

The Mathematical Association of Victoria

Trial Exam 2015

MATHEMATICAL METHODS (CAS)

WRITTEN EXAMINATION 1

STUDENT NAME _____

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 8 pages, with a detachable sheet of miscellaneous formulas in the centerfold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **name** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Question 1 (4 marks)

a. If $y = xe^{2x}$ find $\frac{dy}{dx}$.

2 marks

b. Hence find an expression for $\int (xe^{2x}) dx$.

2 marks

Question 2 (2 marks)

For what value(s) of k is there a unique solution to the following system of equations?

$$\begin{aligned}(k-1)x + 2y &= 1 \\ x + (k-1)y &= -k\end{aligned}$$

Question 3 (5 marks)

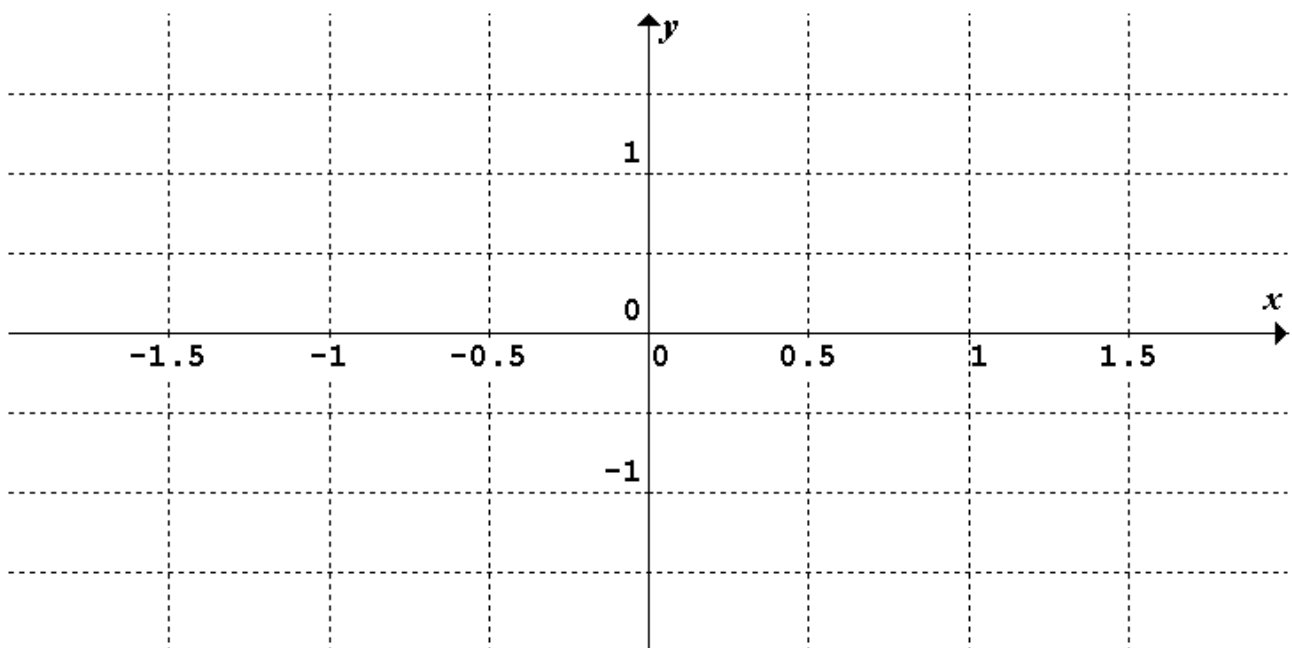
Consider $f: \left[-\frac{1}{3}, \infty\right) \rightarrow \mathbb{R}, f(x) = \log_e(3x+2)$ and $g: [0, \infty) \rightarrow \mathbb{R}, g(x) = |x-1|$.

- a. Write down the rule and the domain for $g(f(x))$.

2 marks

- b. Hence sketch the graph of $y = g(f(x))$, labelling the endpoint and axial intercepts with their coordinates.

3 marks



Question 4 (4 marks)

Consider the function $h:[0,14] \rightarrow \mathbb{R}$, $h(t) = 2\sin\left(\frac{\pi}{30}(t+1)\right) + 2$ where h is the height of water, in cm, in a container at time t seconds. The water is being poured into the container at a rate of $2 \text{ cm}^3/\text{s}$.

- a. Find $h'(t)$.

1 mark

- b. Find the rate of change of the volume with respect to the height of the water in the container when $h = 3$.

3 marks

Question 5 (3 marks)

Let $f(x) = \frac{1}{2} \log_e(x(x+1)) \log_e(2x-1)$.

a. Find $f'(x)$.

1 mark

b. Hence evaluate $f'(2)$, expressing your answer in the form of $\log_e(a^m b^n)$ where a and b are prime numbers and m and n are real numbers.

2 marks

Question 6 (3 marks)

Find the general solution of $2\sqrt{3} \cos(2x) = -3$.

Question 7 (5 marks)

- a.** Given $f(x) = 2(x-1)^{\frac{1}{3}}$, find an approximate value for $f(27.99)$ using $f(x+h) \approx f(x) + hf'(x)$.

3 marks

- b.** Will the answer to **part a.** be an under or over estimate of the actual value of $f(27.99)$? Explain.

2 marks

A random variable X has a probability density function

$$f(x) = \begin{cases} -(x-1)(x-2) & 1 \leq x \leq 2 \\ \frac{1}{2}x - 1 & 2 < x \leq a \\ 0 & \text{elsewhere} \end{cases}$$

Find the value of a , where a is a real constant.

[illegible]

Question 9 (9 marks)

An examination consists of 22 multiple choice questions. Each question has five possible answers. Max has not done any work and he has to guess his answers.

- a. What is the probability he will get the first three answers correct. 1 mark

- b. What is the probability he will get two of the first five questions correct? 2 marks

- c. Calculate $\mu \pm 2\sigma$ for the number of questions Max will get right. Interpret the result. 3 marks

- d. What is the least number of multiple choice questions Max must attempt to ensure that the probability of getting at least one correct is more than $\frac{369}{625}$? 3 marks

END OF QUESTION AND ANSWER BOOKLET