdot \frac{4}{10} \cdot \frac{6}{9} \cdot \frac{5}{8}$	$\frac{5}{11}$
$4 \cdot \frac{5}{11} \cdot \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{4}{8}$	$\frac{10}{33}$	$4 \cdot \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{6}{8}$	$\frac{2}{11}$
$6 \cdot \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{6}{9} \cdot \frac{5}{8}$	$\frac{5}{11}$	$\frac{5}{11} \cdot \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{2}{8}$	$\frac{1}{66}$
$4 \cdot \frac{5}{11} \cdot \frac{4}{10} \cdot \frac{3}{9} \cdot \frac{6}{8}$	$\frac{2}{11}$		

$$\text{h. } \Pr\left(\hat{P}=1 \mid \hat{P} > \frac{1}{2}\right) = \frac{\Pr\left(\hat{P}=1 \cap \hat{P} > \frac{1}{2}\right)}{\Pr\left(\hat{P} > \frac{1}{2}\right)} \quad 1\text{M}$$

$$= \frac{\frac{1}{66}}{\frac{13}{66}} = \frac{1}{13} \quad 1\text{A}$$

$$\text{i. } \Pr(X > 200) = 0.1, \Pr(X < 185) = 0.05 \quad 1\text{M}$$

$$\Pr\left(Z > \frac{200 - \mu}{\sigma}\right) = 0.1, \Pr\left(Z < \frac{175 - \mu}{\sigma}\right) = 0.05$$

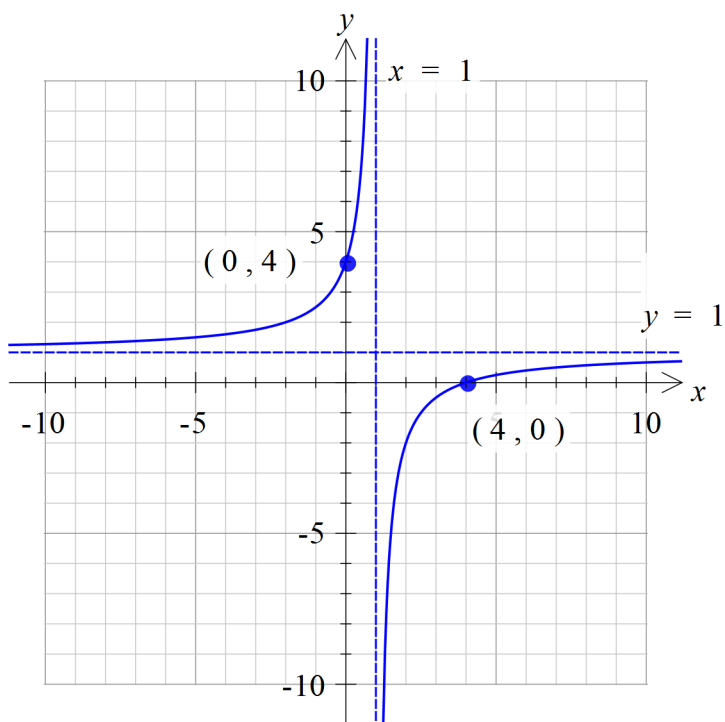
$$\frac{200 - \mu}{\sigma} = 1.2815..., \frac{175 - \mu}{\sigma} = -1.6448... \quad 1\text{M}$$

$$\mu = 189 \text{ days}, \sigma = 9 \text{ days} \quad 1\text{A}$$

$\text{invNorm}(0.9, 0, 1)$	1.28155
$\text{invNorm}(0.05, 0, 1)$	-1.64485
$\text{solve}\left(\frac{200-a}{b} = 1.2815515665787 \text{ and } \frac{175-a}{b} = -1.64485\right)$	$a=189.052 \text{ and } b=8.5429$

Question 4 (14 marks)

- a.** shape **1A**
 asymptotes with equations **1A**
 coordinates of axial-intercepts **1A**



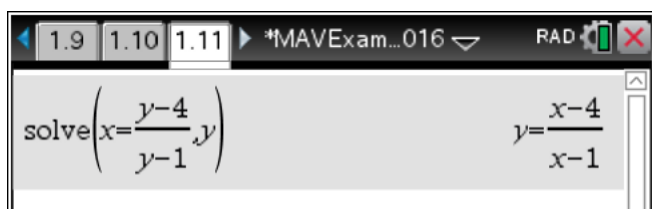
b. Let $y = \frac{x-4}{x-1}$

Inverse swap x and y .

