

Year 2016

VCE

Mathematical Methods

Trial Examination 1



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Victorian Certificate of Education 2016

STUDENT NUMBER

Figures
Words

Letter

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MATHEMATICAL METHODS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software) notes of any kind, blank sheets of paper, and/or correction fluid/tape.

Materials supplied

- Question and answer book of 18 pages.
- Detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

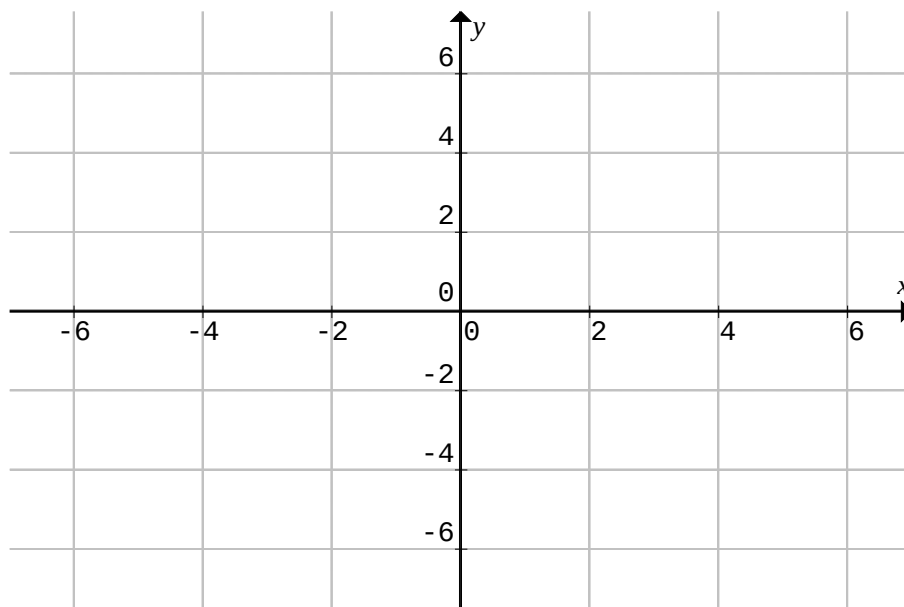
Question 1 (3 marks)

- a. Differentiate $\sqrt{16 - x^2}$ with respect to x . 1 mark

- b. If $f(x) = \log_e(\tan(2x))$, find $f'\left(\frac{\pi}{6}\right)$. Give your answer in the form $\frac{a\sqrt{b}}{c}$ where a, b, c are integers. 2 marks

Question 3 (4 marks)

- a. Sketch the graph of $y = \frac{8}{(x-2)^2} - 2$ on the axes below, clearly indicating all axial intercepts and the equation of any asymptotes, stating the maximal domain and range. 2 marks



- b. Find the area bounded by the graph of $y = \frac{8}{(x-2)^2} - 2$ the x-axis and the lines $x = 3$ and $x = 5$. 2 marks

Question 4 (4 marks)

The function f has the rule $f(x) = \log_4(x)$ and the function g has the rule

$g(x) = \sqrt{4x^2 - 1}$. State the maximal domain for which

a. $f(g(x))$ is defined.

2 marks

b. $g(f(x))$ is defined.

2 marks

Question 5 (4 marks)

- a. Find the general solution of the equation $2\cos^2(x) - \cos(x) - 1 = 0$

2 marks

- b. Solve $2\cos^2(x) - \cos(x) - 1 = 0$ for x , where $0 \leq x \leq 2\pi$

2 marks

Question 6 (4 marks)

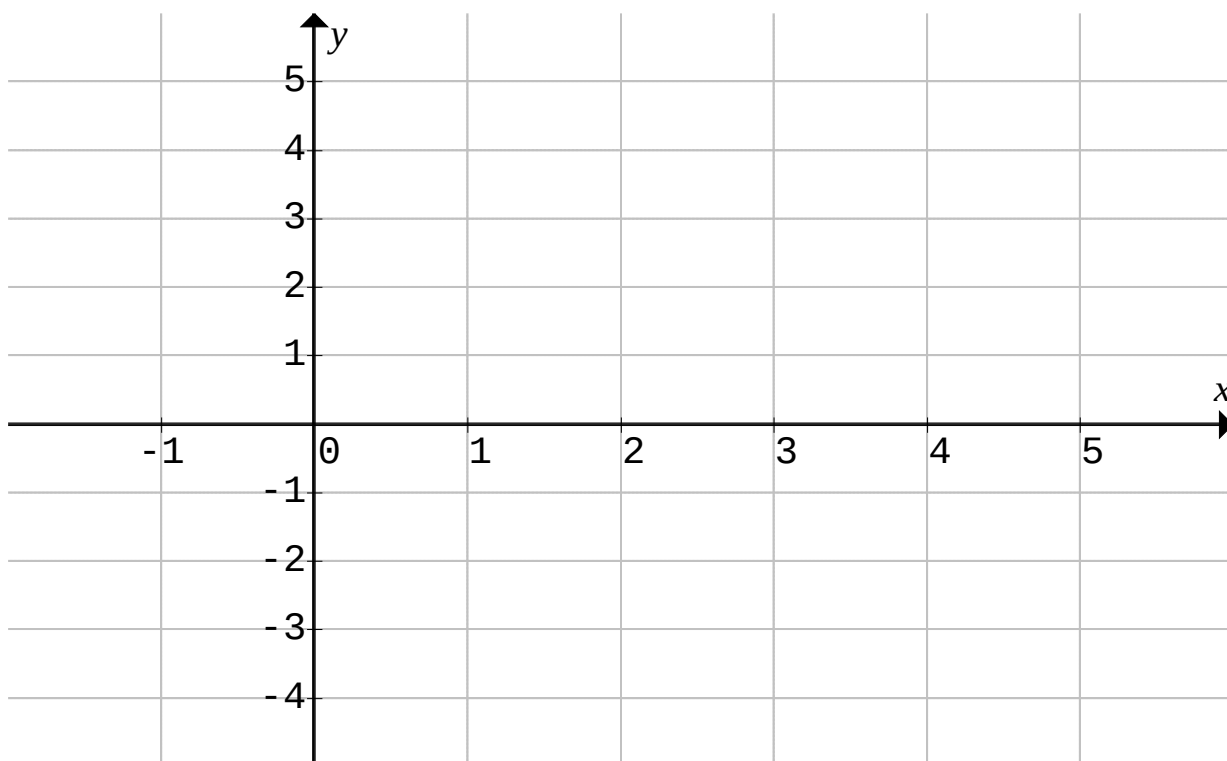
a. On the axes below, sketch the graphs of the functions

