

2015 VCAA Math Methods CAS Exam 1 Solutions
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Q1a $y = (5x+1)^7$, $\frac{dy}{dx} = 7 \times 5 \times (5x+1)^6 = 35(5x+1)^6$

Q1bi $f(x) = \frac{\log_e(x)}{x^2}$,
 $f'(x) = \frac{(x^2)(\frac{1}{x}) - (2x)(\log_e(x))}{(x^2)^2} = \frac{x(1-2\log_e(x))}{x^4} = \frac{1-2\log_e(x)}{x^3}$

Q1bii $f'(1) = \frac{1-2\log_e(1)}{1^3} = 1$

Q2 $f'(x) = 1 - \frac{3}{x}$, $f(x) = \int \left(1 - \frac{3}{x}\right) dx = x - 3\log_e|x| + c$

$f(e) = e - 3\log_e|e| + c = -2$, $\therefore c = 1 - e$

$\therefore f(x) = x - 3\log_e|x| + 1 - e$

Q3 $\int_1^4 \left(\frac{1}{\sqrt{x}}\right) dx = \int_1^4 x^{-\frac{1}{2}} dx = \left[2x^{\frac{1}{2}}\right]_1^4 = 2\sqrt{4} - 2\sqrt{1} = 2$

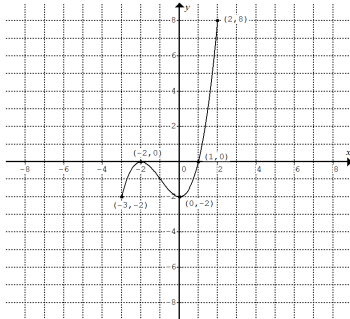
Q4a $f(x) = \frac{1}{2}(x^3 + 3x^2 - 4)$, $f'(x) = \frac{1}{2}(3x^2 + 6x)$

Let $f'(x) = 0$, $\frac{1}{2}(3x^2 + 6x) = \frac{3}{2}x(x+2) = 0$

$\therefore x = -2, 0$ and the corresponding ordinates are $y = 0, -2$

The stationary points are: $(-2, 0)$ and $(0, -2)$.

Q4b



Q4c Av. value = $\frac{\int_0^2 \frac{1}{2}(x^3 + 3x^2 - 4) dx}{2-0} = \frac{1}{4} \left[\frac{x^4}{4} + x^3 - 4x \right]_0^2 = 1$

Q5a $h(t) = 14 + 8\sin\left(\frac{\pi t}{12}\right)$, minimum depth = $14 - 8 = 6$ m

Q5b $14 + 8\sin\left(\frac{\pi t}{12}\right) = 10$ where $0 \leq t \leq 24$

$\sin\left(\frac{\pi t}{12}\right) = -\frac{1}{2}$, $\frac{\pi t}{12} = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\therefore t = 14, 22$

Q6a $\Pr(X > 3.1) = \Pr\left(Z > \frac{3.1-2.5}{0.3}\right) = \Pr(Z > 2) = \Pr(Z < b)$, $b = -2$

Q6b $\Pr(Z < -1) = \Pr(Z > 1) = 0.16$, $\therefore \Pr(Z < 1) = 0.84$

$\therefore \Pr(0 < Z < 1) = 0.84 - 0.5 = 0.34$

$\Pr(X < 2.8 | X > 2.5) = \Pr(Z < 1 | Z > 0) = \frac{\Pr(0 < Z < 1)}{\Pr(Z > 0)} = \frac{0.34}{0.5} = 0.68$

Q7a $\log_2(6-x) - \log_2(4-x) = 2$, $\log_2\left(\frac{6-x}{4-x}\right) = 2$, $\frac{6-x}{4-x} = 2^2$

$6-x = 16-4x$, $x = \frac{10}{3}$

Q7b $3e^t = 5 + 8e^{-t}$, $3e^t - 5 - 8e^{-t} = 0$, $(3e^t - 5 - 8e^{-t})e^t = 0$
 $3(e^t)^2 - 5e^t - 8 = 0$ $\therefore (3e^t - 8)(e^t + 1) = 0$

Since $e^t + 1 > 0$, $\therefore 3e^t - 8 = 0$, $e^t = \frac{8}{3}$, $t = \log_e\left(\frac{8}{3}\right)$

Q8a $\Pr(A \cap B) = \Pr(A|B)\Pr(B) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

Q8b $\Pr(A \cap B) + \Pr(A' \cap B) = \Pr(B)$

$\therefore \Pr(A' \cap B) = \Pr(B) - \Pr(A \cap B) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

Q8c If A and B are independent, $\Pr(A) = \Pr(A|B) = \frac{3}{4}$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) = \frac{3}{4} + \frac{1}{3} - \frac{1}{4} = \frac{5}{6}$

Q9a $\Pr(\text{white}) = \frac{3}{5}p + \frac{1}{5}(1-p) = \frac{1}{5}(2p+1)$

Q9bi $\Pr(B|\text{white}) = \frac{\Pr(B \cap \text{white})}{\Pr(\text{white})} = \frac{\frac{1}{5}(1-p)}{\frac{1}{5}(2p+1)} = \frac{1-p}{1+2p}$

Q9bii $\frac{1-p}{1+2p} = 0.3$, $p = \frac{7}{16}$

Q10a $T(2+2\cos\theta, 2\sin\theta)$

Q10b Gradient of $TC = \tan\theta$

$\therefore$ gradient of tangent = $-\frac{1}{m_N} = -\frac{1}{\tan\theta}$

Alternatively, gradient of tangent = $\tan\angle TXx = \tan\left(\frac{\pi}{2} + \theta\right)$

Q10ci $B(2, b)$ is on the line $x\cos\theta + y\sin\theta = 2 + 2\cos\theta$

$\therefore 2\cos\theta + b\sin\theta = 2 + 2\cos\theta$, $\therefore b = \frac{2}{\sin\theta}$

Q10cii $D(4, d)$ is on the line $x\cos\theta + y\sin\theta = 2 + 2\cos\theta$

$\therefore 4\cos\theta + d\sin\theta = 2 + 2\cos\theta$, $\therefore d = \frac{2-2\cos\theta}{\sin\theta}$

Q10d Area $A = \frac{1}{2}(d+b) \times 2 = d+b = \frac{4-2\cos\theta}{\sin\theta}$

$\frac{dA}{d\theta} = \frac{\sin\theta(2\sin\theta) - (4-2\cos\theta)\cos\theta}{\sin^2\theta}$
 $= \frac{2(\sin^2\theta + \cos^2\theta) - 4\cos\theta}{\sin^2\theta} = \frac{2-4\cos\theta}{\sin^2\theta}$

Let $\frac{dA}{d\theta} = 0$, $2-4\cos\theta = 0$, $\cos\theta = \frac{1}{2}$, $\theta = \frac{\pi}{3}$

$\therefore$ minimum $A = \frac{4-1}{\frac{\sqrt{3}}{2}} = 2\sqrt{3}$ square units

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