



Trial Examination 2015

# VCE Mathematical Methods (CAS) Units 3&4

## Written Examination 1

### Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: \_\_\_\_\_

Teacher's Name: \_\_\_\_\_

#### Structure of Booklet

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 10                         | 10                                        | 40                     |

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

#### Materials supplied

Question and answer booklet of 13 pages and a sheet of miscellaneous formulas.

Working space is provided throughout the booklet.

#### Instructions

Write your **name** and **teacher's name** in the space provided above on this page.

All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2015 VCE Mathematical Methods (CAS) Units 3&4 Written Examination 1.

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**Instructions**

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

**Question 1 (3 marks)**

If  $h(x) = x \sin(x^2)$ , find

a.  $h'(x)$ .

2 marks

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b.  $h'\left(\sqrt{\frac{\pi}{2}}\right)$ .

1 mark

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**Question 3 (4 marks)**

a. Solve  $2 \sin\left(2x - \frac{\pi}{6}\right) = \sqrt{3}$  for  $-\pi < x < \pi$ .

2 marks

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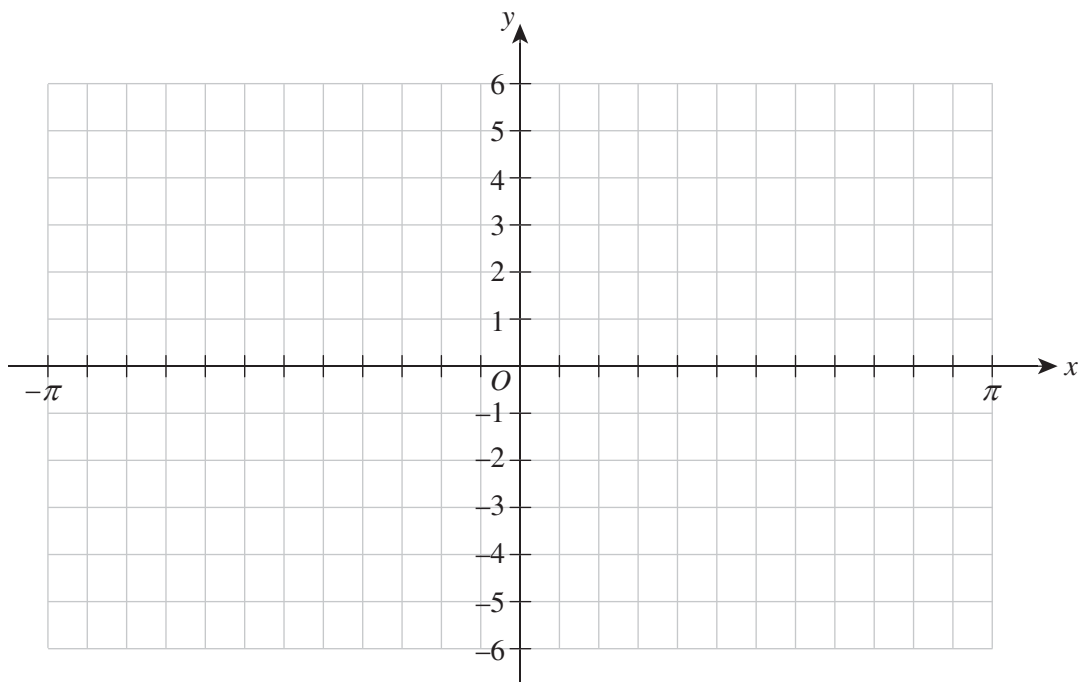
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b. Hence sketch the graph of  $y = 2 \sin\left(2x - \frac{\pi}{6}\right) - \sqrt{3}$  for  $-\pi < x < \pi$ , showing all axial intercepts and endpoints.

2 marks



**Question 4 (3 marks)**

Solve the equation  $3 \times 9^{x+1} + 1 = 5^2 \times 3^x + 3^{x+1}$ .

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**Question 5 (5 marks)**

Consider  $f: [-2, 3] \rightarrow \mathbb{R}$ ,  $f(x) = 6 + x - x^2$  and  $g: (-3, 3) \rightarrow \mathbb{R}$ ,  $g(x) = |x|$ .

- a.** Find the coordinates of the maximum point of  $f$ . 2 marks

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- b. i.** Find the function  $f(g(x))$ . 1 mark

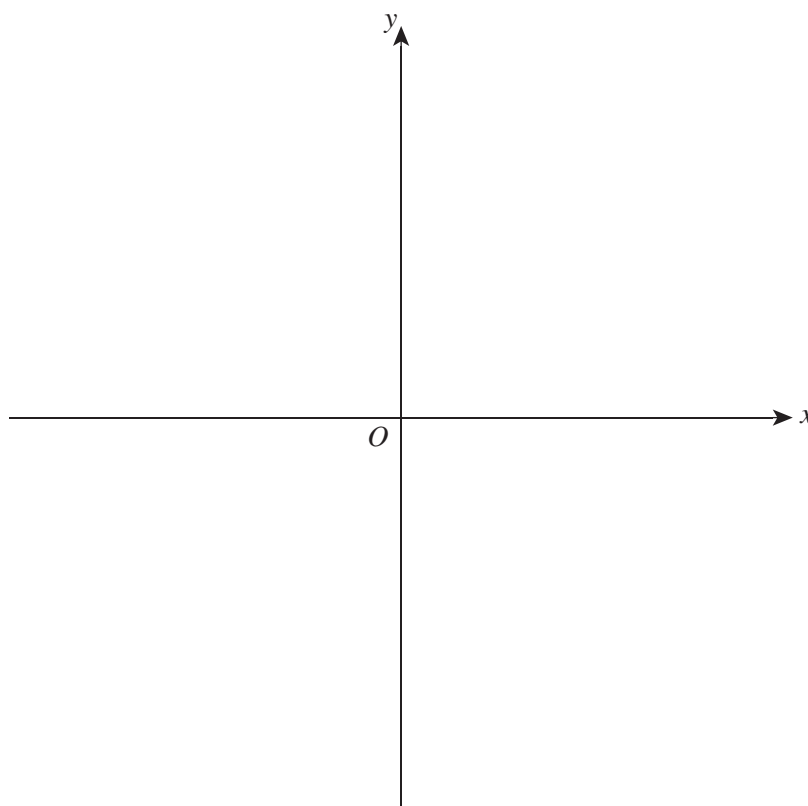
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- ii.** Sketch the graph of  $f(g(x))$ , clearly labelling any axial intercepts and turning points with their coordinates. 2 marks



**Question 6 (4 marks)**

- a. Given that  $y = \log_e(x^3 + 1)$  for  $x > -1$ , find  $\frac{dy}{dx}$ . 2 marks

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- b. Hence find  $\int (x + x^{-2})^{-1} dx$ . 2 marks

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**Question 7 (4 marks)**

A farmer is building a storage shed for wheat. He decides on a square-shaped floor of side length  $s$  metres, vertical walls of height  $h$  metres and a flat roof. The floor is to be built from concrete at a cost of  $\frac{\$80}{9}$  per square metre. The roof is to be made of steel at a cost of  $\frac{\$40}{9}$  per square metre. The walls will be built of reinforced concrete at a cost of \$10 per square metre and will require foundations around the perimeter of the square floor at a cost of \$10 per lineal metre.

- a.** Find an expression for  $C$ , the total cost of the project. 1 mark

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- b.** Given that the total cost of the shed is to be \$4800, deduce an expression for  $h$  as a function of  $s$ . 1 mark

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- c.** Hence, find an expression for the volume,  $V$ , of the shed that depends only on  $s$ . 1 mark

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- d.** Find the height  $h$  and the side length  $s$  that will result in the maximum volume.

1 mark

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**Question 8 (3 marks)**

A transformation  $T: R^2 \rightarrow R^2$  that maps the graph of  $g: R \setminus \left\{-\frac{1}{3}\right\} \rightarrow R, g(x) = \frac{1}{(3x+1)^2} + 2$  to the graph  $h: R \setminus \{0\} \rightarrow R, h(x) = \frac{1}{x^2}$  has the rule  $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix}\right) + \begin{bmatrix} b \\ c \end{bmatrix}$ , where  $a, b$  and  $c$  are non-zero real numbers.

Find the values of  $a, b$  and  $c$ .

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**Question 9 (5 marks)**

The number of customers,  $X$ , waiting to be served in a bakery at 9:00 am has the probability distribution given in the table below.

|        |                      |                |                |                |               |
|--------|----------------------|----------------|----------------|----------------|---------------|
| $X$    | 0                    | 1              | 2              | 3              | 4             |
| $p(x)$ | $\frac{3k^2 - 1}{7}$ | $\frac{3k}{7}$ | $\frac{4k}{7}$ | $\frac{2k}{7}$ | $\frac{k}{7}$ |

- a. Find the value of  $k$ . 3 marks

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- b. Calculate the probability that there is at least one customer in the shop at 9:00 am. 2 marks

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**Question 10 (6 marks)**

A continuous random variable has the following probability density function:

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 1, \text{ where } k \text{ is a constant} \\ \frac{7k-3x}{4} & 1 < x \leq \frac{7}{3} \\ 0 & \text{elsewhere} \end{cases}$$

- a.** Find the value of  $k$ .

2 marks

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- b.** Find the exact value of the mode.

2 marks

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- c. Find the exact value of the mean.

2 marks

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**END OF QUESTION AND ANSWER BOOKLET**