

MATHEMATICAL METHODS (CAS)

Units 3 & 4 – Written examination 1



2015 Trial Examination

SOLUTIONS

Question 1

a. $3 - 5x \geq 0$

$$x \leq \frac{3}{5}$$

$$\text{Domain: } \left(-\infty, \frac{3}{5}\right]$$

A1
1 mark

b. $f'(x) = \frac{1}{2}(3 - 5x)^{-\frac{1}{2}} \times -5$

$$f'(x) = -\frac{5}{2\sqrt{3-5x}}$$

M1+A1
2 marks

c. $f'\left(\frac{1}{5}\right) = -\frac{5}{2\sqrt{3-1}} = -\frac{5}{2\sqrt{2}} = -\frac{5\sqrt{2}}{4}$

A1
1 mark

Question 2

a. $\int \sin(3x) dx = -\frac{\cos(3x)}{3} + c$

$$0 = -\frac{1}{3} + c \text{ which gives } c = \frac{1}{3}$$

$$F(x) = -\frac{\cos(3x)}{3} + \frac{1}{3}$$

M2+A1
3 marks

$$\begin{aligned} \text{b. } -\frac{\cos(3x)}{3} + \frac{1}{3} &= \frac{1}{2} \\ \cos(3x) &= -\frac{1}{2} \\ 3x &= \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3} \\ x &= \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9} \end{aligned}$$

M2+A1

3 marks

Question 3

$$\begin{aligned} \text{a. } y &= \frac{x-2}{x+2} \\ x &= \frac{y-2}{y+2} \\ yx + 2x &= y - 2 \\ y(x-1) &= -2 - 2x \\ y &= \frac{2+2x}{1-x} \\ f^{-1}(x) &= \frac{2+2x}{1-x} \end{aligned}$$

M2+A1

3 marks

$$\begin{aligned} \text{b. } \text{Domain: } R \setminus \{1\} \\ \text{Range: } R \setminus \{-2\} \end{aligned}$$

A2

2 marks

c. Using long division,

$$f^{-1}(x) = -2 + \frac{4}{1-x}$$

$$\int_0^{\frac{1}{2}} \left(-2 + \frac{4}{1-x}\right) dx = \left(-2x - 4\log_e(1-x)\right)\Big|_0^{\frac{1}{2}} = -1 - 4\ln\left(\frac{1}{2}\right) = -1 + 4\ln 2$$

M3+A1

4 marks

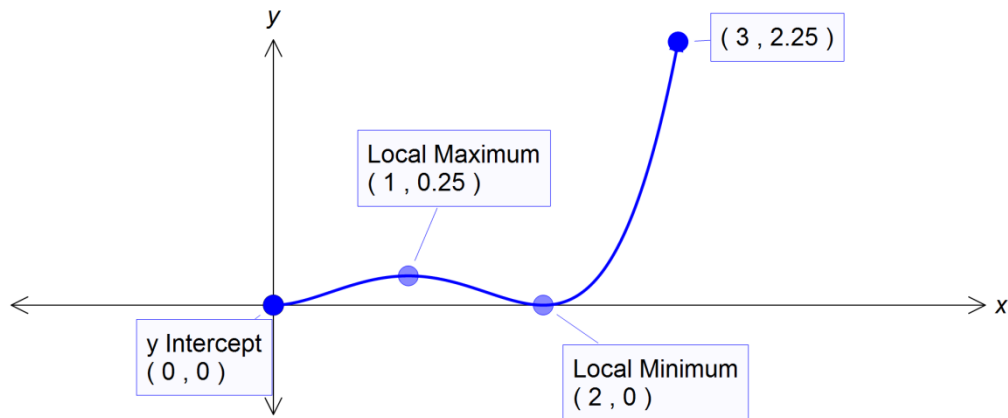
Question 4

$$\begin{aligned} \text{a. } f'(x) &= x^3 - 3x^2 + 2x \\ f'(x) = 0 &\text{ gives } x(x^2 - 3x + 2) = 0 \\ x(x-2)(x-1) &= 0 \text{ gives } x = 0, 1, 2 \\ (0, 0), \left(1, \frac{1}{4}\right), (2, 0) \end{aligned}$$

M2+A1

3 marks

b.



1 for shape, 1 for stationary points, 1 for end points.

3 marks

$$\text{c. } \text{Area} = \int_0^2 \left(\frac{1}{4}x^4 - x^3 + x^2 \right) dx = \left(\frac{x^5}{20} - \frac{x^4}{4} + \frac{x^3}{3} \right)_0^2$$

$$\text{Area} = \frac{8}{5} - 4 + \frac{8}{3} = \frac{4}{15} \text{ square units}$$

M1+A1

2 marks

Question 5

$$\text{a. } 4000 = 5(2 + 7^{3x})$$

$$800 = 2 + 7^{3x}$$

$$798 = 7^{3x}$$

$$3x = \log_7(798)$$

$$x = \frac{1}{3} \log_7(798)$$

M1+A1

2 marks

$$\text{b. } 2 \times 2^{2x} + 2^x - 1 = 0$$

$$2y^2 + y - 1 = 0, \text{ where } y = 2^x$$

$$(2y - 1)(y + 1) = 0$$

$$y = \frac{1}{2}, -1$$

$$2^x = \frac{1}{2}, 2^x = -1$$

$$x = -1 \quad (2^x = -1 \text{ has no solution})$$

M2+A1

3 marks

Question 6

a. $\frac{1}{5} + \frac{1}{10} + \frac{1}{3} + k = 1$
 $k = \frac{11}{30}$

A1
1 mark

b. $\Pr(X < 2) = \frac{1}{5} + \frac{1}{3} = \frac{8}{15}$

A1
1 mark

c. $\text{Mean} = \sum x\Pr(X = x) = 0 + \frac{1}{3} + \frac{1}{5} + \frac{11}{10} = \frac{49}{30}$

M1+A1
2 marks

Question 7

$$\frac{dy}{dx} = -\frac{3}{x^2}$$

grad of tangent $= -\frac{3}{a^2}$

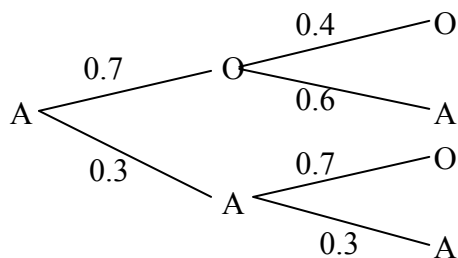
$$-\frac{3}{a^2} = -9$$

$$a = \pm \frac{1}{\sqrt{3}}$$

$$a = \frac{\sqrt{3}}{3}$$

M1+A1
2 marks

Question 8



$$\Pr(\text{Orange on Wed}) = 0.7 \times 0.4 + 0.3 \times 0.7 = \frac{28}{100} + \frac{21}{100} = \frac{49}{100} = 0.49$$

M1+A1
2 marks