Year 2015

VCE

Mathematical Methods

Trial Examination 2

Solutions

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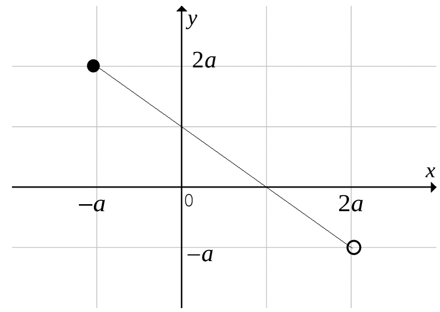
SECTION 1

ANSWERS

1	**A**	**B**	**C**	**D**	**E**
2	**A**	**B**	**C**	**D**	**E**
3	**A**	**B**	**C**	**D**	**E**
4	**A**	**B**	**C**	**D**	**E**
5	**A**	**B**	**C**	**D**	**E**
6	**A**	**B**	**C**	**D**	**E**
7	**A**	**B**	**C**	**D**	**E**
8	**A**	**B**	**C**	**D**	**E**
9	**A**	**B**	**C**	**D**	**E**
10	**A**	**B**	**C**	**D**	**E**
11	**A**	**B**	**C**	**D**	**E**
12	**A**	**B**	**C**	**D**	**E**
13	**A**	**B**	**C**	**D**	**E**
14	**A**	**B**	**C**	**D**	**E**
15	**A**	**B**	**C**	**D**	**E**
16	**A**	**B**	**C**	**D**	**E**
17	**A**	**B**	**C**	**D**	**E**
18	**A**	**B**	**C**	**D**	**E**
19	**A**	**B**	**C**	**D**	**E**
20	**A**	**B**	**C**	**D**	**E**
21	**A**	**B**	**C**	**D**	**E**
22	**A**	**B**	**C**	**D**	**E**

SECTION 1

Question 1 **Answer C**



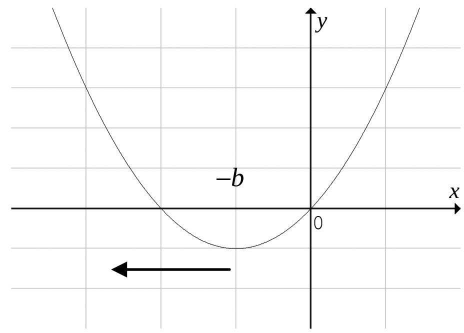
$f : [-a, 2a) \to R$, $f(x) = a - x$

$f(-a) = 2a$ included

$f(2a) = -a$ not included

The range is $(-a, 2a]$

Question 2 **Answer B**



$f : (-\infty, a] \to R$, $f(x) = x^2 + 2bx$

$f(x) = x^2 + 2bx = (x+b)^2 - b^2$

The quadratic has a turning point at $x = -b$
For the inverse function to exist,
the function must be one-one, we must
restrict the domain, to be less than $-b$,
so $a < -b$ or $a + b < 0$

Question 3 **Answer C**

$|k - x| < 2k$

$-2k < k - x < 2k$

$-3k < -x < k$

$-k < x < 3k$

alternatively

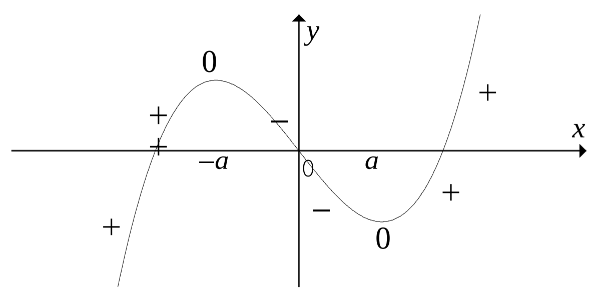
$|k - x| = |x - k|$

$|x - k| < 2k$

$-2k < x - k < 2k$

$-k < x < 3k$

Question 4 **Answer B**



$f'(x) < 0$ for $|x| < a$ that is $-a < x < a$

$f'(x) > 0$ for $|x| > a$ that is $x > a$ or $x < -a$

$f'(-a) = 0$ and $f'(a) = 0$

the signs of the gradient are shown.

the graph of f has stationary points at
both $x = \pm a$. A local maximum at $x = -a$
and a local minimum at $x = a$.

Question 5 **Answer D**

$\frac{d}{dx}\left[f(x)\right]=g(x)$ so that $\frac{d}{du}\left[f(u)\right]=g(u)$ and $h(x)=x^2$

$\frac{d}{dx}\left[f(h(x))\right]=\frac{d}{dx}\left[f(x^2)\right]$ let $u=x^2$

$=\frac{d}{dx}\left[f(u)\right]=\frac{d}{du}\left[f(u)\right]\frac{du}{dx}=2x\frac{d}{du}\left[f(u)\right]=2x\,g(u)=2x\,g(x^2)$

Question 6 **Answer B**

$f(x)=\sqrt{x}\,g(x)$ using the product rule

$$f'(x)=g(x)\frac{d}{dx}\left[\sqrt{x}\right]+\sqrt{x}\frac{d}{dx}\left[g(x)\right]$$

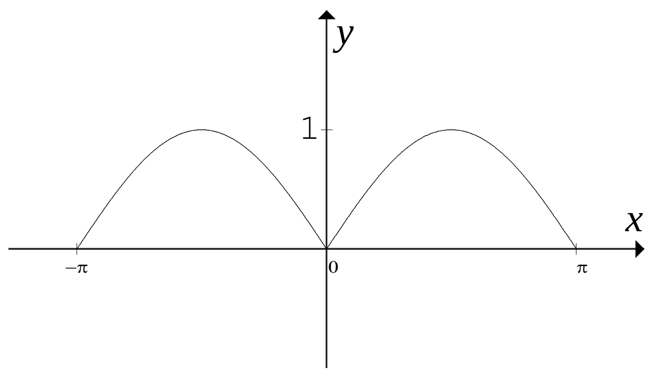
$$=\frac{g(x)}{2\sqrt{x}}+\sqrt{x}\,g'(x)$$

$f'(4)=\frac{g(4)}{2\sqrt{4}}+\sqrt{4}\,g'(4)$ substitute $g(4)=8$ and $g'(4)=-1$

$$=\frac{8}{4}+2\times{}^{-}1$$

$$=0$$

Question 7 **Answer E**



$f:(-\pi,\pi)\to R$, $f(x)=\sin(x)$ and $g(x)=|x|$.

$f(g(x))=f(|x|)=\sin(|x|)$

$g(f(x))=f(\sin(x))=|\sin(x)|$

now both have a domain of $(-\pi,\pi)$ and a range of $[0,1]$.

The graphs are the same.

$\lim\limits_{x\to 0} f(g(x))=0$ and $\lim\limits_{x\to 0} g(f(x))=0$

$f(g(x))$ and $g(f(x))$ are both continuous at $x=0$.

But there is a cusp at $x=0$.

Both $f(g(x))$ and $g(f(x))$ are not differentiable at $x=0$

Question 8 **Answer E**

$f \ : \quad y = x^2 - 4$

$f^{-1}: \ x = y^2 - 4$

$y^2 = x + 4$

$y = \pm\sqrt{x+4}$

since the dom $f = R^- =$ ran f^{-1} we must take the negative sign.

since the ran $f = (-4, \infty) =$ dom f^{-1}, so that $f^{-1}: (-4, \infty) \to R$, $f^{-1}(x) = -\sqrt{x+4}$

Question 9 **Answer A**

$(3, -4)$ lies on the graph of the function $y = f(x)$, so that $f(3) = -4$

$(-5, 2)$ lies on the graph of the function $y = g(x)$, so that $g(-5) = 2$

$g(x) = -f(-x-2) - 2$

$g(-5) = -f(5-2) - 2 = -f(3) - 2 = 4 - 2 = 2$

Alternatively take the point $(3, -4)$, reflect in the *x*-axis, $-f(x)$ it becomes, $(3, 4)$, reflect in the *y*-axis $-f(-x)$ it becomes, $(-3, 4)$ translate 2 units down parallel to the *y*-axis, $-f(-x) - 2$ it becomes $(-3, 2)$, translate 2 units to the left parallel to the *x*-axis $-f(-(x+2)) - 2$ it becomes $(-5, 2)$. These transformations are $-f(-(x+2)) - 2$

Question 10 **Answer C**

$\int_1^3 (4 - 3f(x))\,dx = 2$

$[4x]_1^3 - 3\int_1^3 f(x)\,dx = 2$

$12 - 4 - 3\int_1^3 f(x)\,dx = 2$

$8 - 3\int_1^3 f(x)\,dx = 2$

$3\int_1^3 f(x)\,dx = 6$

$\int_1^3 f(x)\,dx = 2 \quad \Rightarrow \quad \int_3^1 f(x)\,dx = -2$

Question 11 **Answer E**

Since $f(x) = x^3 + bx^2 + c^2x = x\left(x^2 + bx + c^2\right)$ crosses the x-axis only once at the origin, the quadratic factor has no real roots, hence its discriminant $\Delta = b^2 - 4c^2 < 0$, so that $b^2 < 4c^2$ or $\frac{b^2}{c^2} < 4$ since both b and c are non-zero positive constants then $0 < \frac{b}{c} < 2$.

Now $f'(x) = 3x^2 + 2bx + c^2$ and there are two stationary points, so this discriminant $\Delta = (2b)^2 - 4 \times 3c^2 > 0$ so that $4b^2 > 12c^2$ or $\frac{b^2}{c^2} > 3$, since both b and c are non-zero positive constants then $\frac{b}{c} > \sqrt{3}$, so overall we require $\sqrt{3} < \frac{b}{c} < 2$

Question 12 **Answer B**

gradient $\frac{dy}{dx} = 6\cos\left(\frac{x}{3}\right)$ integrating $y = \int 6\cos\left(\frac{x}{3}\right)dx = 18\sin\left(\frac{x}{3}\right) + c$. To find c, use $x = \frac{\pi}{2}$ when $y = 0$, so that $0 = 18\sin\left(\frac{\pi}{6}\right) + c \Rightarrow c = -9$. The curve is $y = 18\sin\left(\frac{x}{3}\right) - 9$. This crosses the y-axis when $x = 0$ so that $y = -9$.

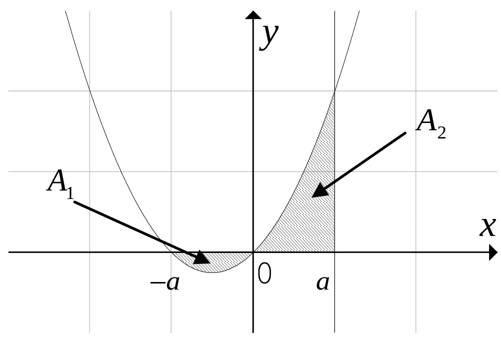
Question 13 **Answer A**

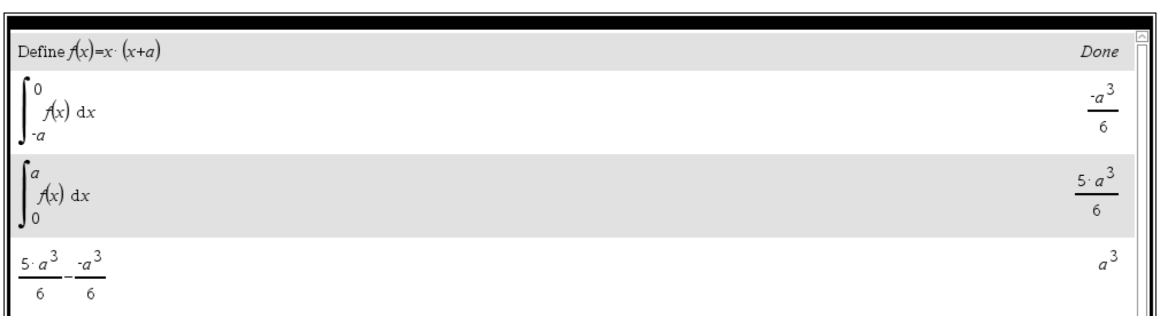
$y = x(x+a)$ crosses the x-axis at $x = -a$ and $x = 0$.

