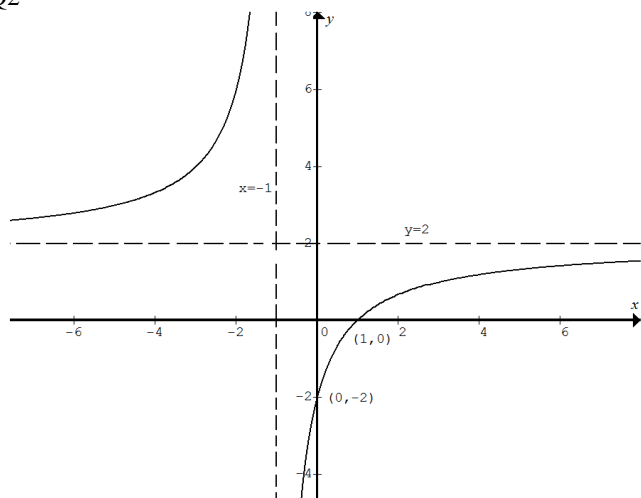


Q1a $\frac{d}{dx} \sqrt{4-x} = -\frac{1}{2\sqrt{4-x}}$

Q1b $f(x) = \frac{x}{\sin x}$, $f'(x) = \frac{(\sin x)(1) - x(\cos x)}{\sin^2 x}$
 $f'\left(\frac{\pi}{2}\right) = \frac{(\sin \frac{\pi}{2})(1) - \frac{\pi}{2}(\cos \frac{\pi}{2})}{\sin^2 \frac{\pi}{2}} = 1$

Q2



Q3a $\int \frac{1}{(2x-1)^3} dx = \frac{1}{2(-2)(2x-1)^2} = -\frac{1}{4(2x-1)^2}$

Q3b $g'(x) = \sin(2\pi x)$, $g(x) = \int \sin(2\pi x) dx = -\frac{\cos(2\pi x)}{2\pi} + c$

Given $g(1) = \frac{1}{\pi}$, $g(1) = -\frac{\cos(2\pi)}{2\pi} + c = \frac{1}{\pi}$, $\therefore c = \frac{3}{2\pi}$

$\therefore g(x) = \frac{3 - \cos(2\pi x)}{2\pi}$

Q4a $\bar{X} = 0 \times 0.2 + 1 \times 0.2 + 2 \times 0.5 + 3 \times 0.1 = 1.5$

Q4b $0.2^3 = 0.008$

Q5a At the point of intersection, $\cos \frac{\pi}{3} = a \sin \frac{\pi}{3}$

$\therefore \frac{1}{a} = \tan \frac{\pi}{3} = \sqrt{3}$, $\therefore a = \frac{1}{\sqrt{3}}$

Q5b $\tan x = \sqrt{3}$, $x = \frac{\pi}{3}, \frac{4\pi}{3}$, $\therefore$ the other point of intersection

is at $x = \frac{4\pi}{3}$.

Q6a $2 \log_3 5 - \log_3 2 + \log_3 x = 2$, $\log_3 \frac{5^2 x}{2} = 2$

$\frac{5^2 x}{2} = 3^2$, $x = \frac{18}{25}$

Q6b $3e^t = 5 + 8e^{-t}$, equation $\times e^t$, $3(e^t)^2 - 5e^t - 8 = 0$
 $(3e^t - 8)(e^t + 1) = 0$.

Since $e^t + 1 \neq 0$, $\therefore 3e^t - 8 = 0$, $t = \log_e \frac{8}{3}$

Q7a $sd(\hat{p}) \leq \frac{1}{100}$, $\sqrt{\frac{p(1-p)}{n}} \leq \frac{1}{100}$ where $p = \frac{1}{5}$

$\therefore \frac{2}{5\sqrt{n}} \leq \frac{1}{100}$, $\sqrt{n} \geq 40$, $n \geq 1600$, $\therefore$ smallest $n = 1600$

Q7b $\Pr(\text{exactly one success}) = \Pr(SF) + \Pr(FS)$
 $= \Pr(S)\Pr(F|S) + \Pr(F)\Pr(S|F) = \frac{1}{23} \times \frac{22}{22} + \frac{22}{23} \times \frac{1}{22} = \frac{2}{23}$

Q8a $\int_0^m \frac{1}{5} e^{-\frac{x}{5}} dx = 0.5$, $\left[-e^{-\frac{x}{5}}\right]_0^m = 0.5$, $-e^{-\frac{m}{5}} + e^0 = 0.5$

$e^{-\frac{m}{5}} = 0.5$, $m = 5 \log_e 2$

Q8b $\Pr(X < 1 | X \leq 5 \log_e 2) = \frac{\Pr(X < 1)}{\Pr(X \leq 5 \log_e 2)}$

$= \frac{\int_0^1 \frac{1}{5} e^{-\frac{x}{5}} dx}{0.5} = \frac{\left[-e^{-\frac{x}{5}}\right]_0^1}{0.5} = \frac{-e^{-\frac{1}{5}} + e^0}{0.5} = 2\left(1 - e^{-\frac{1}{5}}\right)$

Q9a $\frac{d}{dx} x^2 \log_e x = x^2 \left(\frac{1}{x}\right) + 2x \log_e x = x + 2x \log_e x$

Q9b $\frac{d}{dx} x^2 \log_e x = x + 2x \log_e x$

$\therefore x \log_e x = \frac{1}{2} \left(\frac{d}{dx} x^2 \log_e x - x \right)$

Area of the shaded region

$= \int_1^3 x \log_e x dx = \frac{1}{2} \left(\int_1^3 \frac{d}{dx} x^2 \log_e x dx - \int_1^3 x dx \right)$
 $= \frac{1}{2} \left(\left[x^2 \log_e x \right]_1^3 - \left[\frac{x^2}{2} \right]_1^3 \right) = \frac{1}{2} \left(9 \log_e 3 - \frac{9}{2} + \frac{1}{2} \right) = \frac{9}{2} \log_e 3 - 4$

Q10a $y = ax^2 + bx$, $\frac{dy}{dx} = 2ax + b$

At $(2, 4)$, $4 = a(2^2) + b(2)$, $\therefore 4a + 2b = 4$, $\therefore 2a + b = 2 \dots (1)$

also gradient of the tangent $= 2a(2) + b = 4a + b = \frac{0-4}{6-2}$,

$\therefore 4a + b = -1 \dots (2)$

Solve (1) and (2) simultaneously, $a = -\frac{3}{2}$ and $b = 5$

Q10bi Gradient of VQ = gradient of QU

$\therefore \frac{v-4}{0-2} = \frac{4-0}{2-u}$, $v-4 = \frac{8}{u-2}$, $v = 4 + \frac{8}{u-2}$, $v = \frac{4u}{u-2}$

Q10bii Shaded area $A(u) = \frac{1}{2}uv - 8 = \frac{2u^2}{u-2} - 8$

Let $\frac{dA}{du} = 0$ to find the turning point(s).

$\frac{(u-2)(4u) - (2u^2)(1)}{(u-2)^2} = 0 \therefore 2u^2 - 8u = 0$, $2u(u-4) = 0$.

Since $u > 0$, $\therefore u = 4$ and $A = \frac{2(4^2)}{4-2} - 8 = 8$

First end point at $u = \frac{5}{2}$, $A = \frac{2(\frac{5}{2})^2}{\frac{5}{2}-2} - 8 = 17$

Second end point at $u = 6$, $A = \frac{2(6^2)}{6-2} - 8 = 10$

$\therefore A_{\min} = 8$ square units

Q10biii From part ii, $A_{\max} = 17$ square units at $u = \frac{5}{2}$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors