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Mathematical Methods (CAS)

2015

Trial Examination 1 (1 hour)

Instructions

Answer **all** questions. Do **not** use CAS/calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Question 1

Consider the parabolas $y = -x^2$ and $y = 2(x - a)^2 + b$ where $a, b \in \mathbb{R} \setminus \{0\}$.

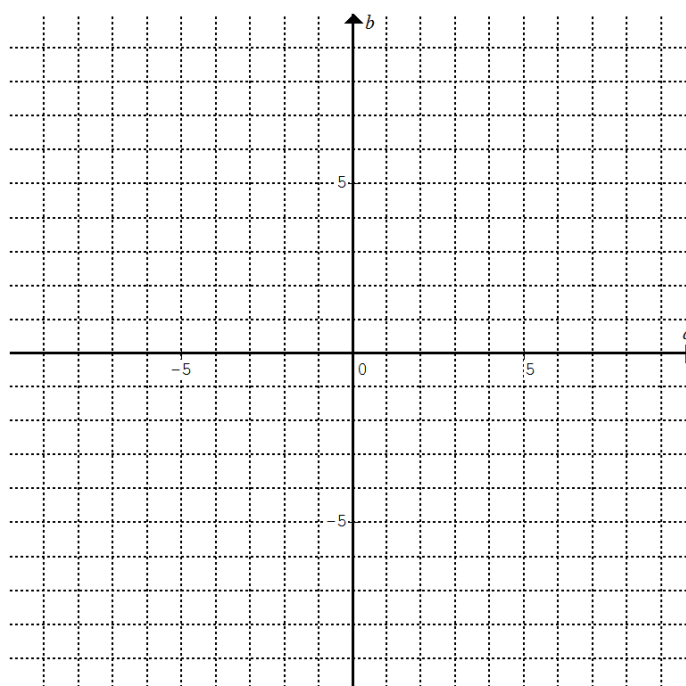
The two parabolas **touch** each other

- a. Find a value for each of a and b .

2 marks

- b. Sketch the graph of b versus a .

2 marks



Question 2

- a. The graph of $y = \frac{1}{x+1} + 1$ is reflected in the line $y = x$, followed by a translation of 2 units in the negative x -direction and then a translation of 2 units in the positive y -direction.

Find the equation of the resulting graph.

2 marks

Given $h: D \rightarrow \mathbb{R}$, $h(x) = \sqrt{\frac{1}{x+1} + 1}$ and $g: \left(a, \frac{5\pi}{6}\right) \rightarrow \mathbb{R}$, $g(x) = 2\sin x$, where D is the maximal domain of function h , and $a \in \mathbb{R}$.

- b. Find D , the maximal domain of function h .

2 marks

- c. Find the minimum value of a such that $h \circ g$ is defined.

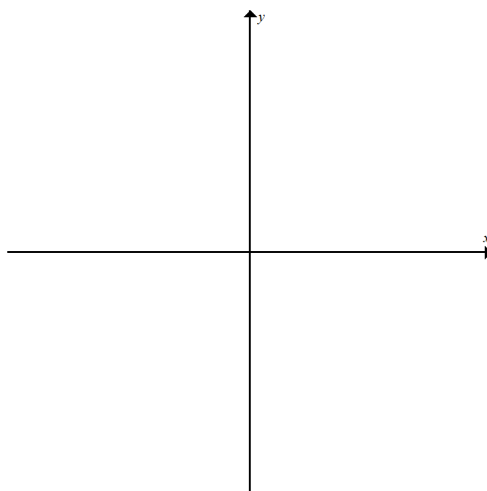
2 marks

Question 3

Consider the function $f(x) = e^{-x} - \frac{x}{e}$, given $e \approx 2.7$.