$$A_1 = \int_{-a}^{0} x(x+a)dx = -\frac{a^3}{6} < 0$$

$$A_2 = \int_{0}^{a} x(x+a)dx = \frac{5a^3}{6}$$

The total area is $A = \frac{5a^3}{6} + \left|-\frac{a^3}{6}\right| = a^3$





Question 14 **Answer E**

Image curve $y' = 4 - \dfrac{2}{x'-3} \Rightarrow \dfrac{y'-4}{2} = -\dfrac{1}{x'-3} \Rightarrow \dfrac{y'-4}{2} = \dfrac{1}{3-x'}$

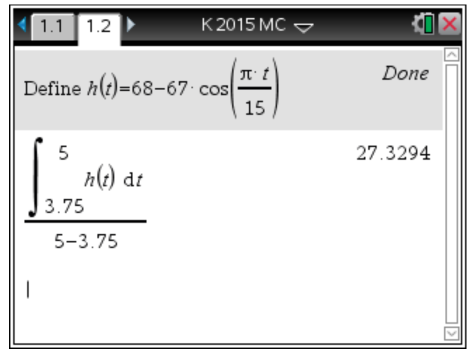
The original curve is $y = \dfrac{1}{x}$ so that $y = \dfrac{y'-4}{2} \Rightarrow y' = 2y+4$ and

$x = 3 - x' \Rightarrow x' = -x+3$, in matrix form $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Question 15 **Answer D**

$h(t) = 68 - 67\cos\left(\dfrac{\pi t}{15}\right)$. The average height over $t = 3.75$ and $t = 5$

$$\frac{\int_{3.75}^{5}\left(68-67\cos\left(\frac{\pi t}{15}\right)\right)dt}{5-3.75} = 27.33$$



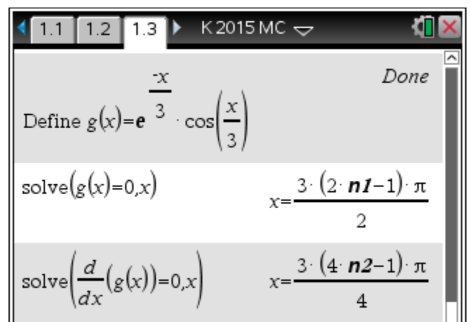
Question 16 **Answer D**

The graph $y = e^{-\frac{x}{3}}\cos\left(\dfrac{x}{3}\right)$ crosses the x-axis

when $\cos\left(\dfrac{x}{3}\right) = 0 \Rightarrow x = 3(2n-1)\dfrac{\pi}{2}$

The graph has turning points when

$\dfrac{dy}{dx} = 0 \Rightarrow x = 3(4n-1)\dfrac{\pi}{4}$



Question 17 **Answer A**

$y = f(x) = \dfrac{1}{2}(x-2)^2 + 1$

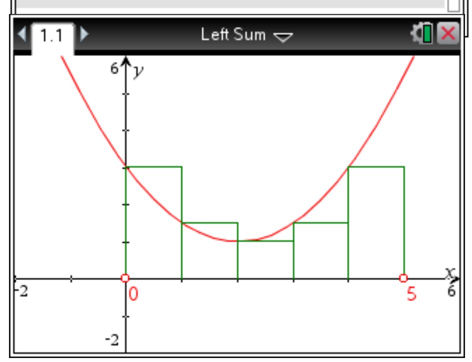
x	0	1	2	3	4	5
$f(x)$	3	$\frac{3}{2}$	1	$\frac{3}{2}$	3	$\frac{11}{2}$

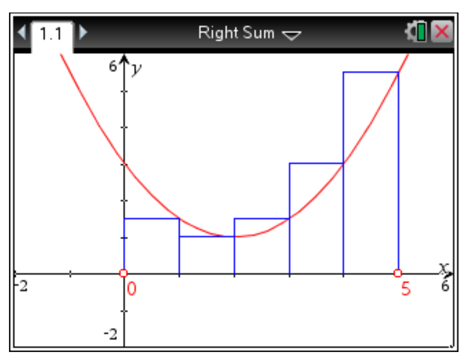
$L = 3 + \dfrac{3}{2} + 1 + \dfrac{3}{2} + 3 = 10$

$R = \dfrac{3}{2} + 1 + \dfrac{3}{2} + 3 + \dfrac{11}{2} = \dfrac{25}{2} = 12.5$

$A = \int_0^5 \left(\dfrac{1}{2}(x-2)^2 + 1\right)dx = 10\dfrac{5}{6}$,

So that $L < A < R$



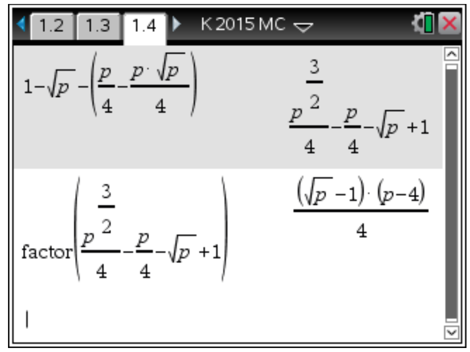


Question 18 **Answer C**

$\Pr(A)=\sqrt{p}$ and $\Pr(B)=\dfrac{p}{4}$, since A and B are independent events

$$\Pr(A\cap B)=\Pr(A)\Pr(B)=\frac{p\sqrt{p}}{4}$$

	A	A'	
B	$\dfrac{p\sqrt{p}}{4}$	$\dfrac{p}{4}-\dfrac{p\sqrt{p}}{4}$	$\dfrac{p}{4}$
B'	$\sqrt{p}-\dfrac{p\sqrt{p}}{4}$		$1-\dfrac{p}{4}$
	$\sqrt{p}$	$1-\sqrt{p}$	

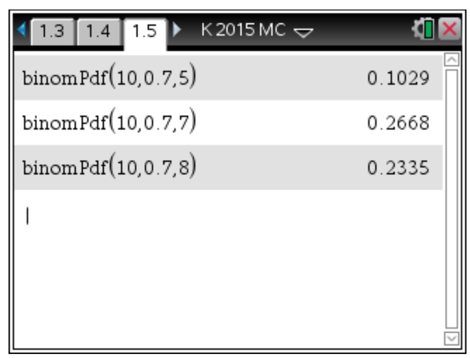


$\Pr(A'\cap B')=1-\sqrt{p}-\left(\dfrac{p}{4}-\dfrac{p\sqrt{p}}{4}\right)$ alternatively $\Pr(A'\cap B')=1-\dfrac{p}{4}-\left(\sqrt{p}-\dfrac{p\sqrt{p}}{4}\right)$

$\Pr(A'\cap B')=\dfrac{1}{4}\left(\sqrt{p}-1\right)(p-4)$ alternatively A' and B' are independant

$$\Pr(A'\cap B')=\Pr(A')\Pr(B')=\left(1-\sqrt{p}\right)\left(1-\frac{p}{4}\right)=\frac{1}{4}\left(\sqrt{p}-1\right)(p-4)$$

Question 19 **Answer D**



The graph is left skewed so $0.5<p<1$

$p=0.7$ is the most likely value.

$X\overset{d}{=}Bi(n=10,p=0.7)$

$\Pr(X=5)\approx 0.1,\Pr(X=7)\approx 0.27,\Pr(X=8)\approx 0.23$

Question 20 **Answer A**

$X\overset{d}{=}Bi(n,p)$, $\Pr(X=x)=\dbinom{n}{x}p^x q^{n-x}$

$$\Pr(X=0)=\binom{n}{0}p^0q^n=q^n=(1-p)^n=A$$

$$\Pr(X=2)=\binom{n}{2}p^2q^{n-2}=\frac{n(n-1)}{2}p^2(1-p)^{n-2}=\frac{n(n-1)p^2(1-p)^n}{2(1-p)^2}$$

$$=\frac{n(n-1)p^2A}{2(1-p)^2}$$

Question 21 **Answer A**

$Z \stackrel{d}{=} N\left(\mu = 0, \sigma^2 = 1\right)$

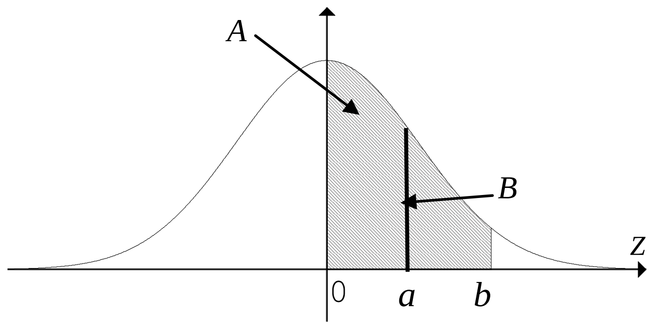
$\Pr(0 < Z < a) = A$

$\Pr(0 < Z < b) = B$, where $b > a$

$$\Pr(Z < b \mid Z > a) = \frac{\Pr(a < Z < b)}{\Pr(Z > a)}$$

$$= \frac{\Pr(Z < b) - \Pr(Z < a)}{0.5 - \Pr(0 < Z < a)}$$

$$= \frac{B - A}{0.5 - A}$$



Question 22 **Answer B**

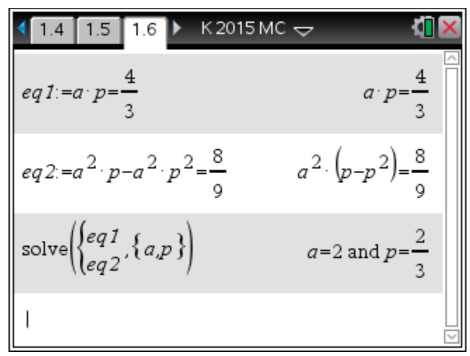
$$E(X) = \sum x \Pr(X = x) = 0 \times (1 - p) + A \times p = Ap = \frac{4}{3}$$

$$E\left(X^2\right) = \sum x^2 \Pr(X = x) = 0^2 \times (1 - p) + A^2 \times p$$

$$\operatorname{Var}(X) = E\left(X^2\right) - \left(E(X)\right)^2$$

$$= A^2 p - (Ap)^2 = A^2 p - A^2 p^2$$

$$= A^2 p(1 - p) = \frac{A^2 p^2 (1 - p)}{p} = \frac{8}{9}$$



solving the equations for A and p (or use CAS)

$$\frac{16(1 - p)}{9p} = \frac{8}{9}$$

$2(1 - p) = p$

$2 - 2p = p$

$3p = 2$

$p = \frac{2}{3}$ so $A = 2$

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

a. $R(t) = 200\sqrt{t}\cos\left(\frac{t^2}{6}\right)$

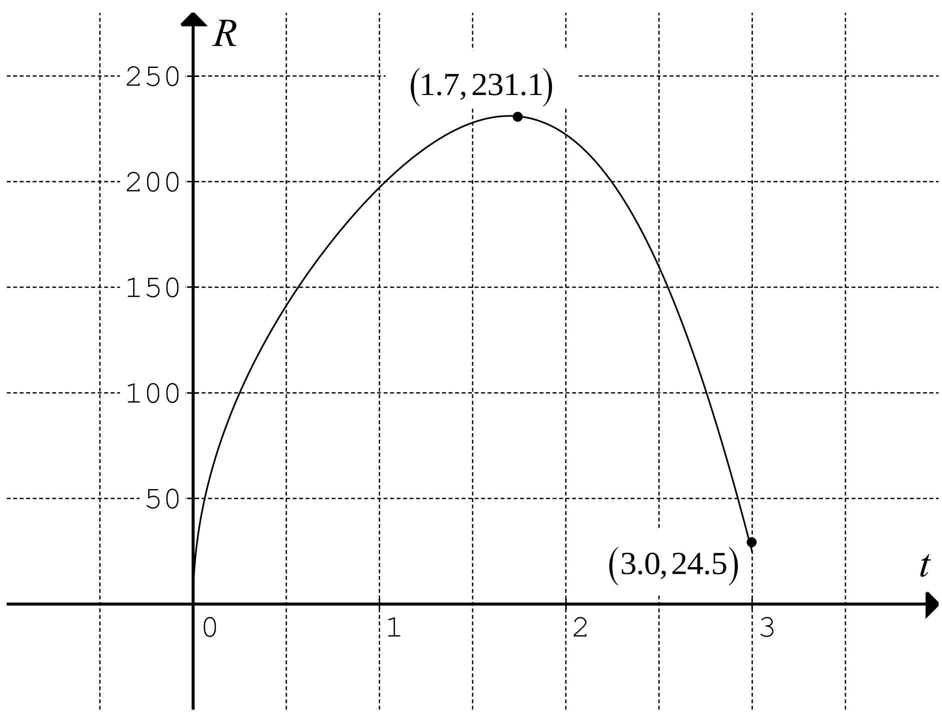
$R(3) = 200\sqrt{3}\cos\left(\frac{3}{2}\right) \approx 24.5$ cars/hr A1

b. $\frac{dR}{dt} = 0 \Rightarrow t = 1.697$ at $t = 1.7$ A1

$R(1.697) \approx 231.1$ cars/hr A1

c. total number of cars is $\int_0^3 R(t)\,dt = 502$ cars A1

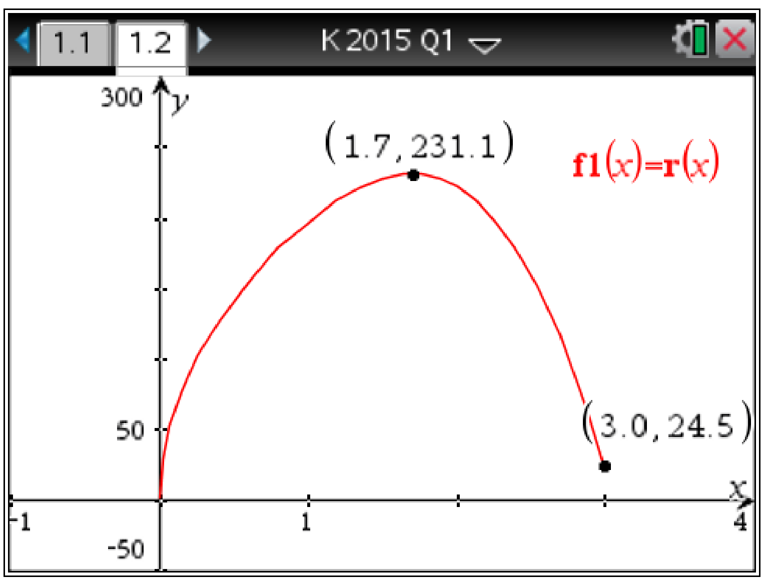
d. correct graph, shape, scale, restricted domain, end-point G1



e. $R(t) = 150 \Rightarrow t = t_1 = 0.564$, $t = t_2 = 2.547$ A1

$\bar{R} = \frac{\int_{t_1}^{t_2} R(t)\,dt}{t_2 - t_1} = \frac{\int_{0.564}^{2.547} R(t)\,dt}{2.547 - 0.564} = 203.7$ cars/hr A1

Define $r(t)=200\cdot\sqrt{t}\cdot\cos\left(\frac{t^2}{6}\right)\mid 0\le t\le 3$	*Done*
$r(3)$	24.5041
$\text{solve}\left(\frac{d}{dt}(r(t))=0,t\right)$	$t=1.6972$
$r(1.6972)$	231.0996
$\int_0^3 r(t)\,dt$	502.1446
$\text{solve}(r(t)=150,t)$	$t=0.5641$ or $t=2.5475$
$t2:=2.5475$	2.5475
$t1:=0.5641$	0.5641
$\dfrac{\int_{t1}^{t2} r(t)\,dt}{t2-t1}$	203.6555
Define $f1(x)=r(x)$	*Done*



Question 2

a.i. $y = x^p \quad \dfrac{dy}{dx} = px^{p-1}$ at $P(1,1),\ p > 1$

when $x = 1 \quad \left.\dfrac{dy}{dx}\right|_{x=1} = m_T = p$, tangent $\quad y - 1 = p(x-1)$

T: $\quad y = px + 1 - p$ A1

ii. at R, $\ y = 0,\quad 0 = px + 1 - p \quad \Rightarrow x_R = \dfrac{p-1}{p}$

so coordinates of R are $\left(\dfrac{p-1}{p}, 0\right)$ A1

iii. area $C = \dfrac{1}{2}\text{base} \times \text{height} = \dfrac{1}{2}\left(1 - \dfrac{p-1}{p}\right) \times 1 = \dfrac{1}{2}\left(\dfrac{p-(p-1)}{p}\right)$

$C = \dfrac{1}{2p}$ A1

iv. $B = \int_0^1 x^p\, dx - C = \int_0^1 x^p\, dx - \dfrac{1}{2p}$ A1

v. $B = \left[\dfrac{1}{p+1} x^{p+1}\right]_0^1 - \dfrac{1}{2p}$

$B = \dfrac{1}{p+1} - \dfrac{1}{2p}$ A1

vi. solving $\dfrac{dB}{dp} = -\dfrac{1}{(p+1)^2} + \dfrac{1}{2p^2} = 0$ M1

$\Rightarrow (p+1)^2 = 2p^2$

$p^2 + 2p + 1 = 2p^2$

$p^2 - 2p + 1 = 2$

$(p-1)^2 = 2$

$p - 1 = \pm\sqrt{2}$ but $p > 1$

$p = 1 + \sqrt{2}$ A1

b.i. normal $m_N = -\dfrac{1}{p}$ equation of the normal $\quad y - 1 = -\dfrac{1}{p}(x-1)$ M1

$y = -\dfrac{x}{p} + 1 + \dfrac{1}{p} = \dfrac{p+1}{p} - \dfrac{x}{p}$

ii. at Q, $x=0$, $y_Q = \dfrac{p+1}{p}$

at S, $y=0$, $0=\dfrac{p+1}{p}-\dfrac{x}{p} \Rightarrow x_S = p+1$

so coordinates $Q\left(0,\dfrac{p+1}{p}\right)$, $S(p+1,0)$ A1

iii. $A=\displaystyle\int_0^1\left(\frac{p+1}{p}-\frac{x}{p}-x^p\right)dx$ A1

iv. $A=\left[\dfrac{(p+1)x}{p}-\dfrac{x^2}{2p}-\dfrac{x^{p+1}}{p+1}\right]_0^1$ A1

$A=\dfrac{p+1}{p}-\dfrac{1}{2p}-\dfrac{1}{p+1}=1-\dfrac{1}{p+1}+\dfrac{1}{2p}$

solving $\dfrac{dA}{dp}=\dfrac{1}{(p+1)^2}-\dfrac{1}{2p^2}=0$ as in **a.vi.**

$\Rightarrow$ $p=1+\sqrt{2}$, it is a minimum (since **a.vi.** was a maximum) A1

Define $f(x)=x^p$	*Done*
tangentLine$(f(x),x,1)$	$p\cdot x-p+1$
Define $t(x)=p\cdot x-p+1$	*Done*
solve$(t(x)=0,x)$	$x=\dfrac{p-1}{p}$
$b:=\displaystyle\int_0^1 f(x)\,dx-\frac{1}{2\cdot p}\mid p>1$	$\dfrac{1}{p+1}-\dfrac{1}{2\cdot p}$
$\dfrac{d}{dp}\left(\dfrac{1}{p+1}-\dfrac{1}{2\cdot p}\right)$	$\dfrac{1}{2\cdot p^2}-\dfrac{1}{(p+1)^2}$
solve$\left(\dfrac{1}{2\cdot p^2}-\dfrac{1}{(p+1)^2}=0,p\right)\mid p>1$	$p=\sqrt{2}+1$
normalLine$(f(x),x,1)$	$\dfrac{p+1}{p}-\dfrac{x}{p}$
Define $n(x)=\dfrac{p+1}{p}-\dfrac{x}{p}$	*Done*
$n(0)$	$\dfrac{p+1}{p}$
solve$(n(x)=0,x)$	$x=p+1$
$a:=\displaystyle\int_0^1 (n(x)-f(x))\,dx\mid p>1$	$\dfrac{-1}{p+1}+\dfrac{1}{2\cdot p}+1$

Question 3

a. Total 20 pens

$$\Pr(\text{one of each color}) = 3! \times \frac{8\times 7\times 5}{20\times 19\times 18} = \frac{14}{57}$$ A1

b. Let R be Lilly uses red pen, and B be Lilly uses a blue pen

$B \to R = 0.6 \quad \Rightarrow B \to B = 0.4$ and $R \to R = 0.3 \quad \Rightarrow R \to B = 0.7$

i. $\Pr(\text{red three times}) = \Pr(RBRR) + \Pr(RRBR) + \Pr(RRRB)$ M1

$= 0.7\times 0.6\times 0.3 + 0.3\times 0.7\times 0.6 + 0.3^2\times 0.7$

$= 0.315$ A1

ii.
$$\begin{array}{cc} & \begin{matrix} R & B \end{matrix} \\ \begin{matrix} R \\ B \end{matrix} & \begin{bmatrix} 0.3 & 0.6 \\ 0.7 & 0.4 \end{bmatrix}^3 \end{array} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.447 \\ 0.553 \end{bmatrix}$$ M1

red pen on Thursday 0.447 A1

OR $\Pr(RXXR) = \Pr(RRRR) + \Pr(RBRR) + \Pr(RRBR) + \Pr(RBBR)$

$= 0.3^3 + 0.7\times 0.6\times 0.3 + 0.3\times 0.7\times 0.6 + 0.7\times 0.4\times 0.6 = 0.447$

iii. $$\begin{bmatrix} 0.3 & 0.6 \\ 0.7 & 0.4 \end{bmatrix}^{100} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.4615 \\ 0.5385 \end{bmatrix}$$

long run probability of red is 0.4615, probability of blue 0.5385

long run, more likely to use blue A1

c.i. Since the total area under the curve is equal to one.

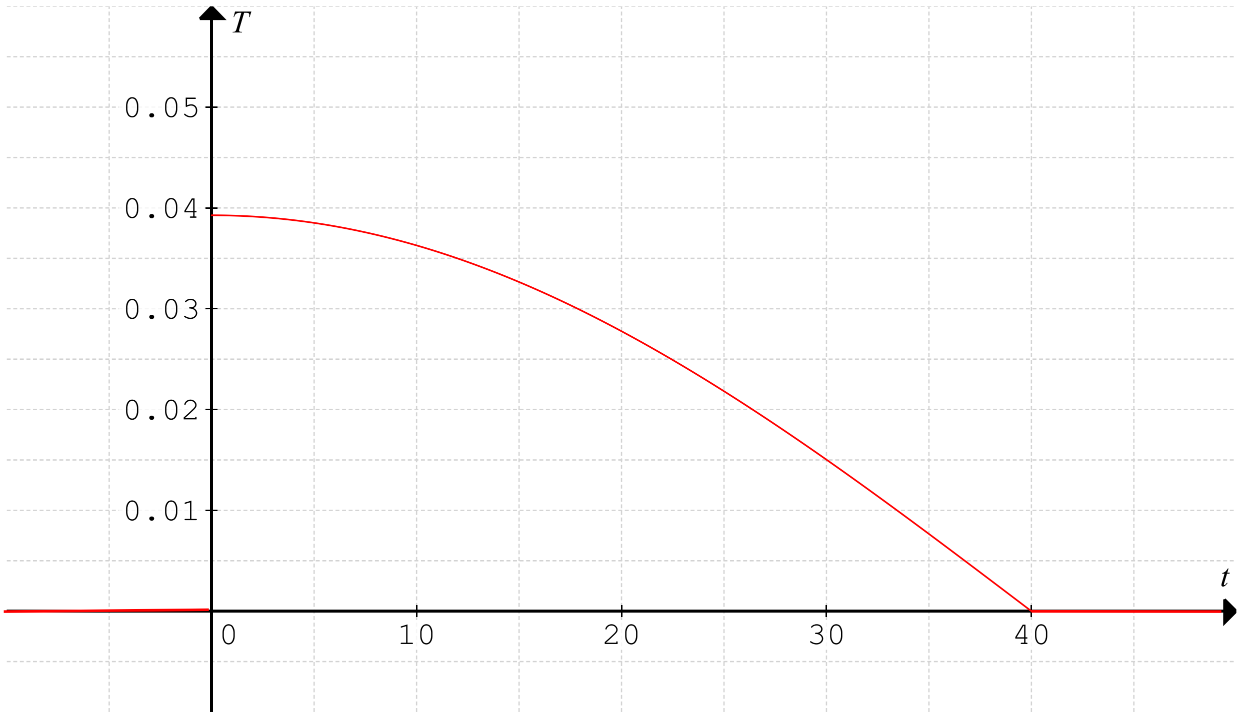
$$a\int_0^{40} \cos\left(\frac{\pi t}{80}\right) dt = 1$$ M1

$$a\left[\frac{80}{\pi}\sin\left(\frac{\pi t}{80}\right)\right]_0^{40} = 1$$ A1

$$\frac{80a}{\pi}\left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right) = 1$$

$$\frac{80a}{\pi} = 1 \quad \Rightarrow \quad a = \frac{\pi}{80}$$

ii. Since $\frac{\pi}{80} \approx 0.039$, correct graph scales, restricted domain A1



iii. $\Pr(T < 30) = \frac{\pi}{80}\int_0^{30} \cos\left(\frac{\pi t}{80}\right) dt = 0.9239$ A1

iv. $E(T) = \frac{\pi}{80}\int_0^{40} t\cos\left(\frac{\pi t}{80}\right) dt$

$= \frac{40(\pi - 2)}{\pi}$ minutes A1

v. the median m, satisfies $\frac{\pi}{80}\int_0^{m} \cos\left(\frac{\pi t}{80}\right) dt = \frac{1}{2}$ A1

solving gives $m = 13.33$ minutes A1

d. $X \stackrel{d}{=} N\left(\mu = ?, \sigma^2 = ?\right)$

$\Pr(X > 2600) = 0.091 \quad \Rightarrow \frac{2600 - \mu}{\sigma} = 1.3346$ M1

$\Pr(X < 1800) = 0.328 \quad \Rightarrow \frac{1800 - \mu}{\sigma} = -0.4454$ A2

solving $\mu = 2000.2$, $\sigma = 449.4$

the mean is 2000 metres, the standard deviation is 449 metres. A1

e. $Y \overset{d}{=} N\left(\mu = 2500, \sigma^2 = 300^2\right)$

$\Pr\left(Y > 3000\right) = 0.04779$

$Z \overset{d}{=} Bi\left(n = 3, p = 0.04779\right)$

$\Pr\left(Z \geq 1\right) = 1 - \Pr\left(Z = 0\right)$

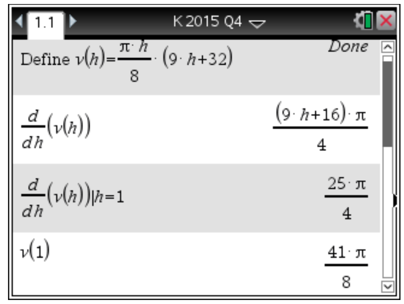
$= 1 - \left(1 - 0.04779\right)^3$

$= 0.1366$ A1

Define $f(t) = a \cdot \cos\left(\frac{\pi \cdot t}{80}\right)$	*Done*
$\int_0^{40} f(t)\, dt$	$\frac{80 \cdot a}{\pi}$
solve$\left(\frac{80 \cdot a}{\pi} = 1, a\right)$	$a = \frac{\pi}{80}$
Define $f(t) = a \cdot \cos\left(\frac{\pi \cdot t}{80}\right) \mid a = \frac{\pi}{80}$ and $0 \leq t \leq 40$	*Done*
$\int_0^{30} f(t)\, dt$	0.9239
$\int_0^{40} \left(t \cdot f(t)\right) dt$	$\frac{40 \cdot (\pi - 2)}{\pi}$
solve$\left(\int_0^{m} f(t)\, dt = \frac{1}{2}, m\right)$	$m = 13.3333$
invNorm$(1 - 0.091, 0, 1)$	1.3346
invNorm$(0.328, 0, 1)$	-0.4454
solve$\left(\frac{2600 - \mu}{\sigma} = 1.3346 \text{ and } \frac{1800 - \mu}{\sigma} = -0.4454, \{\mu, \sigma\}\right)$	$\mu = 2000.1798$ and $\sigma = 449.4382$
normCdf$(3000, \infty, 2500, 300)$	0.04779
binomCdf$(3, 0.04779, 1, 3)$	0.13663

Question 4

a.i. $V=\frac{\pi h}{8}(9h+32)$, $\Delta h=-0.1$



$V=\frac{\pi}{8}(9h^2+32h)$

$\frac{dV}{dh}=\frac{\pi}{8}(18h+32)=\frac{\pi}{4}(9h+16)$ M1

when $h=1$ $\frac{dV}{dh}=\frac{25\pi}{4}$

$\frac{dV}{dh}\approx\frac{\Delta V}{\Delta h}$ $\Rightarrow$ $\Delta V\approx\frac{dV}{dh}\times\Delta h=\frac{25\pi}{4}\times\frac{-1}{10}=-\frac{25\pi}{40}$

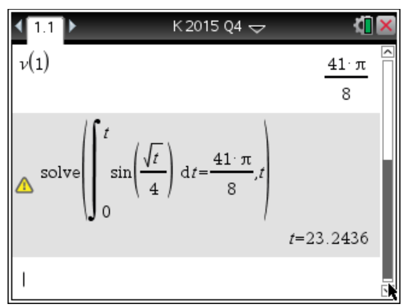
Volume decreases by $\frac{5\pi}{8}$ m^3 A1

ii. $\frac{dV}{dt}=-k$ so $V=-kt+c$ when $t=0$, $V=\frac{41\pi}{8}$ $\Rightarrow c=\frac{41\pi}{8}$

four hours is 240 minutes, when $t=240$, $V=0$ $\Rightarrow$ $0=-240k+\frac{41\pi}{8}$

