



Units 3 and 4 Maths Methods (CAS): Exam 2

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Section A – Multiple-choice questions

Question 1

The correct answer is A.

This can be observed by graphing $f(x)$.

Question 2

The correct answer is B.

$$f(1) = -6$$

$$f(3) = -38$$

$$\text{Average rate of change} = \frac{\Delta f(x)}{\Delta x} = \frac{f(3) - f(1)}{3 - 1} = -16$$

Question 3

The correct answer is C.

Let $f(x) = y$. Swap x and y , then solve for y .

Question 4

The correct answer is A.

$$f(-1) = 5$$

$$f(2) = -1$$

The turning point of the function $f(x)$ occurs at $(0, 7)$. Hence the range of the function is $[-1, 7]$.

Question 5

The correct answer is A.

We can construct a 2×2 matrix and find its determinant. If the determinant is zero, the system of simultaneous equations has either no or infinite number of solutions.

$$\begin{bmatrix} m+2 & 1 \\ 4 & m-1 \end{bmatrix}$$

$$\text{Determinant} = (m+2)(m-1) - 4 = 0$$

$$m = 2 \text{ or } -3$$

Substituting $m = -3$ and $n = 2$ yields the infinite number of solutions.

Question 6

The correct answer is D.

$$-\int_{-1}^3 (3f(x) - 2)dx = -3 \int_{-1}^3 f(x)dx + \int_{-1}^3 2dx = 2$$

Question 7

The correct answer is D.

The derivative of a positive parabola yields a positive linear graph. Likewise, a negative linear graph yields a negative horizontal gradient graph.

Question 8

The correct answer is C.

This can be observed from the graph of $f(x)$.

Question 9

The correct answer is B.

$$(x, y) \rightarrow \left(-\frac{x}{2}, y - 3\right) \rightarrow (x', y')$$

$$x = -2x'$$

$$y = y' + 3$$

$$\therefore y' = -4x'^2 - 3$$

Question 10

The correct answer is B.

$$g(x) = -x^3 + \frac{1}{2}x^2 - x + c$$

$$g(1) = -1 + \frac{1}{2} - 1 + c = \frac{9}{2}$$

$$c = 6$$

$$\therefore g(x) = -x^3 + \frac{1}{2}x^2 - x + 6$$

$$g(-2) = 18$$

Question 11

The correct answer is A.

$$\text{Average value} = \frac{2}{5} \int_{\frac{1}{2}}^3 -\frac{\sin(x)}{x} dx = -0.0175$$

Question 12

The correct answer is B.

A signed area can have a negative value. Evaluating $\int_{-\infty}^0 f(x) dx$ gives $-\frac{5}{2}$.

Question 13

The correct answer is A.

$$\frac{dy}{dx} = 2 \cos(2x)$$

Hence, the gradient of the tangent is, $2 \cos\left(2 * \frac{\pi}{6}\right) = 1$.

$y = \frac{\sqrt{3}}{2}$ when $x = \frac{\pi}{6}$. The equation of the tangent is therefore $y = x - \frac{\pi}{6} + \frac{\sqrt{3}}{2}$.

Question 14

The correct answer is E.

$$\text{Var}(X) = sd^2 = np(1 - p) = 81$$

$$900p(1 - p) = 81$$

$$p = 0.1 \text{ or } 0.9$$

$$p = 0.1 \text{ since } p < 0.5$$

$$\text{mean} = np = 900 * 0.1 = 90$$

Question 15

The correct answer is B.

$\Pr(B) = 0.65$ can be obtained by sketching a Venn diagram.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{0.30}{0.65} = 0.4615$$

Question 16

The correct answer is C.

$$X \sim N(21, 16)$$

$$\Pr(X < 14) = 0.0401$$

Question 17

The correct answer is E.

$$\Pr(Z < z) = 0.7$$

$$z = 0.5244$$

$$z = \frac{x - \mu}{\sigma}$$

$$0.5244 = \frac{25.6 - 20}{\sigma}$$

$$\therefore \sigma = 10.7$$

Question 18

The correct answer is D.

The area under the graph $f(x)$ between $x = 0$ and $x = a$ must be 1.

$$\begin{aligned} \int_0^a e^{-2x} + 3 \, dx &= 1 \\ -\frac{1}{2}e^{-2a} + 3a + \frac{1}{2} &= 1 \\ a &\approx 0.26 \end{aligned}$$

Question 19

The correct answer is A.

Year 11 Male: 248

Year 11 Female: 248

Year 12 Male: 209

Year 12 Female: 171

Total number of Year 11 and 12 students: 876

Year 11 and 12 boys to be selected:

$$\frac{248}{876} * 100 \approx 28 \text{ Year 11 boys}$$

$$\frac{209}{876} * 100 \approx 24 \text{ Year 12 boys}$$

The values should be rounded to whole numbers due to the nature of the sample.

Question 20

The correct answer is B.

$$\hat{p} = 0.17$$

$$n = 20$$

z value for 99% confidence interval = 2.58

Substitute the values into the confidence interval formula $(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}})$.

Section B – Short-answer questions

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a i

Amplitude = 2 [1]

Question 1a ii

Period = $\frac{2\pi}{\frac{\pi}{6}} = 12$ [1]

Question 1b

A sinusoidal graph with 2 complete cycles should be drawn over the domain, $t \in [0, 24]$ [2]

End points are at $(0, 4 - \sqrt{2})$ and $(24, 4 - \sqrt{2})$ [1]

Maximum and minimum at $(\frac{3}{2}, 2), (\frac{15}{2}, 6), (\frac{27}{2}, 2), (\frac{39}{2}, 6)$ [1]

Question 1c

$h'(t) = \frac{\pi}{3} \cos(\frac{\pi t}{6} - \frac{3\pi}{4})$ [1]

$h'(2.5) = \frac{\pi}{6}$ metre/hour [1]

Question 1d i

$h'(t) = \frac{\pi}{3} \cos(\frac{\pi t}{6} - \frac{3\pi}{4}) = 0$

$t = \frac{27}{2}, \frac{39}{2}, t \in [12, 24]$

Lowest depth at $t = \frac{27}{2}$ can be determined using double derivative or graph [1]

Question 1d ii

Highest point at $t = \frac{39}{2}$ [1]

Question 1e

$h(t) = 2.6$ [1]

$t = 3.02, 11.98, 15.02, 23.98$

The ferry operates between 6 am and 8 pm, $t \in [6, 20]$ [1]

Hence the ferry can enter the water when $t \in [6, 11.98]$ and $t \in [15.02, 20]$.

$(11.98 - 6) + (20 - 15.02) = 10.96$ hours [1]

45 minutes = 0.75 hours

The number of round trip = $\frac{10.96}{0.75} = 14.62$.

Round down to a whole number = 14 round trips per day [1]

Question 2a

$f(x) = -\frac{1}{3}(x^3 - 12x + 16)$

Perform long division $(x^3 - 12x + 16) \div (x - 2)$ [2] for showing long division.

$$f(x) = -\frac{1}{3}(x - 2)^2(x + 4) \text{ [1]}$$

Question 2b

$$f'(x) = -x^2 + 4 \text{ [1]}$$

$$f'(x) = 0$$

$$x = -2 \text{ or } 2 \text{ [1]}$$

$$f(-2) = -\frac{32}{3}$$

$$f(2) = 0$$

$$\therefore (2, 0) \text{ local maximum, } \left(-2, -\frac{32}{3}\right) \text{ local minimum [1]}$$

Question 2c

The graph should be a negative cubic with local maximum and x-intercept at $(2, 0)$, local minimum at $\left(-2, -\frac{32}{3}\right)$. The other x-intercept is located at $(-4, 0)$ and the y-intercept at $\left(0, -\frac{16}{3}\right)$ [3]

Question 2d

$a = 2$ for an inverse function to exist.

Question 2e

$$f'(x) = -x^2 + 4$$

$$f'(-3) = -5 \text{ [1]}$$

$$f(-3) = -\frac{25}{3} \text{ [1]}$$

$$y - \left(-\frac{25}{3}\right) = -5(x - (-3))$$

$$y = -5x - \frac{70}{3} \text{ [1]}$$

Question 2f

$$y = -5x - \frac{70}{3}$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(-5x - \frac{70}{3}\right)^2} \text{ [1]}$$

$$\frac{dL}{dx} = \frac{\sqrt{2}(39x+175)}{\sqrt{117x^2+1050x+2450}} = 0 \text{ for minimum distance [1].}$$

$$\therefore x = a = -\frac{175}{39} \text{ and } y = b = -\frac{35}{39}$$

$$\text{Hence, } a = -4.487, b = -0.897 \text{ [1]}$$

$$L = \sqrt{\left(-\frac{175}{39}\right)^2 + \left(-\frac{35}{39}\right)^2} = 4.576 \text{ to 3 decimal places [1]}$$

Question 2g

$$y = -5x - \frac{70}{3}$$

$$f(x) = -\frac{x^3}{3} + 4x - \frac{16}{3}$$

$$-5x - \frac{70}{3} = -\frac{x^3}{3} + 4x - \frac{16}{3} \quad [1]$$

$$x = -3 \text{ or } 6$$

$$\text{Area} = \int_{-3}^6 \left(-\frac{x^3}{3} + 4x - \frac{16}{3} \right) - \left(-5x - \frac{70}{3} \right) dx \quad [1]$$

$$\text{Area} = 182.25 \quad [1]$$

Question 3a

Let X = the number of times that Jen won the race.

$$\Pr(X > 3.5) \quad [1]$$

$$= \Pr(X = 4) + \Pr(X = 5) + \Pr(X = 6) + \Pr(X = 7)$$

$$= \binom{7}{4}(0.48)^4(0.52)^3 + \binom{7}{5}(0.48)^5(0.52)^2 + \binom{7}{6}(0.48)^6(0.52)^1 + \binom{7}{7}(0.48)^7(0.52)^0 \quad [1]$$

$$= 0.4563 \quad [1]$$

Question 3a ii

$$\Pr(X = 29 | X > 25)$$

$$= \frac{\Pr(X=29)}{\Pr(X>25)} \quad [1]$$

$$= \frac{0.0791}{0.5569}$$

$$= 0.1421 \quad [1]$$

Question 3b i

$$M = 60 \quad [1]$$

Question 3b ii

$$\int_{60}^k -\frac{1}{800}(x - 60)^2 + \frac{1}{10} dx = \frac{1}{2} \quad [1]$$

$$k = 42.440, 65.822, 71.737$$

$$\therefore k = 65.822, k \in (60, 68.944) \quad [1]$$

$$\therefore a = 65.822 - 60 = 5.822 \quad [1]$$

Question 3b iii

$$\mu = \int_{54.178}^{65.822} -\frac{x}{800}(x - 60)^2 + \frac{1}{10} dx \quad [1]$$

$$= 59.997 \text{ seconds} \quad [1]$$

Question 3b iv

The lower limit for the probability density function: $M - a = 54.178 \quad [1]$

$$\int_{54.178}^{56} f(x) dx = 0.127 \text{ [1]}$$

Question 4a

$$N(0) = 215 \text{ [1]}$$

Question 4b

$$N\left(\frac{1}{3}\right) \approx 221 \text{ [1]}$$

Question 4c

$$N_{max} = 215 * 100 = 21500$$

$$15 * e^t + 200 = 21500$$

$$t = 7.258 \text{ hours [1]}$$

$$D(T) = ae^{-\frac{1}{100}T} + 500$$

$$21500 = ae^{-\frac{1}{100} * 7.258} + 500 \text{ [1]}$$

$$a = 22581 \text{ [1]}$$

Question 4d

Criteria for a large population:

$$np \geq 10$$

$$nq \geq 10$$

$$10n \leq N$$

Let $np = 10$ for the smallest sample size.

$$p = 0.02$$

$$\therefore n = 500 \text{ [1]}$$

Question 4e

$$\mu_{\hat{p}} = p = 0.02 \text{ [1]}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.02 * 0.98}{5000}} = 0.002 \text{ [2]}$$

Question 4f

$$n = 100\,000, \hat{p} = 0.035 \text{ [1]}$$

For the 90% confidence interval, the area under the curve to the left of z value is 0.95.

Hence the relevant z-value is 1.64 using the inverse normal [1]

$$(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}) = (0.0340, 0.0360) \text{ [1]}$$

The microbiologist can be 90% confident that a 3.4-3.6% of the *E.coli* population is pathogenic [1]