

The Mathematical Association of Victoria

Trial Examination 2016

MATHEMATICAL METHODS

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer book of 12 pages
- Formula sheet
- Working space is provided throughout the book.

Instructions

- Write your **name** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Question 1 (4 marks)

- a.** If $f(x) = \log_e(\cos(4x))$ find $f'(x)$. Express your answer in the form $A \tan(Bx)$ where A and B are real numbers.

2 marks

- b. i.** Factorise $x^3 - 3x^2 + 3x - 1$.

1 mark

- ii. Hence**, antidifferentiate $\frac{1}{(1-x)(x^3 - 3x^2 + 3x - 1)}$.

1 mark

TURN OVER

Question 2 (3 marks)

The depth, $d(t)$ m, of water at a pier t hours after midnight on a particular 24 hour day is given by

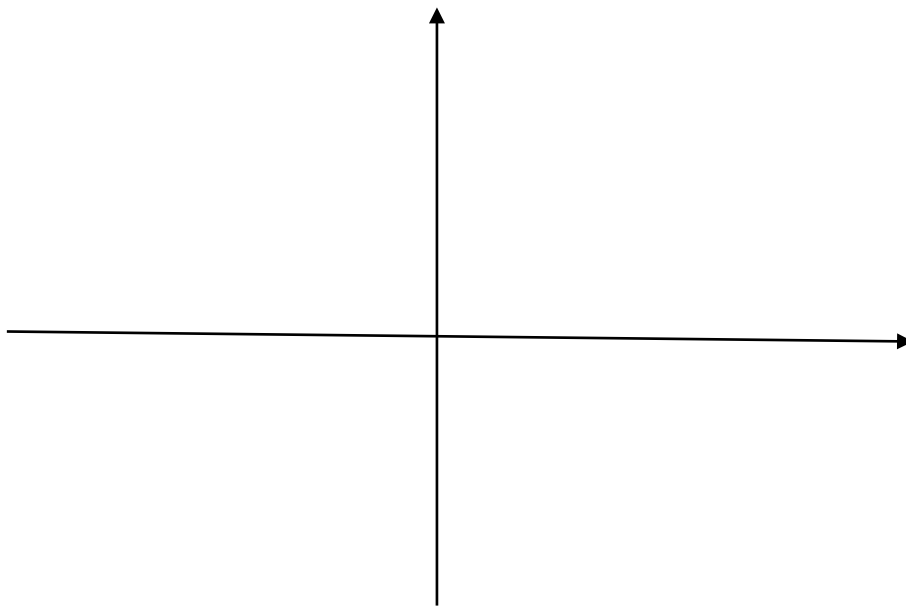
$d(t) = 2\sin\left(\frac{\pi t}{6}\right) + 5$. Find the values of t for which the depth is greater than 6 m.

Question 3 (5 marks)

Consider the function $g : [0, 2] \rightarrow \mathbb{R}, g(x) = 2x^5 - 10x^4 + 20x^3 - 20x^2 + 10x + 2$.

- a. Find A , B and C given $g(x) = A(x+B)^5 + C$, where A , B and C are real constants. 3 marks

- b. Sketch the graph of g on the set of axes below. Label the endpoints and any stationary points with their coordinates. 2 marks

**TURN OVER**

Question 4 (3 marks)

- a. Show that $x = \log_e(3)$ is a solution of the equation $\frac{9}{e^{2x}} - \frac{6}{e^x} + 2e^x = 5$.

1 mark

- b. Hence, or otherwise, solve the equation $\frac{9}{e^{2x}} - \frac{6}{e^x} + 2e^x = 5$ for x , given that there are only two real solutions.

2 marks

Let $f(x) = xe^{2x}$.

- 1 mark

- 3 marks

[illegible]

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Question 6 (3 marks)

Find the values of A and B , where A and B are real constants, if the graph of $y = A \log_e(x - B)$ passes through the points $(2, 10)$ and $(8, 20)$.

Question 7 (4 marks)

Two linear equations can be written in the form of

$$\begin{aligned} -2(m-1)x + my &= -m + 4 \\ mx - 3y &= 2m + 1 \end{aligned} \quad \text{where } m \text{ is a real constant.}$$

- a. Find the value(s) of m such that the graphs of the two lines have a unique solution.

2 marks

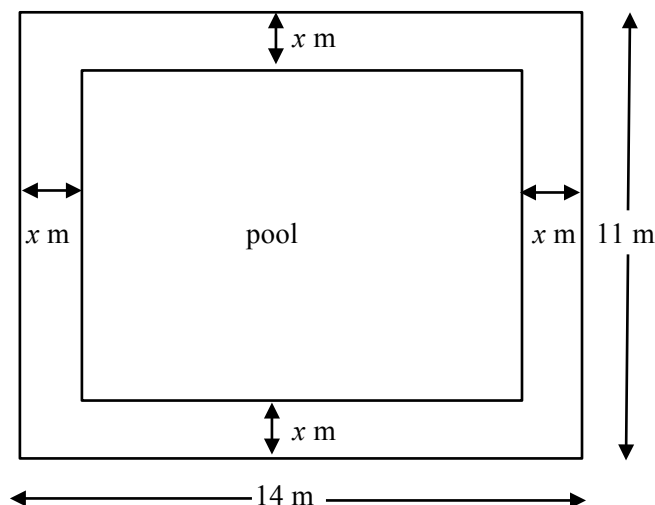
- b. If $m = -1$, show that the line with equation $-2(m-1)x + my = -m + 4$ is a tangent to the parabola with the equation $y = mx^2 + 2x - 6$.

2 marks

TURN OVER

Question 8 (6 marks)

Taren and Yao have a rectangular garden. It is 14 metres long and 11 metres wide. They want to put a rectangular swimming pool in the middle of the garden and a path of width x metres around the edge, as shown below.



- a. Show that an expression for the length of the diagonal of the pool in terms of x is

$$\sqrt{8x^2 - 100x + 317}.$$

2 marks

Taren's swimming instructor insists that the length of the diagonal of the pool is at least 15 metres for their pool dancing lessons.

- b. For what value(s) of x will the diagonal be at least 15 metres in length?

3 marks

- c. Yao wants the surface area of the floor of the pool to be at least 155 square metres. Show that this is not possible.

1 mark

Question 9 (8 marks)

- a. Suppose that 80% of all 16 year olds play basketball. If a sample of size 4 is taken find the probability that the sample proportion lies within and including one standard deviation of the population proportion.

3 marks

Sam plays basketball. The probability that Sam scores a goal every time she has a shot is 0.2.

- b. Given that she scores no more than one goal in four shots, what is the probability the first two shots were **not** goals?

2 marks

- c. What is the least number of shots she needs to make to ensure the probability that she gets at least one goal is more than 0.9, given $\log_{10}(8) \approx 0.903$?

3 marks

END OF QUESTION AND ANSWER BOOKLET