$$f : [0, 4] \rightarrow R, f(x) = 3\sin\left(\frac{\pi x}{2}\right) \text{ and } g : [0, 4] \rightarrow R, g(x) = x$$

$$\text{Hence sketch the graph of } h : [0, 4] \rightarrow R, h(x) = 3\sin\left(\frac{\pi x}{2}\right) + x,$$

stating the co-ordinates of the endpoints.

3 marks



b. Define completely the function $h'(x)$.

1 mark

Question 7 (3 marks)

A discrete random variable X has a probability distribution given by

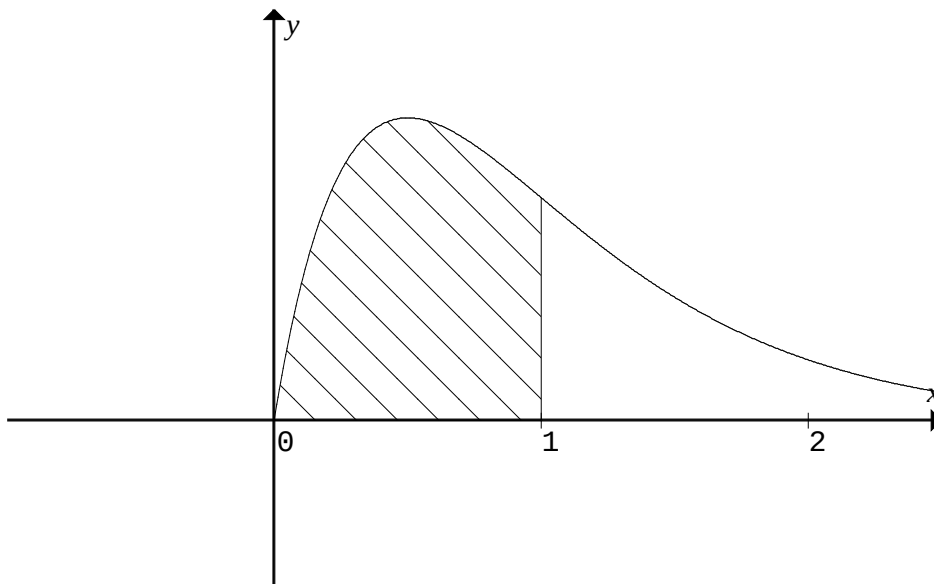
X	1	2
$\Pr(X = x)$	$\frac{1}{2e^k}$	$\frac{e^k}{4}$

Find the value(s) of k .

Question 9 (5 marks)

- a. Differentiate $x e^{-2x}$ with respect to x . 1 mark

The diagram shows part of the graph of $f : [0, \infty) \rightarrow \mathbb{R}$, $f(x) = x e^{-2x}$.



- b. Find the co-ordinates of the turning point on the graph. 1 mark

The graph of the tangent line to the curve at the point $P(p, f(p))$, where $1 \leq p \leq 3$ is also shown.

- a.** Determine the equation of this tangent line in terms of p .

1 mark

- b.** If this tangent line crosses the x -axis at the point S , and crosses the y -axis at the point R , find the co-ordinates of the points S and R in terms of p .

2 marks

If O is the origin, then the area A of the triangle OSR is given by $A = \frac{(9 + p^2)^2}{4p}$.

- c. Find the **minimum** area of the triangle OSR and the value of p for which the area is a minimum.

3 marks

- d. Find the **maximum** area of the triangle OSR and the value of p for which the area is a maximum.

1 mark

END OF QUESTION AND ANSWER BOOKLET**END OF EXAMINATION**

MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Mathematical Methods formulas

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$	volume of a pyramid	$\frac{1}{3}Ah$
curved surface area of a cylinder	$2\pi rh$	volume of a sphere	$\frac{4}{3}\pi r^3$
volume of a cylinder	$\pi r^2 h$	area of triangle	$\frac{1}{2}bc \sin(A)$
volume of a cone	$\frac{1}{3}\pi r^2 h$		

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$
$\frac{d}{dx}((ax+b)^n) = na(ax+b)^{n-1}$	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c, x > 0$
$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$	$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$	$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$	
product rule $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$	quotient rule $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
chain rule $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$	

Probability

$\Pr(A) = 1 - \Pr(A')$		$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
$\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$			
mean	$\mu = E(X)$	variance	$\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x p(x)$	$\sigma^2 = \sum (x - \mu)^2 p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x) dx$	$\mu = \int_{-\infty}^{\infty} x f(x) dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$

Sample proportions

$\hat{p} = \frac{X}{n}$		mean	$E(\hat{p}) = p$
standard deviation	$\text{sd}(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$	approximate confidence interval	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$

END OF FORMULA SHEET