



Units 3 and 4 Maths Methods (CAS): Exam 1

Practice Exam Solutions

Stop!

Don't look at these solutions until you have attempted the exam.

Any questions?

Check the Engage website for updated solutions, then email practiceexams@ee.org.au.

Marks allocated are indicated by a number in square brackets, for example, [1] indicates that the line is worth one mark.

Question 1a

$$\frac{dy}{dx} = 2x \sin(3x) + 3x^2 \cos(3x) [2]$$

[1] for an application of chain rule

Question 1b

$$f(x) = xe^{-x}$$

$$f'(x) = e^{-x} - xe^{-x}$$

$$= (1 - x)e^{-x} [1]$$

$$f'(2) = (1 - 2)e^{-2}$$

$$= -e^{-2} [1]$$

Question 2

$$\int_1^2 \frac{1}{\sqrt{2x-1}} dx = \int_1^2 (2x-1)^{-1/2} dx$$

$$= [(2x-1)^{1/2}]_1^2 [1]$$

$$= (2(2)-1)^{1/2} - (2(1)-1)^{1/2}$$

$$= \sqrt{3} - 1 [1]$$

Question 3

$$\text{Let } a = \frac{\theta}{2} + \frac{\pi}{4}$$

$$a \in \left[\frac{\pi}{4}, \frac{11\pi}{4}\right]$$

[1] for acknowledging the domain

Substitute a for $\frac{\theta}{2} + \frac{\pi}{4}$:

$$\tan(a) = \sqrt{3}$$

$$a = \tan^{-1} \sqrt{3}$$

$$= \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3} [1]$$

$$\text{Hence, } \frac{\theta}{2} + \frac{\pi}{4} = \frac{\pi}{3}, \frac{4\pi}{3}, \frac{7\pi}{3}$$

$$\therefore \theta = \frac{\pi}{6}, \frac{13\pi}{6}, \frac{25\pi}{6} [1]$$

Question 4

$$e^{2x} - 6e^x + 8 = 0$$

$$(e^x - 2)(e^x - 4) = 0 [1]$$

$$e^x = 2 \text{ or } 4$$

$$\therefore x = \ln(2) \text{ or } \ln(4) \text{ [1]}$$

Question 5a

$$\text{ran}_f = \mathbb{R}$$

$$\text{dom}_g \in \left[\frac{1}{2}, \infty\right)$$

$$\therefore \text{ran}_f \not\subseteq \text{dom}_g$$

Hence, $g(f(x))$ does not exist. [1]

Question 5b

$$f(g(x)) = -4(2x - 1)^{3/2} \text{ [1]}$$

$$\therefore \text{dom}_{f \circ g} \in \left[\frac{1}{2}, \infty\right) \text{ [1]}$$

$$\text{ran}_{f \circ g} \in (-\infty, 0] \text{ [1]}$$

Question 6a

Find the y- intercept:

$$y = \ln(3 - 0) = \ln(3) \text{ [1]}$$

Rearrange the equation in terms of x:

$$y = \ln(3 - x)$$

$$e^y = 3 - x$$

$$x = 3 - e^y \text{ [1]}$$

Find the area enclosed:

$$\int_0^{\ln(3)} 3 - e^y \, dy$$

$$= [3y - e^y]_0^{\ln(3)} \text{ [1]}$$

$$= (3 \ln(3) - e^{\ln(3)}) - (0 - e^0)$$

$$= 3 \ln(3) - 3 + 1$$

$$= 3 \ln(3) - 2 \text{ [1]}$$

Question 6b

$$(x, y) \rightarrow (x, -y) \rightarrow (x, -2y) \rightarrow (x + 1, -2y) \rightarrow (x', y') \text{ [1]}$$

$$x' = x + 1$$

$$x = x' - 1$$

$$y' = -2y$$

$$y = -\frac{y'}{2}$$

$$-\frac{y'}{2} = \log_e(3 - (x' - 1))$$

$$\therefore y' = -2\log_e(4 - x') [1]$$

Question 7a i

$$E(X) = 0.2 \cdot 4 + 0.4 \cdot 5 + 0.3 \cdot 6 + 0.1 \cdot 7$$

$$= 5.3 [1]$$

Question 7a ii

$$\Pr(X > 5.3) = \Pr(X=6) + \Pr(X=7)$$

$$= 0.3 + 0.1$$

$$= 0.4 [1]$$

Question 7b

$$\text{Var}(X) = 0.81$$

$$\text{Sd}(X) = \sqrt{\text{Var}(X)} = 0.9 [1]$$

$$\text{sd}(2X-1) = 4\text{sd}(X)$$

$$= 4 \cdot 0.9$$

$$= 3.6 [1]$$

Question 8

$$f'(x) = 6x - 4$$

$$f'(2) = 12 - 4 = 8 [1]$$

Gradient of the tangent at $x=2$ is 8.

Hence the gradient of the normal is $-\frac{1}{8}$ [1]

$$f(2) = 4$$

$\therefore$ the equation of normal:

$$y - 4 = -\frac{1}{8}(x - 2)$$

$$y = -\frac{1}{8}x + \frac{17}{4} [1]$$

Question 9

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$0.2^2 = \frac{0.1 \cdot 0.9}{n} [1]$$

$$n = 2.25 [1]$$

Question 10a

$$= \sqrt{9} = 3$$

$$\Pr(33 < X < 42) = \Pr(36 - < X < 36 + 2) [1]$$

$$= 0.68 * 0.5 + 0.95 * 0.5$$

$$= 0.815 [1]$$

Question 10b

$$\Pr(X < 39 | X > 36) = \frac{0.68 * 0.5}{0.5} = 0.68 [1]$$

Question 11a

Let radius of the oil in container = r

$$h:r = 20:5 = 4:1$$

$$\text{Hence, } r = \frac{h}{4} [1]$$

$$V = \frac{1}{3} \text{ base} * \text{height}$$

$$= \frac{1}{3} * \pi r^2 * h$$

$$= \frac{1}{3} * \pi \left(\frac{h}{4}\right)^2 * h$$

$$V = \frac{h^3 \pi}{48} [1]$$

Question 11b

$$V(15) = \frac{15^3 \pi}{48}$$

$$V(15) = \frac{1125\pi}{16} \text{ cm}^3 [1]$$

Question 11c

$$V = \frac{h^3 \pi}{48}$$

$$\frac{dV}{dh} = \frac{h^2 \pi}{16} [1]$$

$$\pi = \frac{h^2 \pi}{16}$$

$$h^2 = 16$$

$$h = \pm 4$$

$$h = 4 \text{ since } h > 0 [1]$$