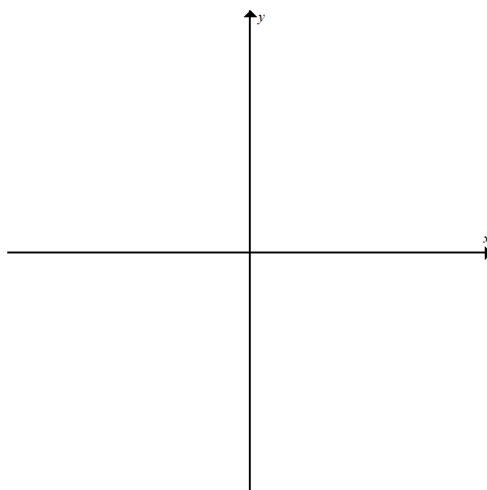
- a. Use the method of addition of ordinates to sketch the graph of $y = f(x)$.
Show the coordinates of the axis-intercepts.

3 marks



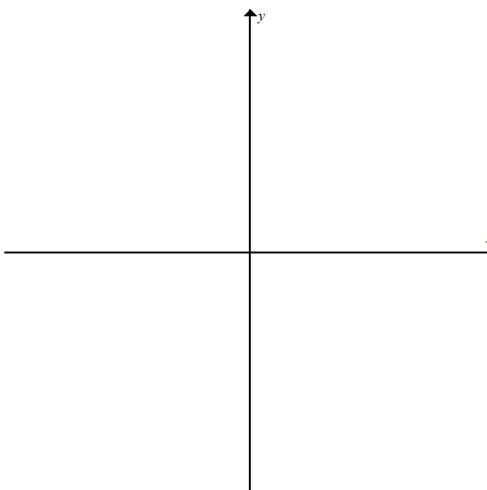
- b. Sketch the graph of $y = |f(x)|$.

1 mark



- c. Sketch the graph of $y = |f(|x|)|$.

2 marks



Question 4

Consider the graphs of $y = \sqrt{x} - 1$ and $y = \sqrt{2x+2} - 2$. $\frac{x}{a} + \frac{y}{b} = 1$ is common tangent to the two graphs.

- a. Find the coordinates of the intersection(s) of the two curves. 3 marks

- b. Find the value(s) of a and the corresponding value(s) of b . 2 marks

Question 5

Consider $f(x) = 2\log_{10}(2x-1) - \log_{10}(x+1)$.

- a. State the domain of the function. 1 mark

- b. Write down the equation(s) of the asymptote(s). 1 mark

- c. Find the coordinates of the x -intercept(s) of the graph of $y = f(x)$. 2 marks

Question 6

Use $f(a+h) \approx f(a) + hf'(a)$ to estimate the value of $\sin 46^\circ$. Leave your answer in terms of π and surds.

3 marks

Question 7

a. Given $y = e^x(\sin x - \cos x)$, show that $\frac{dy}{dx} = 2e^x \sin x$.

1 mark

b. Use the result in part a to find $\int_0^{\frac{\pi}{3}} e^x \sin x \, dx$.

2 marks

Question 8

- a. Write down the equation of the inverse of $y = (x-1)^2 + 1$. Express y in terms of x . 1 mark

- b. Find the area of the region bounded by the line $x = 2$ and the inverse of $y = (x-1)^2 + 1$. 2 marks

Question 9

There are only two routes for a student to travel to school, Route A or Route B.
The two routes are equally likely to be chosen by the student to travel to school.
The student goes to school five days a week.

The probability that the student travels to school by Route A on n days or more in a week is $\frac{13}{16}$, where $n \in \{1, 2, 3, 4, 5\}$.

Find the probability that the student travels to school by Route B on n days or less in a week.

2 marks

Question 10

$f(x) = \begin{cases} 0, & x < 1 \\ |k(x-2)| + \frac{1}{5}, & 1 \leq x \leq 5 \\ 0, & x > 5 \end{cases}$ is a probability density function of random variable X , where $x \in X$.

a. Show that $|k| = \frac{1}{25}$.

2 marks

b. Find the median of X .

2 marks

End of exam 1