

Year 2015
VCE
Mathematical Methods
Trial Examination 1
Solutions



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9018 5376
FAX: (03) 9817 4334
kilbaha@gmail.com
<http://kilbaha.com.au>

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Multimedia Publishing.
- The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
- For authorised copying within Australia please check that your institution has a licence from <http://copyright.com.au>. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.

Reproduction and communication for educational purposes The Australian Copyright Act 1968 (the Act) allows a maximum of one chapter or 10% of the pages of this work, to be reproduced and/or communicated by any educational institution for its educational purposes provided that educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact
CAL, Level 15, 233 Castlereagh Street, Sydney, NSW, 2000
Tel: (02) 9394 7600
Fax: (02) 9394 7601
Email: info@copyright.com.au

- All of these pages must be counted in Copyright Agency Limited (CAL) surveys
- This file must not be uploaded to the Internet.

These suggested answers have no official status.

While every care has been taken, no guarantee is given that these answers are free from error. Please contact us if you believe you have found an error.

CAUTION NEEDED!

All Web Links when created linked to appropriate Web Sites. Teachers and parents must always check links before using them with students to ensure that students are protected from unsuitable Web Content. Kilbaha Multimedia Publishing is not responsible for links that have been changed in this document or links that have been redirected.

Question 1

a. $y = \frac{\log_e(3x)}{2x}$ using the quotient rule

$$u = \log_e(3x) \quad v = 2x$$

$$\frac{du}{dx} = \frac{1}{x} \quad \frac{dv}{dx} = 2$$

M1

$$\frac{dy}{dx} = \frac{2x \times \frac{1}{x} - 2 \log_e(3x)}{(2x)^2} = \frac{2(1 - \log_e(3x))}{4x^2}$$

$$\frac{dy}{dx} = \frac{1}{2x^2} (1 - \log_e(3x))$$

A1

b. $f(x) = x\sqrt{4x^2 + 9}$

$$y = x\sqrt{4x^2 + 9} \text{ using the product and chain rules}$$

$$u = x \quad v = \sqrt{4x^2 + 9} = (4x^2 + 9)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \frac{1}{2} \times 8x \times (4x^2 + 9)^{-\frac{1}{2}} = \frac{4x}{\sqrt{4x^2 + 9}}$$

$$f'(x) = u \frac{dv}{dx} + v \frac{du}{dx} = \frac{4x^2}{\sqrt{4x^2 + 9}} + \sqrt{4x^2 + 9}$$

M1

$$f'(-2) = \frac{16}{\sqrt{25}} + \sqrt{25} = \frac{16}{5} + 5$$

$$f'(-2) = \frac{41}{5} \text{ or } 8\frac{1}{5} \text{ or } 8.2$$

A1

Question 2

$$3 \times 9^x - 28 \times 3^x + 9 = 0$$

$$\text{let } u = 3^x \text{ then } 9^x = (3^2)^x = 3^{2x} = (3^x)^2 = u^2$$

$$3u^2 - 28u + 9 = 0$$

$$(3u - 1)(u - 9) = 0$$

M1

$$u = 3^x = \frac{1}{3} \quad u = 3^x = 9$$

$$x = -1 \text{ or } x = 2$$

A1

Question 3

$$\begin{array}{lcl}
 kx + 8y = k - 4 & \Delta = \begin{vmatrix} k & 8 \\ 3 & k-2 \end{vmatrix} = k(k-2) - 24 = k^2 - 2k - 24 & \\
 3x + (k-2)y = 1 & \Delta = (k-6)(k+4) & \text{M1}
 \end{array}$$

There is a unique solution when $\Delta \neq 0$ that is $k \in \mathbb{R} \setminus \{-4, 6\}$ A1

When $k = -4$ the equations become $-4x + 8y = -8$
 $3x - 6y = 3$ these lines are parallel

with different y -intercepts, therefore there is no solution when $k = -4$

When $k = 6$ the equations become $6x + 8y = 2$
 $3x + 4y = 1$ these lines are both the same line,
 therefore we have an infinite number of solutions when $k = 6$ A1

