

SECTION 1

1	2	3	4	5	6	7	8	9	10	11
D	E	C	E	C	B	B	A	D	E	B

12	13	14	15	16	17	18	19	20	21	22
E	C	E	E	D	D	A	E	C	D	E

Q1 $x = f(y)$ is the inverse of $y = f(x)$. If they do intersect, the intersecting point is at the line $y = x$. D

Q2 $2^x - 2^{\frac{x+b}{2}} + \frac{1}{2} = 0$, $\left(2^{\frac{x}{2}}\right)^2 - 2^{\frac{b}{2}}\left(2^{\frac{x}{2}}\right) + \frac{1}{2} = 0$
 $\Delta = \left(-2^{\frac{b}{2}}\right)^2 - 4\left(1\right)\left(\frac{1}{2}\right) = 2^b - 2 > 0$ for two distinct solutions
 $\therefore b > 1$ E

Q3 $g(x) < 0$ for $x \in (-2a, -a)$ and $g(-a)$ is undefined,
 $\therefore f(g(x))$ is defined for $x \in \mathbb{R} \setminus (-2a, -a]$ C

Q4 Before the transformations: gradient = $\frac{\text{rise}}{\text{run}} = \frac{m}{1}$
 After the transformations: gradient = $\frac{2 \times m}{0.5 \times 1} = 4m$ E

Q5 $y = \log_e |2x|$, $\frac{dy}{dx} = \frac{1}{x} = \frac{1}{a}$ at $x = a$
 $\therefore$ the gradient of the perpendicular at $x = a$ is $\frac{-1}{\frac{1}{a}} = -a$ C

Q6 Estimate the signed area of the regions bounded by the curve and the x -axis, then divide by 20. B

Q7 For a concave graph, points of inflection exist in pairs. B

Q8 $f(x) = \left(\frac{\sin(nx)}{2}\right)^2 = \frac{1}{8} - \frac{1}{8}\cos(2nx)$, $T = \frac{2\pi}{2n} = \frac{\pi}{n}$ A

Q9 There are $2n-1$ asymptotes inside interval $(0, 2\pi)$ D

Q10 $f(-x) + f(x) = 0$, $\therefore f(x)$ is an odd function. E

Q11 $\int_1^0 f(x-0.5)dx = 2$, $\therefore \int_0^1 f(x-0.5)dx = -2$
 $f(1-2x)$ is the reflection (in the y -axis) and the dilation (by a factor of 0.5 in the x -direction) of $f(x-0.5)$,
 $\therefore \int_{-0.5}^0 f(1-2x)dx = -1$ B

Q12 The local minimum of $f(x)$ is at $x > 0$. For the graph of $y = f(x)$ to cross the negative x -axis only once, the y -intercept must be at or above the origin, i.e. $f(0) = b - a \geq 0$, $\therefore b \geq a$.

Note: $a > \sqrt{1+3a} - 1$ for $a > 1$, $\therefore$ D is not the answer. E

Q13 $1 - 2^{ax+1} - 2 \times 2^{2ax} + 4 \times 2^{3ax} = 0$
 $(1 - 2 \times 2^{ax}) - 2 \times 2^{2ax}(1 - 2 \times 2^{ax}) = 0$
 $(1 - 2 \times 2^{ax})(1 - 2 \times 2^{2ax}) = 0$, $(1 - 2^{ax+1})(1 - 2^{2ax+1}) = 0$
 $\therefore 2^{ax+1} = 1$ or $2^{2ax+1} = 1$
 $\therefore x = -\frac{1}{a}$ or $x = -\frac{1}{2a}$ C

Q14 $\log_e\left(\frac{x}{b}\right) > \log_e(x + a - ab)$, $\therefore \frac{x}{b} > x + a - ab$ where $x > 0$,
 $x + a - ab > 0$ and $b > a > 1$, $\therefore x > ab - a$
 Also, from $\frac{x}{b} > x + a - ab$, $x < ab$
 $\therefore (ab - a) < x < ab$ E

Q15 $y = a \log_e(bx)$ and $y = \frac{1}{b}e^{\frac{x}{a}}$ are inverse of each other.
 Their common point is on the line $y = x$.
 $\therefore \frac{dy}{dx} = \frac{a}{x} = 1$ and $\frac{dy}{dx} = \frac{1}{ab}e^{\frac{x}{a}} = 1$ at the common point
 $\therefore x = a$ and $\therefore ab = e$ E

