



Trial Examination 2015

VCE Mathematical Methods (CAS) Units 3&4

Written Examination 1

Suggested Solutions

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Question 1 (3 marks)

a. $h(x) = x \sin(x^2) = u(x)v(x)$, where $u(x) = x$ and $v(x) = \sin(x^2)$.

Hence $u'(x) = 1$ and $v'(x) = 2x \cos(x^2)$.

$$h'(x) = v(x)u'(x) + u(x)v'(x) \quad \text{M1}$$

$$= \sin(x^2) \times 1 + x \times 2x \cos(x^2)$$

$$= \sin(x^2) + 2x^2 \cos(x^2) \quad \text{A1}$$

b. $h'\left(\sqrt{\frac{\pi}{2}}\right) = \sin\left(\frac{\pi}{2}\right) + 2 \times \frac{\pi}{2} \cos\left(\frac{\pi}{2}\right)$

$$= 1 \quad \text{A1}$$

Question 2 (3 marks)

$$\begin{aligned}
 f(x) &= e^{5x}(e^x + e^{-x}) \\
 &= e^{5x} \times e^x + e^{5x} \times e^{-x} \\
 &= e^{5x+x} + e^{5x-x} \\
 &= e^{6x} + e^{4x}
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence } \int f(x) dx &= \int (e^{6x} + e^{4x}) dx \\
 &= \int e^{6x} dx + \int e^{4x} dx \\
 &= \frac{e^{6x}}{6} + \frac{e^{4x}}{4} + C, \text{ where } C \text{ is a constant.}
 \end{aligned}$$

M1

$$g(x) = \frac{e^{6x}}{6} + \frac{e^{4x}}{4} + C \text{ for a particular value of the constant } C.$$

Since we are given that $g(0) = 0$, we can find the value of C .

$$\begin{aligned}
 g(0) &= \frac{e^{6 \times 0}}{6} + \frac{e^{4 \times 0}}{4} + C \\
 &= \frac{e^0}{6} + \frac{e^0}{4} + C \\
 &= \frac{1}{6} + \frac{1}{4} + C \\
 &= \frac{5}{12} + C
 \end{aligned}$$

M1

$$\text{Thus } 0 = \frac{5}{12} + C$$

$$C = -\frac{5}{12}$$

$$g(x) = \frac{e^{6x}}{6} + \frac{e^{4x}}{4} - \frac{5}{12}$$

A1

Question 3 (4 marks)

a. $\sin\left(2x - \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \quad -\pi < x < \pi$

$$2x - \frac{\pi}{6} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{3}$$

$$2x - \frac{\pi}{6} = \frac{\pi}{3} \quad -\frac{13\pi}{6} < 2x - \frac{\pi}{6} < \frac{11\pi}{6}$$

M1

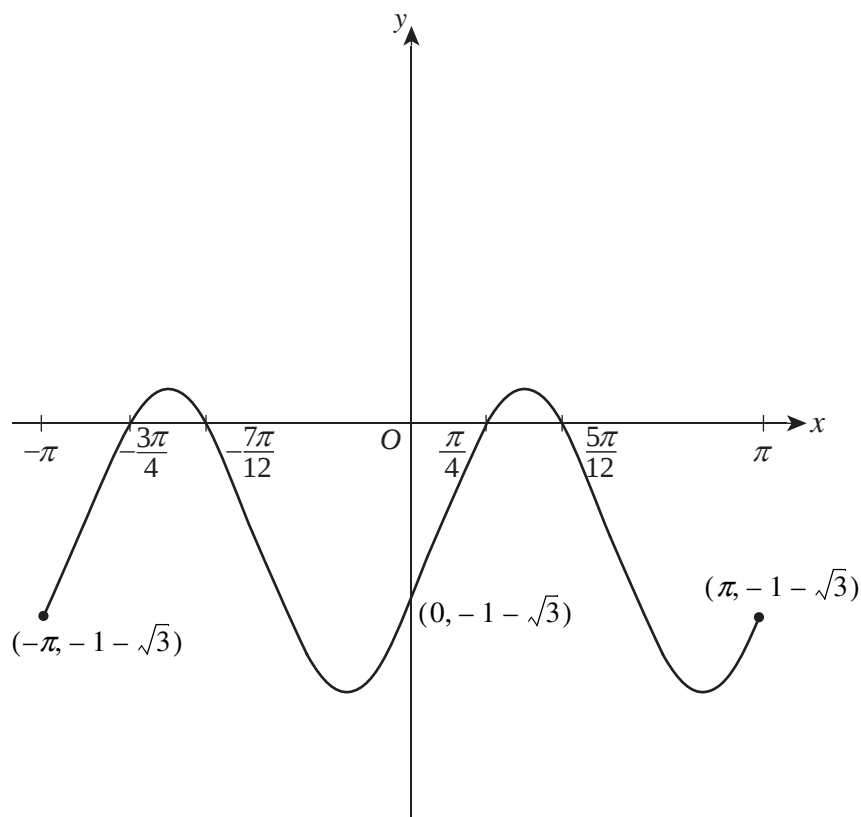
$$2x - \frac{\pi}{6} = -\frac{5\pi}{3}, -\frac{4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$2x = -\frac{3\pi}{2}, -\frac{7\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$$