Solve $x = \frac{y-4}{y-1}$ for y .

$$f^{-1}(x) = \frac{x-4}{x-1}$$

Hence $f(x) = f^{-1}(x)$ **1A**



c. $g: \mathbb{R} \setminus \{-n\} \rightarrow \mathbb{R}, g(x) = \frac{x+m}{x+n}$

$$g^{-1}(x) = \frac{-nx+m}{x-1}$$

$n = -1$ **1A**

$m \in \mathbb{R} \setminus \{-1, 0\}$ **1A**

OR

$$g(x) = 1 + \frac{m+n}{x+n}$$

Horizontal asymptote at $y = 1$

Vertical asymptote at $x = 1$

$$n = -1 \quad \mathbf{1A}$$

$$m \in \mathbb{R} \setminus \{-1, 0\} \quad \mathbf{1A}$$

$$\mathbf{d.} \quad h: \mathbb{R} \setminus \left\{-\frac{b}{n}\right\} \rightarrow \mathbb{R}, h(x) = \frac{ax+m}{bx+n}$$

$$h^{-1}(x) = \frac{-nx+m}{bx-a}$$

$$ax+m = -nx+m, \quad bx-a = bx+n$$

$$a = -n \quad \mathbf{1M \text{ Show that}}$$

$$\mathbf{e.} \quad h_1: \mathbb{R} \setminus \left\{-\frac{n}{4}\right\} \rightarrow \mathbb{R}, h_1(x) = \frac{3x+2}{4x+n}$$

$$n = -3 \quad \mathbf{1A}$$

$$\mathbf{f.} \quad w: \mathbb{R} \rightarrow \mathbb{R}, w(x) = -(2x-1)^3 + \frac{1}{2}$$

$$w^{-1}(x) = \frac{1}{2} \left(\sqrt[3]{\left(\frac{1}{2} - x\right)} + 1 \right)$$

$$\text{Solve } w(x) = w^{-1}(x) \text{ for } x. \quad \mathbf{1M}$$

$$x = \frac{2-\sqrt{2}}{4}, x = \frac{1}{2} \text{ or } x = \frac{2+\sqrt{2}}{4}$$

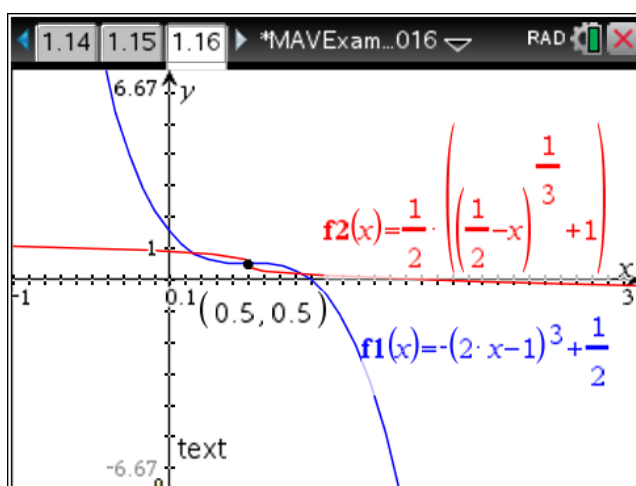
$\mathbf{1A}$

$$\left(\frac{2-\sqrt{2}}{4}, \frac{2+\sqrt{2}}{4}\right), \left(\frac{1}{2}, \frac{1}{2}\right), \left(\frac{2+\sqrt{2}}{4}, \frac{2-\sqrt{2}}{4}\right) \quad \mathbf{1H}$$

$\text{solve} \left(-(2 \cdot x - 1)^3 + \frac{1}{2} = \frac{1}{2} \cdot \left(\left(\frac{1}{2} - x \right)^3 + 1 \right), x \right)$
 $x = \frac{-(\sqrt{2} - 2)}{4} \text{ or } x = \frac{1}{2} \text{ or } x = \frac{\sqrt{2} + 2}{4}$
 $-(2 \cdot x - 1)^3 + \frac{1}{2} \Big|_{x = \frac{-(\sqrt{2} - 2)}{4}} = \frac{\sqrt{2} + 1}{4}$
 $-(2 \cdot x - 1)^3 + \frac{1}{2} \Big|_{x = \frac{\sqrt{2} + 2}{4}} = \frac{1}{2} - \frac{\sqrt{2}}{4}$

g. Area $= 2 \int_{\frac{2-\sqrt{2}}{4}}^{\frac{1}{2}} \left(\frac{1}{2} \left(\sqrt[3]{\frac{1}{2} - x} + 1 \right) - \left(-(2x-1)^3 + \frac{1}{2} \right) \right) dx$ **1M**
 $= \frac{1}{8}$ **1A**

$2 \cdot \left(\frac{1}{2} \cdot \left(\left(\frac{1}{2} - x \right)^{\frac{1}{3}} + 1 \right) - \left(-(2 \cdot x - 1)^3 + \frac{1}{2} \right) \right)$
 $\frac{1}{8}$



h. $w_k : R \rightarrow R, w_k(x) = -(rx-1)^3 + s$
 $s = \frac{1}{r}$ **1A**