Question 4

a. $y = \frac{3}{x+2} - 1$

crosses the x -axis when $y = 0 \Rightarrow x + 2 = 3 \quad x = 1 \quad (1, 0)$

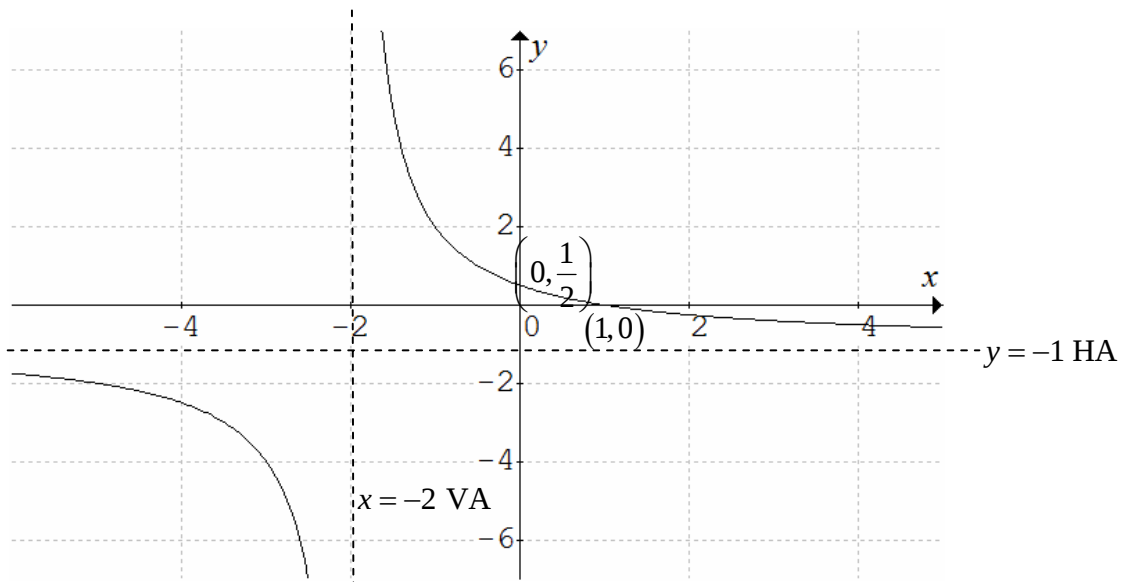
crosses the y -axis when $x = 0 \Rightarrow y = \frac{3}{2} - 1 \quad y = \frac{1}{2} \quad \left(0, \frac{1}{2}\right)$

$x = -2$ is a vertical asymptote, $y = -1$ is a horizontal asymptote

correct graph, shape, asymptotes axial intercepts

A1

G1



b. The area is below the x -axis, the area is $A = -\int_1^4 \left(\frac{3}{x+2} - 1 \right) dx$ A1

$$A = \left[-3\log_e(x+2) + x \right]_1^4$$

$$A = -3\log_e(6) + 4 + 3\log_e 3 - 1$$

$$A = 3\log_e\left(\frac{1}{2}\right) + 3 \text{ or } 3 - 3\log_e(2) \text{ or } 3 - \log_e(8) \quad \text{A1}$$

c. $y = \frac{1}{x}$ into $y = \frac{3}{x+2} - 1$

the translations must come last.

dilate by a factor of 3 parallel to the y -axis (or away from the x -axis)

translate 2 units to the left parallel to the x -axis (or away from the y -axis)

translate 1 unit down parallel to the y -axis (or away from the x -axis) A1

d. $f: y = \frac{3}{x+2} - 1$ swap x and y

$$f^{-1} \quad x = \frac{3}{y+2} - 1$$

$$x+1 = \frac{3}{y+2}$$

$$y+2 = \frac{3}{x+1}$$

$$y = \frac{3}{x+1} - 2$$

M1

but $\text{dom } f = R \setminus \{-2\} = \text{ran } f^{-1}$ and $\text{ran } f = R \setminus \{-1\} = \text{dom } f^{-1}$

To state the function, we must state its domain

$$f^{-1}: R \setminus \{-1\} \rightarrow R, \quad f^{-1}(x) = \frac{3}{x+1} - 2 \quad \text{A1}$$