Q16 $2 \sin\left(\frac{3\pi}{2} - kx\right) + \sqrt{3} = 0$, $\therefore -2 \cos(kx) + \sqrt{3} = 0$
 $\cos(kx) = \frac{\sqrt{3}}{2}$, $kx = 2n\pi \pm \frac{\pi}{6} = \left(\frac{12n \pm 1}{6}\right)\pi$
 $\therefore x = \left(\frac{12n \pm 1}{6k}\right)\pi$ D

Q17 $\begin{bmatrix} -1 & 0 \\ 0 & a \end{bmatrix} \left(\begin{bmatrix} a \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right) = \begin{bmatrix} 1-a \\ a \end{bmatrix}$ D

Q18 $a \sin 0 + a \sin \frac{\pi}{4} + a \sin \frac{\pi}{2} + a \sin \frac{3\pi}{4} + a \sin \pi = 1$
 $\therefore a = \frac{1}{1 + \sqrt{2}}$
 $\therefore \Pr\left(X = \frac{\pi}{2}\right) = a \sin \frac{\pi}{4} = \frac{1}{1 + \sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2 + \sqrt{2}}$
 $= \frac{2 - \sqrt{2}}{2} = 1 - \frac{1}{\sqrt{2}}$ A

Q19 It is a binomial distribution of X with parameters $n = 5$ and $p = 0.9$.
 $\mu = np = 4.5$ and $\sigma = \sqrt{np(1-p)} \approx 0.67$ E

Q20 $\mu - 2\sigma = 1$ and $\mu + \sigma = 4$, $\therefore \mu = 3$ and $\sigma = 1$
 $\Pr\left(\mu - \frac{\sigma}{2} < X < \mu + \frac{3\sigma}{2}\right) = \Pr(2.5 < X < 4.5) \approx 0.6241$

Q21 $(3.5 - 2.5)p + (4.3 - 3.8)3p = 1$, $\therefore p = 0.4$
 $\therefore (3.5 - 2.5)p = 0.4 < 0.5$
 $\therefore$ the median m is in $3.8 \leq x \leq 4.3$
 $\therefore (4.3 - m)3 \times 0.4 = 0.5$, $m \approx 3.88$

Q22

	A	A'	
B	x	$0.7 - x$	0.7
B'	$0.5 - x$	$x - 0.2$	0.3
	0.5	0.5	1

where $x \geq 0$, $0.7 - x \geq 0$, $0.5 - x \geq 0$ and $x - 0.2 \geq 0$
 $\therefore 0.2 \leq x \leq 0.5$

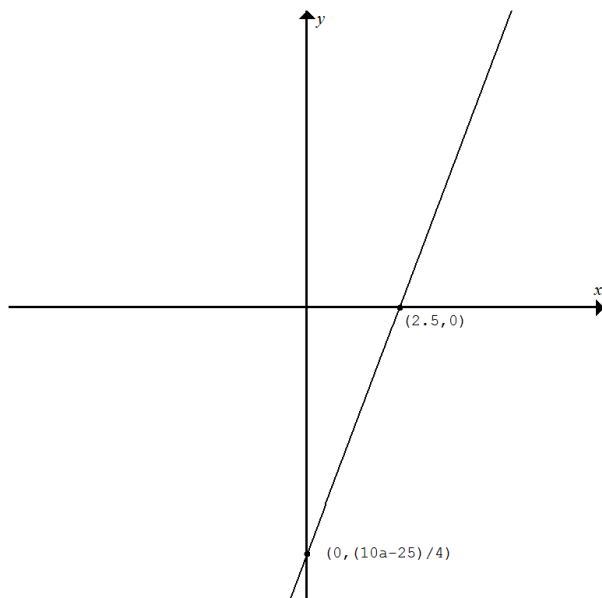
SECTION 2

Q1a i $f(x) - g(x) = (x - a)\left(x - \frac{5}{2}\right) - \left(x^2 - 5x + \frac{25}{4}\right)$
 $= (x - a)\left(x - \frac{5}{2}\right) - \left(x - \frac{5}{2}\right)^2 = \left(\frac{5}{2} - a\right)\left(x - \frac{5}{2}\right)$
 $= \left(\frac{5}{2} - a\right)x + \left(\frac{10a - 25}{4}\right)$ $\therefore B = \frac{5}{2} - a$ and $C = \frac{10a - 25}{4}$

Q1a ii $y = \left(\frac{5}{2} - a\right)x + \left(\frac{10a - 25}{4}\right)$

x-intercept: Let $y = 0$, $x = \frac{5}{2}$

y-intercept: $y = C = \frac{10a - 25}{4}$



C

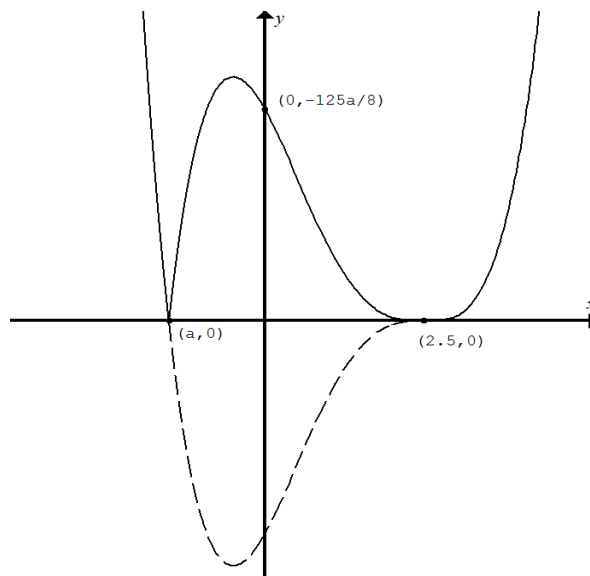
Q1b $y = (x - a)\left(x - \frac{5}{2}\right) + \left(x - \frac{5}{2}\right)^2 = 2\left(x - \frac{5}{4} - \frac{a}{2}\right)\left(x - \frac{5}{2}\right)$

At the turning point, $x = \frac{1}{2}\left(\frac{5}{4} + \frac{a}{2} + \frac{5}{2}\right) = \frac{1}{2}\left(\frac{15}{4} + \frac{a}{2}\right)$

D

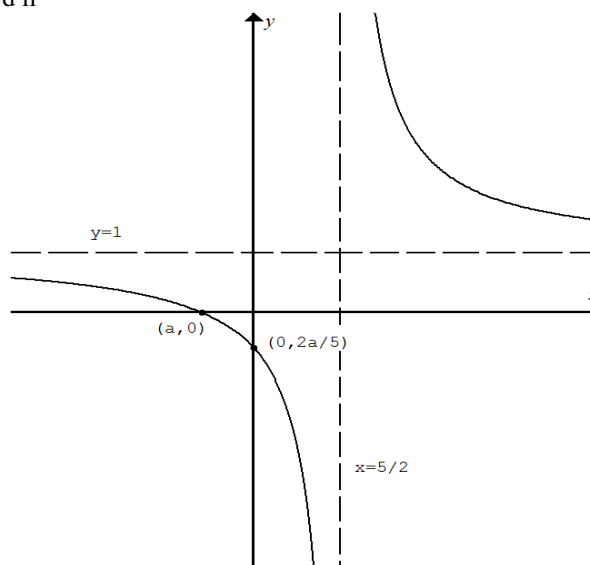
Q1c $y = |f(x)||g(x)| = |f(x)g(x)| = \left|(x - a)\left(x - \frac{5}{2}\right)\right|^3$

