Student Name: ________________________________

MATHEMATICAL METHODS (CAS)

Unit 2
Targeted Evaluation Task for School-assessed Coursework 3

TSSM™
Creating VCE Success

2015 Modelling task on circular functions for Outcomes 2 & 3

Recommended writing time*:1 hour 30 minutes
Total number of marks available: 50 marks

TASK BOOK

* The recommended writing time is a guide to the time students should take to complete this task. Teachers may wish to alter this time and can do so at their own discretion.

Conditions and restrictions

- Students are permitted to bring into the room for this task: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the room for this task: blank sheets of paper and/or white out liquid/tape.
- A CAS calculator is permitted in this task.

Materials supplied

- Question and answer book of 9 pages.

Instructions

- Print your name in the space provided on the top of the front page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic communication devices into the room for this task.

Modelling task questions

Instructions

- Answer each question in the space provided.
- Please provide appropriate workings and use exact answers when required.
- Unless otherwise stated, all decimals should be given correct to 2 decimal places.

Question 1

The population of koalas in a particular area varies according to the rule

$$N(t) = 1000 + 200cos\left(\frac{\pi t}{3}\right)$$

where N is the number of koalas and t is the number of months after 1 Jan 2013.

a. Find the amplitude and period of the function $N(t)$.

2 marks

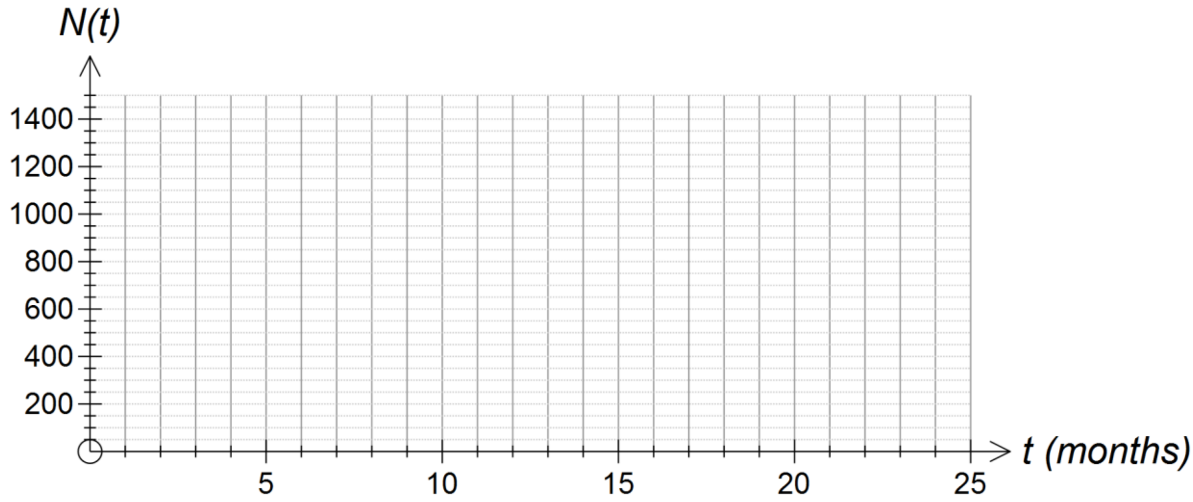
b. Find the maximum and minimum number of koalas in this area.

2 marks

c. After how many months does the population of koalas become minimum for the first time?

2 marks

d. Sketch the graph of $N(t)$ over the first 24 months showing all axes intercepts and turning points correct to four decimal places.



3 marks

e. Find the population of koalas after 10 months.

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1 mark

f. For how long does the population remain more than $N(10)$, before it becomes less than $N(10)$ again?

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2 marks

In the same area, the population of wombats varies according to the rule

$$P(t) = 1000 - 400sin\left(\frac{\pi t}{6}\right)$$

where P is the number of wombats and t is the number of months after 1 Jan 2013.

g. When does the population of wombats become more than koalas?

2 marks

h. How many more koalas than wombats are there in the area after 12 months?

1 mark

Total 15 marks

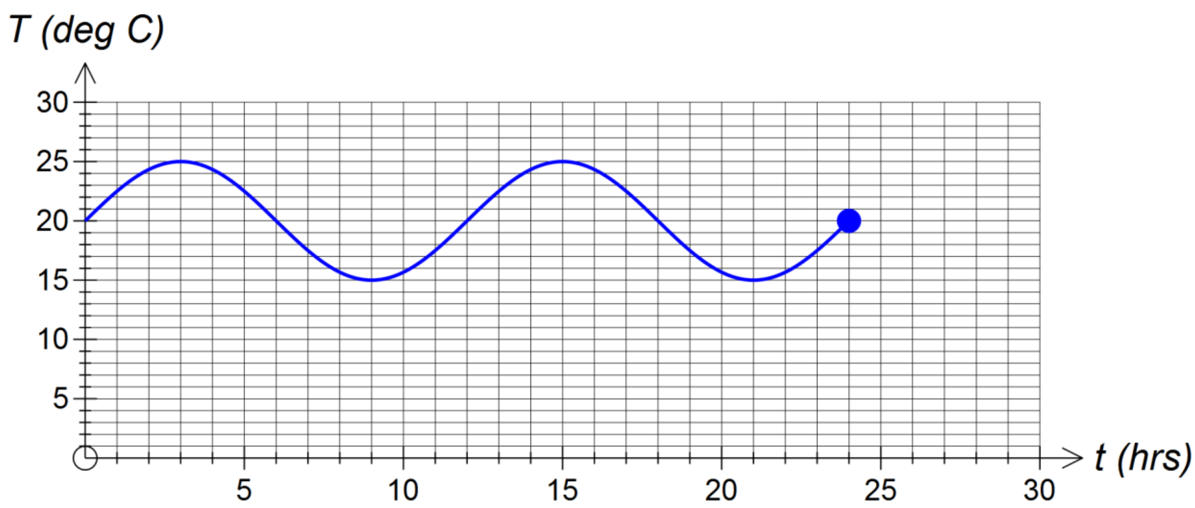
Question 2

The temperature T, of a room is modelled by the equation

$$T(t) = a\sin(nt) + b$$

where t is the number of hours after 7 a.m.

The graph of this model is given below.



a. Find the value of b.

1 mark

b. Find the value of n.

2 marks

c. Find the value of a. Write down the rule of the function.

2 marks

d. Find the range of the function $y = f(x)$.

1 mark

e. Find the time(s) when the temperature is maximum in the first 24 hours.

2 marks

f. Find the maximum temperature between 2pm and 6pm.

2 marks

g. Find the time when the temperature is 18.5 degrees.

3 marks

Total 13 marks

Question 3

The function $f(x)$ has the form $f(x) = a\tan(nx) + b$, where a, n and b are real numbers.

a. Find the period of $f(x)$ in terms of n.

1 mark

b. If $x = \frac{3\pi}{2}$ is an asymptote of $f(x)$, find the value of n.

2 marks

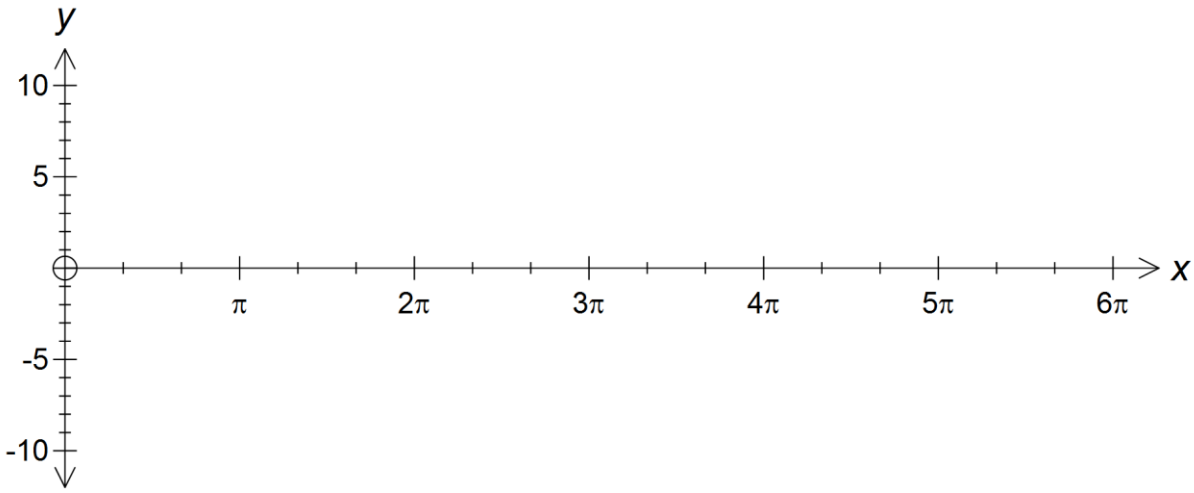
c. If the y-intercept of $f(x)$ is (0, 1), find the value of b.

2 marks

d. If $f(x)$ passes through $\left(\pi, 1 - 2\sqrt{3}\right)$, find the value of a. Also, write down the function.

3 marks

e. Sketch the function $f(x)$ over the interval $[0, 6\pi]$ on the axes below. Label the asymptotes, end-points and axes intercepts, correct to three decimal places.



4 marks

Total 12 marks

Question 4

A ferris wheel with a radius of 25 meters makes one rotation every 36 seconds. At the bottom of the ride, the passenger is 1 metre above the ground.

The height, in metres, of the passenger above the ground can be modelled by the equation

$$h(t) = a + b\cos(nt),$$

where t is the time in seconds after the ride begins to move.

The passenger is at a height of 51m at the commencement of the ride.

a. Find the values of a and n.

3 marks

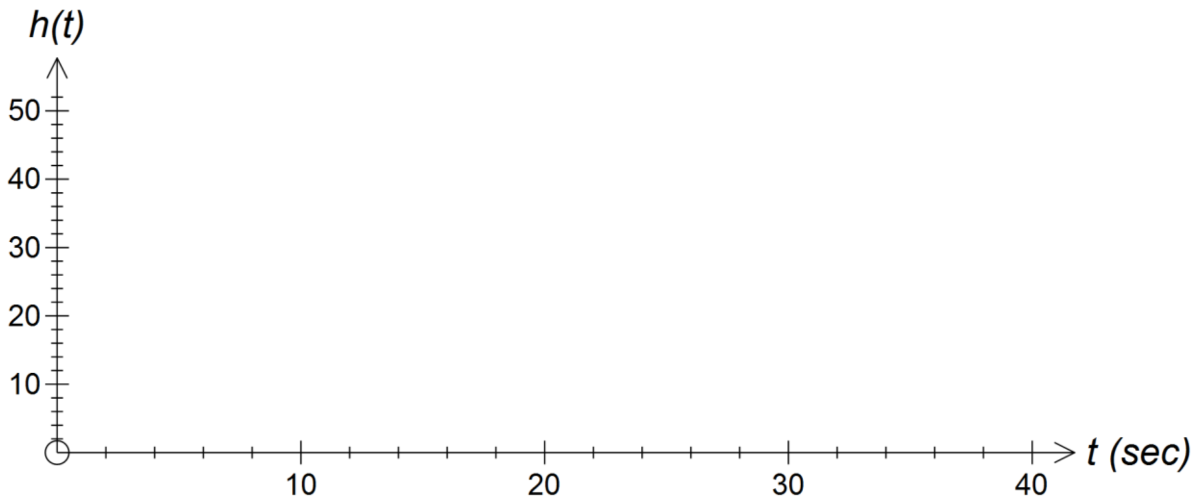
b. Write down the equation to find the height of the passenger.

1 mark

c. Find the height after 45 seconds.

1 mark

d. Sketch the graph of $h(t)$ on the axes below for one rotation.



3 marks

e. After how many seconds, correct to the nearest whole number, is the height of the passenger 12m above the ground?

2 marks

Total 10 marks

END OF TASK BOOK