Question 5

$$f'(x) = 4e^{-2x} - 1$$

$$f(x) = \int (4e^{-2x} - 1) dx = -2e^{-2x} - x + c \quad \text{M1}$$

$$f(0) = 1 \Rightarrow 1 = -2e^{-0} + c \Rightarrow c = 3$$

$$f(x) = 3 - 2e^{-2x} - x \quad \text{A1}$$

Question 6

a. range $-2 \leq y \leq 6 = [-2, 6]$, period $T = \frac{2\pi}{\frac{\pi}{6}} = 12$, amplitude 4 A1

b. $2 - 4\sin\left(\frac{\pi x}{6}\right) = 0 \Rightarrow 4\sin\left(\frac{\pi x}{6}\right) = 2$ or $\sin\left(\frac{\pi x}{6}\right) = \frac{1}{2}$

$$\frac{\pi x}{6} = 2n\pi + \sin^{-1}\left(\frac{1}{2}\right) \quad \text{or} \quad \frac{\pi x}{6} = (2n+1)\pi - \sin^{-1}\left(\frac{1}{2}\right)$$

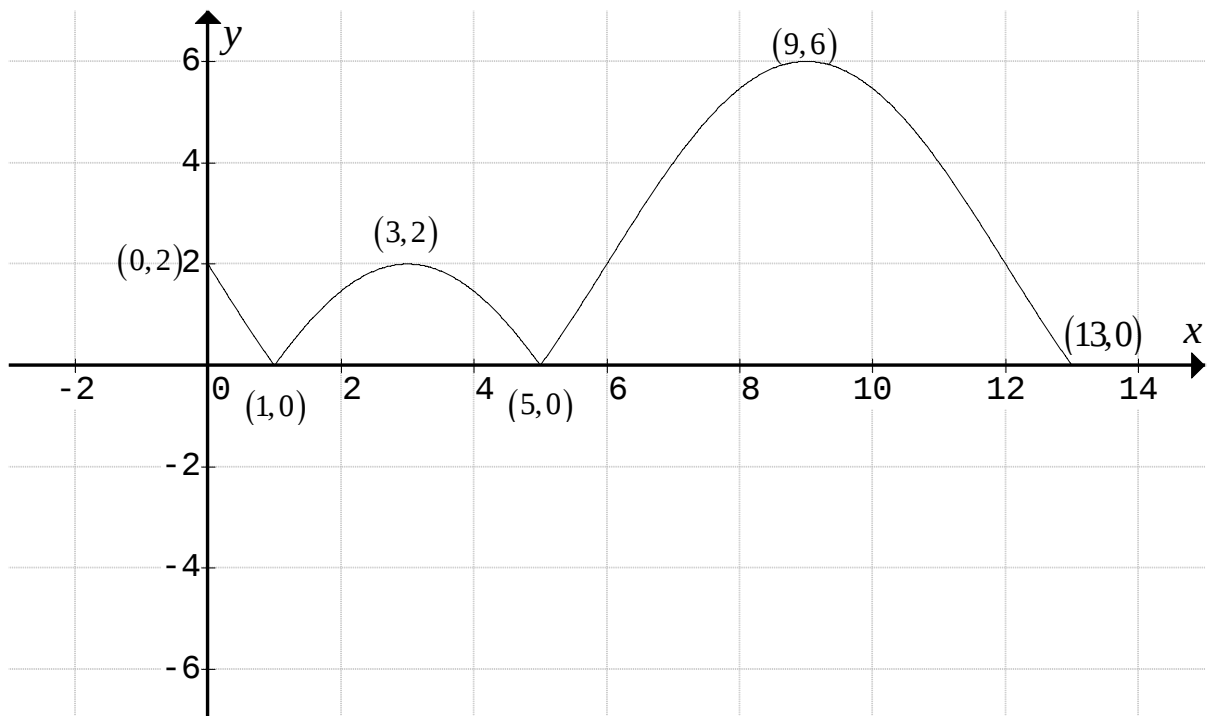
$$\frac{\pi x}{6} = 2n\pi + \frac{\pi}{6} \quad \text{or} \quad \frac{\pi x}{6} = (2n+1)\pi - \frac{\pi}{6}$$

$$\frac{\pi x}{6} = \frac{\pi}{6}(12n+1) \quad \text{or} \quad \frac{\pi x}{6} = \frac{\pi}{6}(12n+5)$$