E



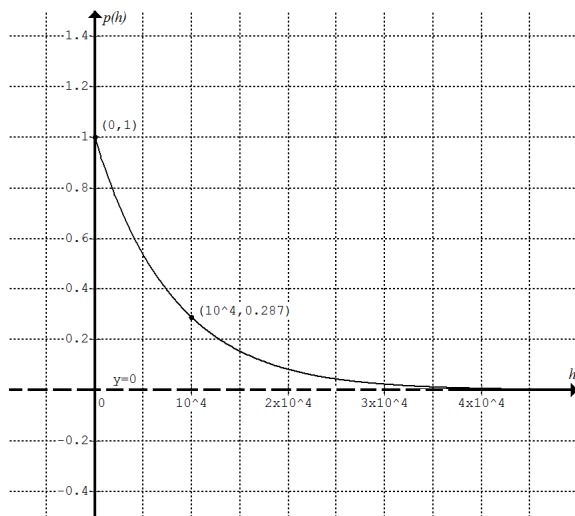
Q1d i $y = \frac{f(x)}{g(x)} = \frac{(x - a)\left(x - \frac{5}{2}\right)}{\left(x - \frac{5}{2}\right)^2} = \frac{x - a}{x - \frac{5}{2}}$
 $= \frac{x - \frac{5}{2} + \frac{5}{2} - a}{x - \frac{5}{2}} = 1 + \frac{\frac{5}{2} - a}{x - \frac{5}{2}}$, $\therefore A = \frac{5}{2} - a$

Q1d ii



Q2a $p(h) = p_0 \times 10^{kh}$, $p(0) = p_0 \times 10^0 = 1.00$, $\therefore p_0 = 1.00$
 $p(1.00 \times 10^4) = 1.00 \times 10^{(1.00 \times 10^4)k} = 0.287$
 $\therefore 10^4 k = \log_{10} 0.287$, $\therefore k = -5.42 \times 10^{-5}$

Q2b



Q2c $p = p_0 \times 10^{kh}$, $10^{kh} = \frac{p}{p_0}$,
 $h = \frac{1}{k} \log_{10} \frac{p}{p_0} \approx 1.84 \times 10^4 \log_{10} p$

Q2d $\frac{p_{\text{cabin}}}{p_0} = \frac{p(1600)}{p_0} = 10^{-5.42 \times 10^{-5} \times 1600} \approx 0.82$

Q2e Average rate of change $= \frac{\Delta p}{\Delta h} = \frac{0.82 - 1.00}{1600} \approx -1.13 \times 10^{-4}$
 Average rate of decrease $\approx 1.13 \times 10^{-4}$ atmosphere per metre

Q2f The two graphs have the same shape, intercept and asymptote.

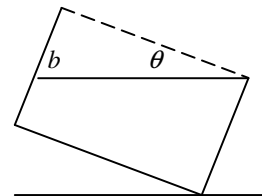
Q2g $p = 1.00 \times 10^{-5.42 \times 10^{-5} h} = e^{ch}$
 $\therefore e^c = 10^{-5.42 \times 10^{-5}}$, $c = \log_e (10^{-5.42 \times 10^{-5}}) = -1.25 \times 10^{-4}$

Q2h $\frac{dp}{dh} = -1.25 \times 10^{-4} \times e^{-1.25 \times 10^{-4} \times 1600} \approx -1.02 \times 10^{-4}$
 Rate of decrease $= 1.02 \times 10^{-4}$ atmosphere per metre

Q2i $p = e^{ch}$, $\frac{dp}{dh} = ce^{ch} = cp$, $\therefore \frac{dp}{dh} \propto p$ and the constant of proportionality $c = -1.25 \times 10^{-4}$.

Q3a $\tan \alpha^\circ = \frac{\sqrt{3}}{3}$, $\alpha = 30$

Q3b

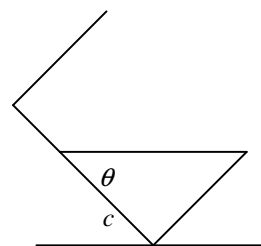


$b = 3 \tan \theta^\circ$

Volume of water spilled out $= \frac{1}{2} \times 3 \times 3 \tan \theta^\circ \times 3$

Volume of water remains $= 3 \times 3 \times \sqrt{3} - \frac{1}{2} \times 3 \times 3 \tan \theta^\circ \times 3$
 $= \frac{27}{2} \left(\frac{2}{\sqrt{3}} - \tan \theta^\circ \right)$

