

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 2



2015 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer: A

Explanation:

Solve the two equations on CAS.

Question 2

Answer: C

Explanation:

It is negative cubic so either C or D. Check the x-intercept.

Question 3

Answer: E

Explanation:

Define the functions on CAS and find $f(g(x))$

Question 4

Answer: D

Explanation:

$$f(x) = 2\left(\sqrt{x} + \frac{1}{2}\right)$$

$$g(x) = 2 \times \frac{1}{2}\left(\sqrt{x} + \frac{1}{2}\right)$$

Question 5

Answer: C

Explanation:

Domain: $4 - x \geq 0$ gives $x \leq 4$ and the graph is above the x-axis.

Question 6

Answer: A

Explanation:

$$Av ROC = \frac{f(8) - f(2)}{8 - 2}$$

Question 7

Answer: C

Explanation:

Note the shaded end-points.

Question 8

Answer: C

Explanation:

$$f(g(x)) = \frac{3}{x+5}, \quad x \neq -2$$

Question 9

Answer: E

Explanation:

Test the validity of the vertical and horizontal line tests.

Question 10

Answer: D

Explanation:

$$\text{Amp} = 2, \text{ Period} = \frac{2\pi}{\frac{1}{5}}.$$

Question 11

Answer: E

Explanation:

$$\frac{dy}{dx} \text{ at } x = 4 \text{ on CAS.}$$

Question 12

Answer: B

Explanation:

$$A_1 = A_2$$

Question 13

Answer: B

Explanation:

$$\text{normline}(f(x), x, 0) \text{ on CAS.}$$

Question 14

Answer: C

Explanation:

$$(f(x))^2 \times (f(y))^2 = e^{2x} \times e^{2y} = e^{2x+2y} = f(2x+2y)$$

Question 15

Answer: A

Explanation:

$$\frac{1}{k} \int_0^k x^3 dx = 9 \text{ gives } k = 6^{\frac{2}{3}} \text{ on CAS.}$$

Question 16

Answer: B

Explanation:

$$\text{binompdf}\left(10, \frac{1}{5}, 6\right)$$

Question 17

Answer: C

Explanation:

$$\text{normcdf}(165, 170, 165, 7.62).$$

Question 18

Answer: A

Explanation:

$$\text{binomcdf}(6, 0.2, 5, 6) \text{ on CAS.}$$

Question 19

Answer: D

Explanation:

50th percentile means she is on average

Question 20

Answer: C

Explanation:

Sketch on CAS and read the maximum value.

Question 21

Answer: C

Explanation:

$$k = 0.2, 2 \times E(X) + 1 = 7.8 + 1$$

Question 22

Answer: B

Explanation:

$$\frac{\pi}{n} = 3 \text{ gives } n = \frac{\pi}{3}$$

SECTION 2: Analysis Questions

Question 1

a. $r = l \sin \alpha, h = l \cos \alpha$

A2
2 marks

b. $V = \frac{1}{3} \pi r^2 h = \frac{\pi}{3} (l \sin \alpha)^2 (l \cos \alpha) = \frac{\pi}{3} l^3 \sin^2 \alpha \cos \alpha$

M1
1 mark

c. $V'(\alpha) = \frac{\pi}{3} l^3 (\sin^2 \alpha \times -\sin \alpha + \cos \alpha \times 2 \sin \alpha \cos \alpha) = 0$

$$\sin \alpha (-\sin^2 \alpha + 2 \cos^2 \alpha) = 0$$

$$\sin \alpha = 0, \tan^2 \alpha = 2$$

$$\alpha = 0, \alpha = \pm \tan^{-1} \sqrt{2}$$

$$\alpha = \tan^{-1} \sqrt{2}, V(\alpha) = \frac{2\sqrt{3}}{27} \pi l^3$$

$$\left(\tan^{-1} \sqrt{2}, \frac{2\sqrt{3}}{27} \pi l^3 \right)$$

M3+A1
4 marks

d. $\alpha = \tan^{-1} \sqrt{2}$ is a point of maximum volume.

$$\text{Max volume} = \frac{2\sqrt{3}}{27} \pi \times 6^3 = 16\sqrt{3} \pi \text{ cm}^3.$$

M1+A1
2 marks

e.