$$x = -\frac{3\pi}{4}, -\frac{7\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{12}$$

A1

b.



correct shape A1
correct intercepts and endpoints A1

Question 4 (3 marks)

$$3 \times 9^{x+1} + 1 - 5^2 \times 3^x - 3^{x+1} = 0$$

$$3 \times (3^2)^{x+1} + 1 - 25 \times 3^x - 3^{x+1} = 0$$

$$3 \times 3^{2x} \times 9 + 1 - 25 \times 3^x - 3 \times 3^x = 0$$

$$27 \times 3^{2x} - 28 \times 3^x + 1 = 0$$

$$27 \times 3^{2x} - 28 \times 3^x + 1 = 0$$

quadratic form M1Let $3^x = a$.

$$27a^2 - 28a + 1 = 0$$

$$(27a - 1)(a - 1) = 0$$

$$a = \frac{1}{27}, 1$$

A1

Therefore $3^x = \frac{1}{27}, 1$. $3^x = 1$ gives $x = 0$ and $3^x = \frac{1}{27}$ gives $x = -3$.

A1

Question 5 (5 marks)**a.** Method 1:

Complete the square to find the turning point:

$$f(x) = -\left(x - \frac{1}{2}\right)^2 + \frac{25}{4}$$

M1

Coordinates are $\left(\frac{1}{2}, \frac{25}{4}\right)$.

A1

Method 2:

$$f'(x) = 1 - 2x$$

There is a maximum when $f'(x) = 0$.

M1

$$1 - 2x = 0$$

$$x = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{25}{4}$$

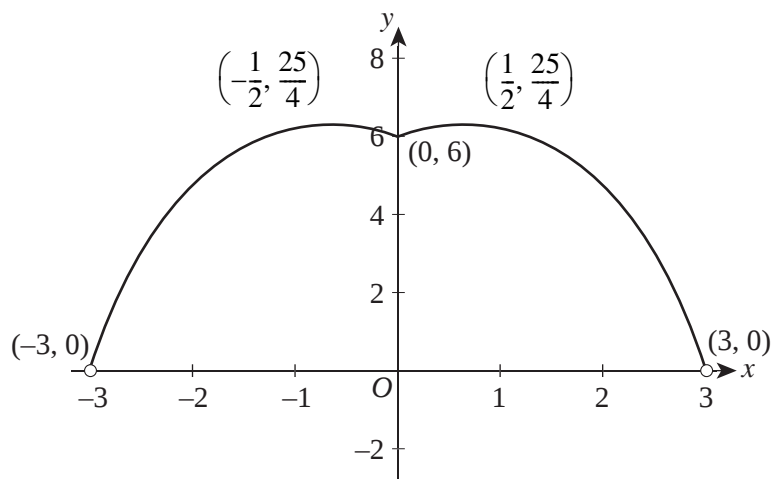
Coordinates are $\left(\frac{1}{2}, \frac{25}{4}\right)$.

A1

b. i. $f(g(x)) = 6 + |x| - |x^2|$ domain: $x \in (-3, 3)$

A1

ii.



correct shape A1
correct intercepts and turning points A1

Question 6 (4 marks)

- a. Let $u = x^3 + 1$ and then $y = \log_e(u)$.

$$\frac{dy}{du} = \frac{1}{u} \text{ and } \frac{du}{dx} = 3x^2.$$

M1

The chain rule states that $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$.

$$\text{Hence } \frac{dy}{dx} = \frac{1}{u} \times 3x^2$$

$$= \frac{1}{x^3 + 1} \times 3x^2$$

$$= \frac{3x^2}{x^3 + 1}$$

A1

- b. By the fundamental theorem of calculus, $\int \frac{3x^2}{x^3 + 1} dx = \log_e(x^3 + 1) + C$, where C is a constant.

$$\text{However, } (x + x^{-2})^{-1} = \frac{1}{x + x^{-2}}$$

$$= \frac{1}{x + x^{-2}}$$

$$= \frac{x^2}{x^3 + 1}$$

M1

We can find the required antiderivative as follows:

$$\int (x + x^{-2})^{-1} dx = \int \frac{x^2}{x^3 + 1} dx$$

$$= \frac{1}{3} \times \int \frac{3x^2}{x^3 + 1} dx$$

$$= \frac{1}{3} \log_e(x^3 + 1) + K, \text{ where } K \text{ is a constant, since } K = \frac{C}{3}.$$

A1

Question 7 (4 marks)

- a. $C = \text{floor} + \text{roof} + \text{walls} + \text{foundations}$

$$= \frac{80}{9}s^2 + \frac{40}{9}s^2 + 40sh + 40s$$

$$= \frac{40}{3}s^2 + 40sh + 40s$$

A1

- b. $4800 = \frac{40}{3}s^2 + 40sh + 40s$

$$120 = s\left(\frac{s}{3} + h + 1\right)$$

$$\frac{120}{s} - \frac{s}{3} - 1 = h$$

$$h = \frac{360 - s^2 - 3s}{3s}$$

A1

- c. $V = s^2h$

$$= s^2 \left(\frac{360 - s^2 - 3s}{3s} \right)$$

$$= 120s - \frac{s^3}{3} - s^2$$

A1

d. $\frac{dV}{ds} = 120 - s^2 - 2s$

$$\begin{aligned}\frac{dV}{ds} = 0 \text{ when } 0 &= 120 - s^2 - 2s \\ &= s^2 + 2s - 120 \\ &= (s - 10)(s + 12)\end{aligned}$$

$$s = 10 \text{ and } s = -12$$

Since $s > 0$, $s = 10$.

$$\begin{aligned}\text{When } s = 10, h &= \frac{360 - 10^2 - 3(10)}{3(10)} \\ &= \frac{230}{30} \\ &= \frac{23}{3} \text{ m}\end{aligned}$$

A1

Question 8 (3 marks)

Let $y = g(x)$.

$$y - 2 = \frac{1}{(3x - 1)^2}$$

From the transformation, $x' = a(x + b)$ gives $a(x + b) = 3x + 1$

$$ax + ab = 3x + 1$$

$$a = 3$$

A1

$$b = \frac{1}{3}$$

A1

$y' = y + c$ gives $y + c = y - 2$

$$c = -2$$

A1

Question 9 (5 marks)

a. $\Sigma p(x) = 1$

$$3k^2 - 1 + 3k + 4k + 2k + k = 7$$

M1

$$3k^2 + 10k - 8 = 0$$

$$(3k - 2)(k + 4) = 0$$

$$k = \frac{2}{3}, -4$$

A1

Since $k > 0$, $k = \frac{2}{3}$.

one solution only A1

b. $\Pr(X \geq 1) = 1 - \Pr(X = 0)$

M1

$$\Pr(X = 0) = \frac{3\left(\frac{2}{3}\right)^2 - 1}{7}$$

$$= \frac{1}{7}\left(\frac{4}{3} - 1\right)$$

$$= \frac{1}{21}$$

$$\Pr(X \geq 1) = 1 - \frac{1}{21}$$

$$= \frac{20}{21}$$

A1

Question 10 (6 marks)

a. Since $f(x)$ is a probability density function, $\int_{-\infty}^{\infty} f(x) dx = 1$.

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^0 f(x) dx + \int_0^1 f(x) dx + \int_1^{\frac{7}{3}} f(x) dx + \int_{\frac{7}{3}}^{\infty} f(x) dx$$

$$1 = \int_{-\infty}^0 0 dx + \int_0^1 kx^2 dx + \int_1^{\frac{7}{3}} \left(\frac{7k-3x}{4}\right) dx + \int_{\frac{7}{3}}^{\infty} 0 dx$$

M1

$$1 = 0 + \left[\frac{kx^3}{3}\right]_0^1 + \left[\frac{7k}{4}x - \frac{3x^2}{8}\right]_1^{\frac{7}{3}} + 0$$

$$1 = \frac{k}{3} + \left(\frac{7k}{4} \times \frac{7}{3} - \frac{3}{8}\left(\frac{7}{3}\right)^2\right) - \left(\frac{7k}{4} - \frac{3}{8}\right)$$

$$1 = \frac{k}{3} + \frac{49}{12}k - \frac{49}{24} - \frac{7k}{4} + \frac{3}{8}$$

$$1 = \frac{64k}{24} - \frac{40}{24}$$

$$\frac{64}{24} = \frac{64k}{24}$$

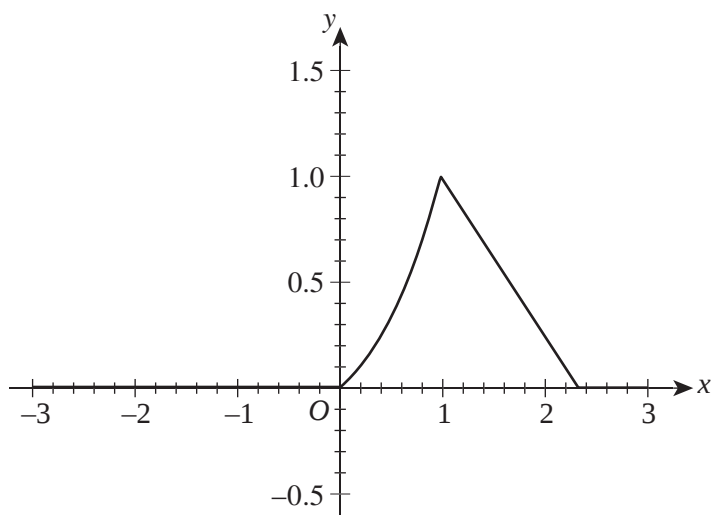
$$k = 1$$

A1

b. Since $k = 1$:

$$f(x) = \begin{cases} x^2 & 0 \leq x \leq 1 \\ \frac{7-3x}{4} & 1 < x \leq \frac{7}{3} \\ 0 & \text{elsewhere} \end{cases}$$

The mode is the value of x for which $f(x)$ is a maximum. This can be found by sketching the graph.



M1

maximum value of $f(x)$ occurs when $x = 1$, therefore mode = 1

A1

c. The mean is given by $\int_{-\infty}^{\infty} xf(x)dx$.

$$\int_{-\infty}^{\infty} xf(x)dx = \int_0^1 xf(x)dx + \int_1^{\frac{7}{3}} xf(x)dx$$

$$= \int_0^1 x^3 dx + \int_1^{\frac{7}{3}} \frac{7x - 3x^2}{4} dx$$

M1

$$= \left[\frac{x^4}{4} \right]_0^1 + \left[\frac{7x^2}{8} - \frac{x^3}{4} \right]_1^{\frac{7}{3}}$$

$$= \frac{1}{4} + \left(\frac{7\left(\frac{7}{3}\right)^2}{8} - \frac{\left(\frac{7}{3}\right)^3}{4} \right) - \left(\frac{7}{8} - \frac{1}{4} \right)$$

$$= \frac{1}{4} + \left(\frac{343}{72} - \frac{343}{108} \right) - \frac{5}{8}$$

$$= \frac{1}{4} + \left(\frac{1029}{216} - \frac{686}{216} \right) - \frac{5}{8}$$

$$= \frac{1}{4} + \frac{343}{216} - \frac{5}{8}$$

$$= \frac{54}{216} + \frac{343}{216} - \frac{135}{216}$$

$$= \frac{262}{216}$$

$$\text{mean} = \frac{131}{108}$$

A1