Q3c

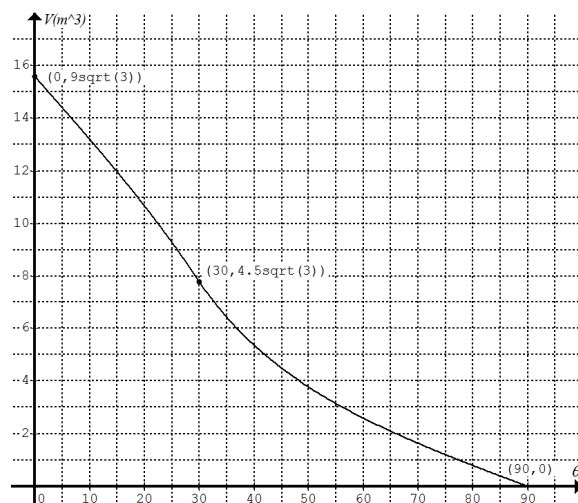


$c = \frac{\sqrt{3}}{\tan \theta^\circ}$

Volume of water remains

$= f(\theta) = \frac{1}{2} \times \frac{\sqrt{3}}{\tan \theta^\circ} \times \sqrt{3} \times 3 = \frac{9}{2 \tan \theta^\circ}$

Q3d



$$\begin{aligned} \text{Q3e } V &= \frac{27}{2} \left(\frac{2}{\sqrt{3}} - \tan \theta^\circ \right) = \frac{27}{2} \left(\frac{2}{\sqrt{3}} - \tan \frac{\pi\theta}{180} \right) \\ \frac{dV}{d\theta} &= -\frac{27}{2} \times \frac{\pi}{180} \sec^2 \frac{\pi\theta}{180} = -\frac{3\pi}{40} \sec^2 \theta^\circ = -\frac{3\pi}{40 \cos^2 \theta^\circ} \\ V &= \frac{9}{2 \tan \theta^\circ} = \frac{9}{2} \left(\tan \frac{\pi\theta}{180} \right)^{-1} \\ \frac{dV}{d\theta} &= -\frac{\pi}{180} \times \frac{9}{2 \tan^2 \frac{\pi\theta}{180}} \times \sec^2 \frac{\pi\theta}{180} = -\frac{\pi \sec^2 \theta^\circ}{40 \tan^2 \theta^\circ} = -\frac{\pi}{40 \sin^2 \theta^\circ} \end{aligned}$$

$$\begin{aligned} \text{Q3f } \text{When } \theta \rightarrow 30, -\frac{3\pi}{40 \cos^2 \theta^\circ} &= -\frac{\pi}{10}, -\frac{\pi}{40 \sin^2 \theta^\circ} = -\frac{\pi}{10} \\ \therefore \text{both derivatives approach the same value } &-\frac{\pi}{10} \text{ m}^3 \text{ per degree} \end{aligned}$$

$$\begin{aligned} \text{Q3g } \frac{dV}{dt} &= \frac{dV}{d\theta} \times \frac{d\theta}{dt} = -\frac{\pi}{10} \times 0.2 = -\frac{\pi}{50} \\ \text{Rate of decrease} &= \frac{\pi}{50} \text{ m}^3 \text{ per second} \end{aligned}$$

$$\text{Q4a } \mu = 65, \sigma = 10$$

Note: Some of the following answers may differ in the second decimal place because of using CAS ($\mu = 65, \sigma = 10$) or working from the given graph.

$$\text{Q4b } \Pr(60 < X < 70) \approx 0.37 \text{ from graph}$$

$$\text{Q4c } 0.31 = 31\% \text{ from graph}$$

$$\text{Q4d } 100\% - 31\% = 69\%$$

$$\text{Q4e } \Pr(\text{B grade}) = \Pr(70 < X < 80) \approx 0.26 \text{ from graph}$$

$$\text{Q4f}$$

$$\Pr(\text{at least one B grade}) = 1 - \Pr(\text{none}) \approx 1 - (1 - 0.26)^3 \approx 0.59$$

$$\text{Q4g } \Pr(\text{B grade or above}) \approx 1 - 0.68 = 0.32$$

$$\text{Mean number of students} = np = 3 \times 0.32 = 0.96$$

$$\text{Q4h } \Pr(\text{B grade or below}) \approx 0.94$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{3 \times 0.94 \times 0.06} \approx 0.41$$

$$\text{Q4i } \Pr(\text{B grade | higher than D grade}) = \frac{0.26}{0.68} \approx 0.382$$

$$\Pr(X = 2) = {}^3C_2 \times 0.382^2 \times (1 - 0.382) \approx 0.27$$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors