



**MATHEMATICS
HIGHER LEVEL
PAPER 1**

1.5 hours

November 2015

NAME: _____

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is not permitted for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the *Mathematics HL information booklet* is required for this paper.

SECTION A: 55 MARKS

SECTION B: 35 MARKS

TOTAL: 90 MARKS

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. (a) Find $f'(x)$ given $f(x) = x^2 \ln x$.

(b) Evaluate $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \cos(2x) dx$

(Total 5 marks)

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2. The polynomial $f(x) = x^3 + 3x^2 + ax + b$ leaves the same remainder when divided by $(x - 2)$ as when divided by $(x + 1)$. Find the value of a .

(Total 6 marks)

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3. The function f' is given by $f'(x) = 2\sin\left(5x - \frac{\pi}{2}\right)$.

(a) Write down $f''(x)$. (2)

(b) Given that $f\left(\frac{\pi}{2}\right) = 1$, find $f(x)$. (4)

(Total 6 marks)

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5. The three terms a , 1 , b are in arithmetic progression. The three terms 1 , a , b are in geometric progression. Find the value of a and of b given that $a \neq b$.

(Total 6 marks)

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6. A, B and C are the angles in a triangle such that $\cos A = \frac{3}{5}$ and $\cos B = \frac{5}{13}$.

Find the value of:

(a) $\tan 2A$,

(b) $\cos(A+B)$

(c) $\cos C$

(Total 9 marks)

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7. (a) Solve $\log_2(6-x) - \log_2(4-x) = 2$ for x , where $x < 4$.

(b) Solve $3e^t = 5 + 8e^{-t}$.

(Total 6 marks)

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8. The function f is defined by $f: x \mapsto x^3$.

Find an expression for $g(x)$ in terms of x in each of the following cases

(i) $(f \circ g)(x) = x + 1$;

(ii) $(g \circ f)(x) = x + 1$.

(Total 4 marks)

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9. In the game of darts, a dart is thrown at a board with concentric circles (circles with a common centre) painted on it. Points are awarded according to how close the centre your dart lands.



The smallest circle on this dartboard (the bullseye) has a radius of 2cm and the radii of the circles increase by 2cm each time. The probability of getting a bullseye is based on the area of the landing space.

(a) Show that the probability of getting a bullseye is $\frac{1}{25}$.

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(b) What would be the probability of getting 2 bullseyes in a row?

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Do **not** write solutions on this page.

Section B

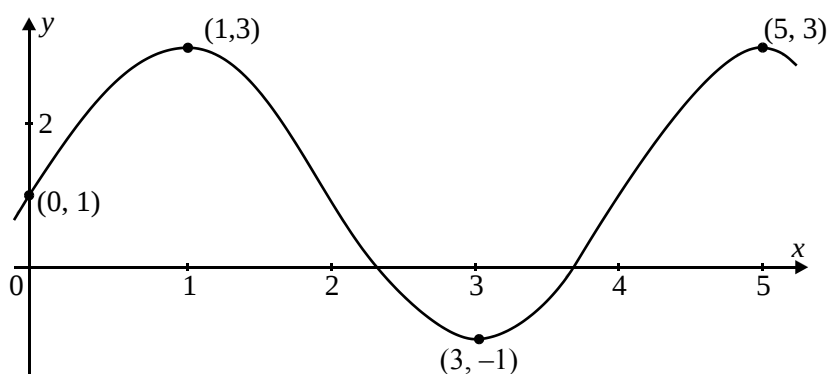
Answer **all** questions on lined paper provided. Please start each question on a new page.

10. [Maximum 10 marks]

The diagram shows the graph of the function f given by

$$f(x) = A \sin\left(\frac{\pi}{2}x\right) + B,$$

for $0 \leq x \leq 5$, where A and B are constants, and x is measured in radians.



The graph includes the points $(1, 3)$ and $(5, 3)$, which are maximum points of the graph.

(a) Write down the values of A and B .

(2)

(b) Show that $f'(x) = \pi \cos\left(\frac{\pi}{2}x\right)$.

(2)

The line $y = k - \pi x$ is a tangent line to the graph for $0 \leq x \leq 5$.

(c) Find

- (i) the point where this tangent meets the curve;
- (ii) the value of k .

(6)
(Total 10 marks)

11. [Maximum 11 marks]

Bag A contains 2 red and 3 green balls.

- (a) Two balls are chosen at random from the bag without replacement. Find the probability that 2 red balls are chosen.

(2)

Bag B contains 4 red and n green balls.

- (b) Two balls are chosen without replacement from this bag. If the probability that two red balls are chosen is $\frac{2}{15}$, show that $n = 6$.

(3)

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

- (c) Calculate the probability that two red balls are chosen.

(4)

- (d) Given that two red balls are chosen, find the probability that a 1 or a 6 was obtained on the die.

(2)

(Total 11 marks)

PTO to Q12

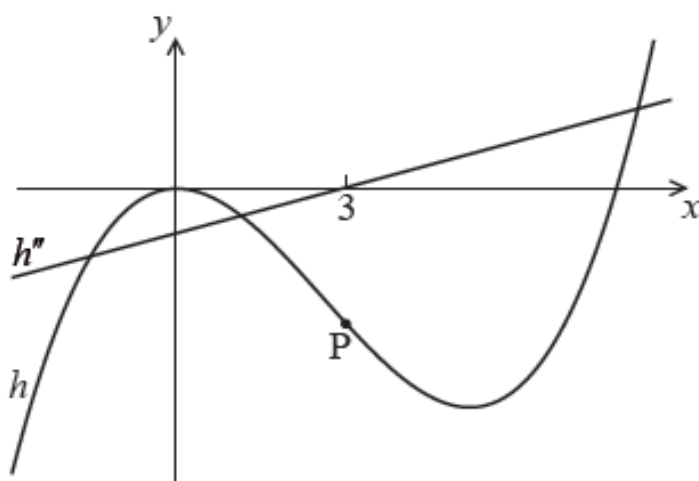
12. [Maximum 14 marks]

Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	-14	-18
$f'(x)$	1	1
$g'(x)$	-5	-3
$h''(x)$	-6	0

(a) Write down the value of $g(3)$, $f'(3)$ and $h''(2)$

The following diagram shows parts of the graphs of h and h'' .



There is a point of inflexion on the graph of h at P, when $x = 3$.

(b) Explain why P is a point of inflexion.

Given that $h(x) = f(x) \times g(x)$,

(c) find the y -coordinate of P.

(d) find the equation of the normal to the graph of $h(x)$ at the point P.