$$\frac{dV}{dt} = 7$$

$$V = \frac{\sqrt{3}}{24} \pi l^3$$

$$\frac{dV}{dl} = \frac{\sqrt{3}}{8} \pi l^2$$

$$\frac{dl}{dt} = \frac{dl}{dV} \times \frac{dV}{dt} = \frac{8}{\sqrt{3} \pi l^2} \times 7 = \frac{8}{\sqrt{3} \pi 8^2} \times 7 = 0.16 \text{ cm/sec}$$

M2+A1
3 marks

Question 2

a. Period = $\frac{2\pi}{\frac{\pi}{2.2}} = 4.4$ years and Amplitude = 300

A2
2 marks

b. Min = 200, Max = 800

A2
2 marks

c. Solve $P(t) = 800$ over $[0, 5]$
 $t = 0.7$. After 8.4 months

M1+A1
2 marks

d. Sketch the graph on CAS and read the domain when $P < 300$
 $2.3 < t < 3.5$ and $6.7 < t < 7.9$

M1+A2
3 marks

Question 3

a. Sketch on CAS and read the max- 0.45 $\mu\text{g/mL}$

A1
1 mark

b. 3.5 minutes

A1
1 mark

c. $C(10) = 0.32 \mu\text{g/mL}$

M1+A1
2 marks

d. $\frac{C(5) - C(\frac{3}{2})}{5 - \frac{3}{2}} = 0.0115 \frac{\mu\text{g}}{\text{mL}}/\text{minute}$

M1+A1
2 marks

- e. Solve $\frac{dC}{dt} < 0$ on CAS
 $t > 3.53$ minutes

M1+A1
 2 marks

- f. $\frac{dC_1}{dt} = 0$ at $t = 120 \dots \dots \dots (1)$
 $C_1(120) = 120 \dots \dots \dots (2)$

Solve the above equations on CAS to get $a = e$ and $b = \frac{1}{120}$

M2+A1
 3 marks

Question 4

- a. $f(x) = x^2 + bx + \frac{b^2}{4} + 3 - \frac{b^2}{4} = \left(x + \frac{b}{2}\right)^2 + 3 - \frac{b^2}{4}$
 $\frac{b}{2} = 5$ gives $b = 10$ ($b > 0$)

M1+A1
 2 marks

- b. Translation of + 5 units parallel to the x – axis
 Translation of + 22 units parallel to the y – axis

A2
 2 marks

- c. Range of g : $[0, \infty)$
 Domain of f : R
 Range of g is a subset of domain of f , hence $f(g(x))$ exists.
 $f(g(x)) = (x^2 + 5)^2 - 22$

M1+A2
 3 marks

- d. $tangentline(h(x), x, k)$
 $y = (4k^3 + 20k)x + (-3k^4 - 10k^2 + 3)$

M1+A1
 2 marks

e. $Area = \int_0^3 ((x^2 + 5)^2 - 22) dx$

A2
2 marks

Question 5

a.

i. $normcdf(-\infty, 11, 7.5, 2.5) = 0.9192$

ii. $normcdf(5.5, 10.5, 7.5, 2.5) = 0.6731$

M1+A1
2 marks

b. $\Pr(D < d) = 0.1$

$d = 4.3 \text{ km}$

M1+A1
2 marks

c. $n = 6, \quad p = \Pr(X \geq 6.8) = 0.6103, \quad r = 4$
 $\Pr(X = 4) = binompdf(6, 0.6103, 4) = 0.3160$

M2+A1
3 marks

d. $\Pr(X \geq 5) = 0.65$

$\Pr\left(Z \geq \frac{5-6.4}{a}\right) = 0.65$

$\Pr\left(Z < \frac{-1.4}{a}\right) = 0.35$

$-0.38532 = -\frac{1.4}{a}$

Standard Deviation = 3.63 km

M2+A1
3 marks

e. Draw a tree diagram to identify the cases.

$(0.45 \times 0.45 \times 0.45) + (0.45 \times 0.45 \times 0.55) + (0.55 \times 0.67 \times 0.45) + (0.45 \times 0.55 \times 0.67) = 0.53415$

M2+A1
3 marks

f. $T = \begin{bmatrix} 0.45 & 0.67 \\ 0.55 & 0.33 \end{bmatrix}$
 $T^{40} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = T^{41} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.54918 \\ 0.45082 \end{bmatrix}$
 Pr(walks in the long term) = 0.55

M1+A1
 2 marks