$x = 12n+1$ or $12n+5$ where $n \in \mathbb{Z}$ M1

c. $n = 0$ or $n = 1 \Rightarrow x = 1, 5, 13$ A1

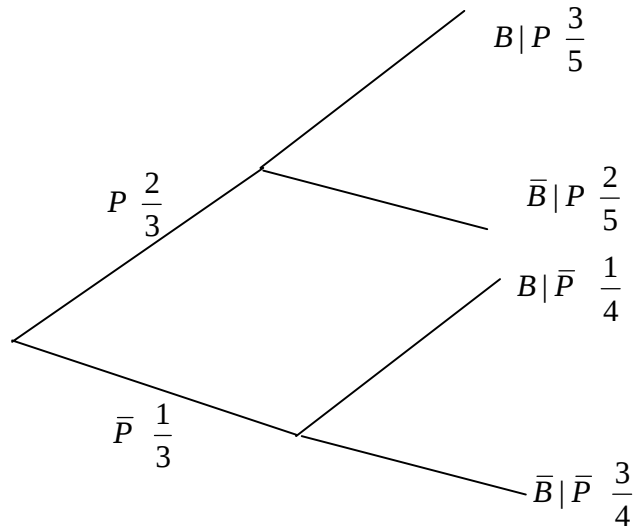
d. correct graph and coordinates G2



e. domain $(0, 13)$ not including 1, 5 or $(0, 1) \cup (1, 5) \cup (5, 13)$ A1

Question 7

- a. P pleasant weather, B uses BBQ, using a tree diagram



$$\Pr(\bar{P} | \bar{B}) = \frac{\Pr(\bar{P} \cap \bar{B})}{\Pr(\bar{B})} = \frac{\frac{1}{3} \times \frac{3}{4}}{\frac{1}{3} \times \frac{3}{4} + \frac{2}{3} \times \frac{2}{5}} = \frac{\frac{1}{4}}{\frac{1}{4} + \frac{4}{15}} = \frac{1}{4} \times \frac{60}{31}$$

M1

$$\Pr(\bar{P} | \bar{B}) = \frac{15}{31}$$

A1

- b. Since the probabilities sum to one. $\sum \Pr(X = x) = 1$

$$\log_8(k-1) + \log_8(k-3) = 1$$

$$\log_8[(k-1)(k-3)] = 1$$

$$(k-1)(k-3) = 8$$

$$k^2 - 4k + 3 = 8$$

M1

$$k^2 - 4k - 5 = 0$$

$$(k-5)(k+1) = 0$$

$$k = 5 \text{ or } k = -1 \text{ but } k > 3$$

$$\text{only solution } k = 5$$

A1

Question 8

a. using the product rule

$$\begin{aligned}\frac{d}{dx}(x \cos(2x)) &= x \frac{d}{dx}(\cos(2x)) + \cos(2x) \frac{d}{dx}(x) \\ &= -2x \sin(2x) + \cos(2x)\end{aligned}$$

A1

b. $f: [0, \infty) \rightarrow \mathbb{R}$, $f(x) = x \sin(2x)$ The graph crosses the x -axis when $\sin(2x) = 0$

$$2x = 0, \pi \text{ so that } x = 0, \frac{\pi}{2}$$

$$\text{The area is } A = \int_0^{\frac{\pi}{2}} x \sin(2x) dx$$

A1

Since $\frac{d}{dx}(x \cos(2x)) = -2x \sin(2x) + \cos(2x)$ from a.

$$\int (-2x \sin(2x) + \cos(2x)) dx = x \cos(2x)$$

$$-2 \int x \sin(2x) dx + \int \cos(2x) dx = x \cos(2x)$$

$$2 \int x \sin(2x) dx = \int \cos(2x) dx - x \cos(2x)$$

M1

$$= \frac{1}{2} \sin(2x) - x \cos(2x)$$

$$\int x \sin(2x) dx = \frac{1}{4} \sin(2x) - \frac{1}{2} x \cos(2x) + c$$

$$A = \left[\frac{1}{4} \sin(2x) - \frac{1}{2} x \cos(2x) \right]_0^{\frac{\pi}{2}}$$

$$A = \frac{1}{4} \sin(\pi) - \frac{1}{2} \times \frac{\pi}{2} \cos(\pi) - \frac{1}{4} \sin(0) - 0$$

$$A = \frac{\pi}{4} \text{ units}^2$$

A1

Question 9

$$\text{The normal } 4y - x + c = 0 \Rightarrow y = \frac{x}{4} - \frac{c}{4} \Rightarrow m_N = \frac{1}{4} \quad m_T = -4$$