$240k=\frac{41\pi}{8}$ $\Rightarrow$ $k=\frac{41\pi}{1920}$ A1

b. $\frac{dV}{dt}=\sin\left(\frac{\sqrt{t}}{4}\right)$

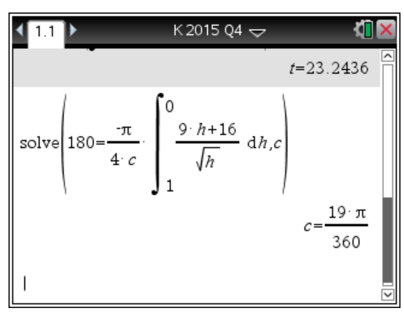


$V=\frac{41\pi}{8}=\int_0^T \sin\left(\frac{\sqrt{t}}{4}\right)dt$ solving M1

$T=23.24$ minutes. A1

c.i. $\frac{dV}{dt}=-c\sqrt{h}$, $\frac{dV}{dh}=\frac{\pi}{4}(9h+16)$ M1

$\frac{dh}{dt}=\frac{dh}{dV}\frac{dV}{dt}=\frac{-4c\sqrt{h}}{\pi(9h+16)}$ A1



ii. $\frac{dt}{dh}=-\frac{\pi(9h+16)}{4c\sqrt{h}}$ A1

$T=180=-\frac{\pi}{4c}\int_1^0\frac{(9h+16)}{\sqrt{h}}dh$ M1

$c=\frac{19\pi}{360}$ A1

Question 5

i. Let $f4(x)=\frac{x^2}{50}-9x+1013$ at the point *H*, $x=220$

$f4(220)=\frac{220^2}{50}-9\times 220+1013=1$

Let $f1(x)=ax^2+bx+c$, at *A*, $x=0$, $f1(0)=c=1$

so that $H(220,1)$ and $A(0,1)$ A1

ii. Let $f2(x)=\frac{x^2}{40}-4x+163$ and $f2'(x)=\frac{x}{20}-4$

at *C*, $x=60$, $f2(60)=\frac{60^2}{40}-4\times 60+163=13$ and $f2'(60)=\frac{60}{20}-4=-1$ A1

since the join is smooth, $f1(60)=13$ and $f1'(60)=-1$ M1

$f1(x)=ax^2+bx+1$ and $f1'(x)=2ax+b$

$f1(60)=13 \Rightarrow 13=3600a+60b+1 \quad (1)\ 3600a+60b=12$

$f1'(60)=-1 \Rightarrow (2) \quad -1=120+b$ M1

solving equations (1) and (2) gives $a=-\frac{1}{50}$, $b=\frac{7}{5}$ A1

iii. $f1(x)=-\frac{x^2}{50}+\frac{7x}{5}+1 \quad f1'(x)=-\frac{2x}{50}+\frac{7}{5}=0 \Rightarrow x=35$

at *B* when $x=35$, $f1(35)=-\frac{35^2}{50}+\frac{7\times 35}{5}+1=25\frac{1}{2}$ so $B\left(35,25\frac{1}{2}\right)$ A1

iv. Let $f3(x)=R\sin\left(\frac{\pi(x-100)}{n}\right)+k$ and $f3'(x)=\frac{R\pi}{n}\cos\left(\frac{\pi(x-100)}{n}\right)$

at *E*, $x=100$, $f2(100)=\frac{100^2}{40}-4\times 100+163=13$ and $f2'(100)=\frac{100}{20}-4=1$ A1

since the join is smooth, $f3(100)=13$ and $f3'(100)=1$ M1

$f3(100)=R\sin(0)+k=13 \Rightarrow k=13$

and $f3'(100)=\frac{R\pi}{n}\cos(0)=\frac{R\pi}{n}=1 \Rightarrow R\pi=n$ M1

at *F*, when $x=150$ it is a maximum, so that $f3'(150)=0$,

$f3'(150)=\frac{R\pi}{n}\cos\left(\frac{50\pi}{n}\right)=0$ but $\cos\left(\frac{\pi}{2}\right)=0$ so $\frac{50\pi}{n}=\frac{\pi}{2}$

$\Rightarrow n=100$ and $R=\frac{100}{\pi}$ A1

v. $f3(x)=\dfrac{100}{\pi}\sin\left(\dfrac{\pi(x-100)}{100}\right)+13$

at F when $x=150$, $f3(150)=\dfrac{100}{\pi}+13$ so $F\left(150,\dfrac{100}{\pi}+13\right)$

the vertical distance BF is $\dfrac{100}{\pi}+13-25\dfrac{1}{2}$

$BF=\dfrac{100}{\pi}-\dfrac{25}{2}$ A1

Define $f4(x)=\dfrac{x^2}{50}-9\cdot x+1013$	*Done*
$f4(220)$	1
Define $f2(x)=\dfrac{x^2}{40}-4\cdot x+163$	*Done*
$f2(60)$	13
$\dfrac{d}{dx}(f2(x))\|x=60$	-1
Define $f1(x)=a\cdot x^2+b\cdot x+1$	*Done*
$f1(60)=13$	$3600\cdot a+60\cdot b+1=13$
$\dfrac{d}{dx}(f1(x))\|x=60$	$120\cdot a+b$
solve$(3600\cdot a+60\cdot b+1=13$ and $120\cdot a+b=-1,\{a,b\})$	$a=\dfrac{-1}{50}$ and $b=\dfrac{7}{5}$
Define $f1(x)=a\cdot x^2+b\cdot x+1\|a=\dfrac{-1}{50}$ and $b=\dfrac{7}{5}$	*Done*
solve$\left(\dfrac{d}{dx}(f1(x))=0,x\right)$	$x=35$
$f1(35)$	$\dfrac{51}{2}$

$f1(35)$	$\frac{51}{2}$
Define $f3(x)=r\cdot\sin\left(\frac{\pi\cdot(x-100)}{n}\right)+k$	*Done*
$f2(100)$	13
$\frac{d}{dx}(f2(x))\vert x=100$	1
$f3(100)=13$	$k=13$
$\frac{d}{dx}(f3(x))\vert x=100$	$\frac{\pi\cdot r}{n}$
$\frac{d}{dx}(f3(x))\vert x=150$	$\frac{\cos\left(\frac{50\cdot\pi}{n}\right)\cdot\pi\cdot r}{n}$
$\text{solve}\left(\frac{50\cdot\pi}{n}=\frac{\pi}{2},n\right)$	$n=100$
Define $f3(x)=\frac{100}{\pi}\cdot\sin\left(\frac{\pi\cdot(x-100)}{100}\right)+13$	*Done*
$f3(150)$	$\frac{100}{\pi}+13$
$\frac{100}{\pi}+13-\frac{51}{2}$	$\frac{100}{\pi}-\frac{25}{2}$

END OF SECTION 2 SUGGESTED ANSWERS