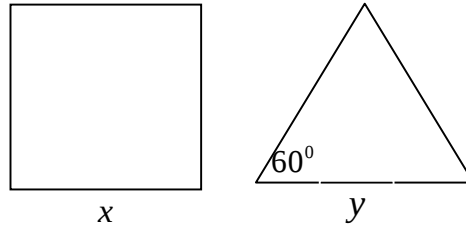
M1

$$y = 2e^{-2x} + 1 \Rightarrow \frac{dy}{dx} = -4e^{-2x} = -4 \Rightarrow x = 0 \text{ so } y = 3 \quad P(0, 3)$$

M1

$$c = x - 4y = -12$$

A1

Question 10**a.**(1) $4x + 3y = 36$ the total length of string $A = x^2 + \frac{1}{2}y^2 \sin(60^\circ)$ total area of square and equilateral triangle

$$(2) A = x^2 + \frac{\sqrt{3}y^2}{4} \quad \text{A1}$$

(1) $\Rightarrow 3y = 36 - 4x = 4(9 - x)$ substitute $y = \frac{4}{3}(9 - x)$ into (2)But since the lengths cannot be negative, we require $0 \leq x \leq 9$ A1

$$A(x) = x^2 + \frac{\sqrt{3}}{4} \left(\frac{4}{3}(9 - x) \right)^2 = x^2 + \frac{4\sqrt{3}}{9} (81 - 18x + x^2)$$

$$A(x) = \left(\frac{4\sqrt{3}}{9} + 1 \right) x^2 - 8\sqrt{3}x + 36\sqrt{3} \quad \text{for } 0 \leq x \leq 9$$

b. For a maximum or a minimum

$$\text{we require } \frac{dA}{dx} = 2x \left(\frac{4\sqrt{3}}{9} + 1 \right) - 8\sqrt{3} = 0$$

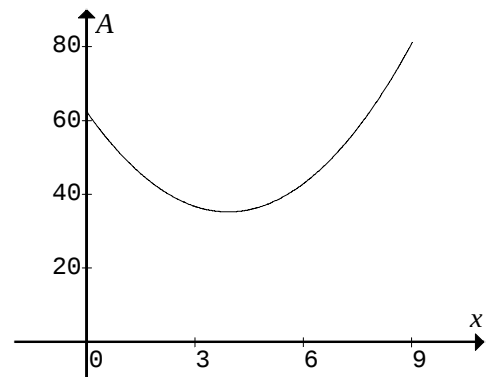
$$\Rightarrow x = \frac{4\sqrt{3}}{\frac{4\sqrt{3}}{9} + 1} = \frac{36\sqrt{3}}{4\sqrt{3} + 9} = \frac{36}{4 + 3\sqrt{3}}$$

$$\text{Now } 1.5 < \sqrt{3} < 2 \text{ so that } 3 < \frac{54}{15} < \frac{36\sqrt{3}}{4\sqrt{3} + 9} < \frac{72}{17} < 5$$

To find the maximum value investigate the end-points of the function

$$A(x) = x^2 + \frac{\sqrt{3}}{4} \left(\frac{4}{3}(9 - x) \right)^2$$

$$A(0) = \frac{\sqrt{3}}{4} \times \left(\frac{4 \times 9}{3} \right)^2 = 36\sqrt{3} \quad \text{and} \quad A(9) = 81 \quad \text{A1}$$

The minimum value of the area occurs when $x = \frac{36\sqrt{3}}{4\sqrt{3} + 9}$ The maximum value of the area occurs when $x = 9$.That is when only the square is formed. A1**END OF SUGGESTED SOLUTIONS**