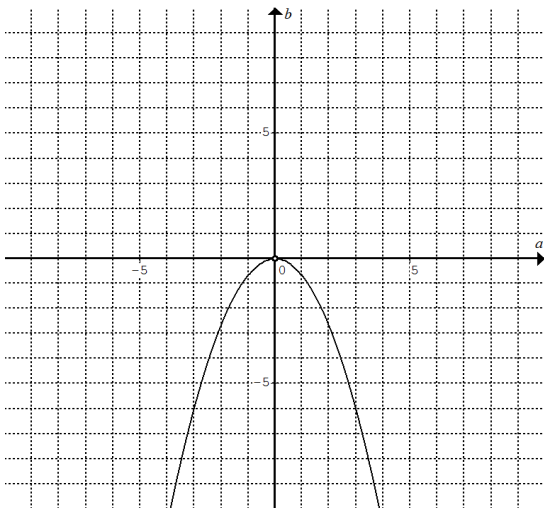


Q1a Let $2(x-a)^2 + b = -x^2$, expand and collect like terms,
 $3x^2 - 4ax + (2a^2 + b) = 0$

To have one point of contact, $\Delta = 0$, $\therefore 2a^2 + 3b = 0$
Pick a value for a , say $a = 3$, then $b = -6$

Q1b $b = -\frac{2}{3}a^2$

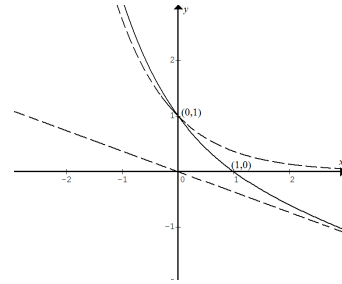


Q2a $y = \frac{1}{x+1} + 1 \rightarrow x = \frac{1}{y+1} + 1 \rightarrow x+2 = \frac{1}{y+1} + 1$
 $\rightarrow x+2 = \frac{1}{y-2+1} + 1$, simplify and write y as the subject of
the equation, $y = \frac{1}{x+1} + 1$

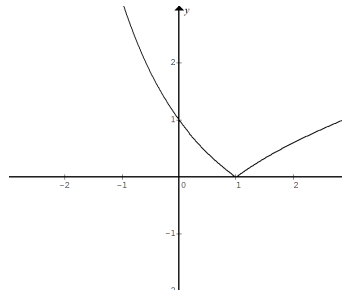
Q2b $\frac{1}{x+1} + 1 \geq 0$, $x \neq -1$ and $\frac{1}{x+1} \geq -1$
If $x+1 > 0$, $x > -1$ and $1 \geq -x-1$, i.e. $x \geq -2$, $\therefore x > -1$
If $x+1 < 0$, $x < -1$ and $1 \leq -x-1$, i.e. $x \leq -2$, $\therefore x \leq -2$
 $\therefore D$ is $(-\infty, -2] \cup (-1, \infty)$

Q2c $\left(a, \frac{5\pi}{6}\right)$ is a continuous interval,
 $\therefore$ the range of g is also a continuous interval
 $h \circ g$ is defined if the range of $g \subseteq D$
 $\therefore$ the range of $g \subseteq (-1, \infty)$
 $\therefore 2 \sin a = -1$, $a = -\frac{\pi}{6}$

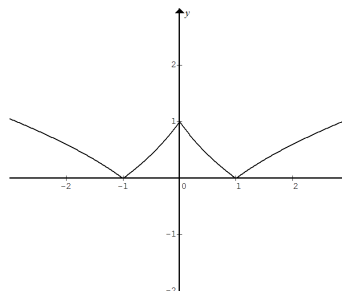
Q3a



Q3b



Q3c



Q4a Let $\sqrt{2x+2} - 2 = \sqrt{x} - 1$, $\sqrt{2x+2} = \sqrt{x} + 1$ where $x \geq 0$
and $2x+2 > 0$, i.e. $x \geq 0$
 $(\sqrt{2x+2})^2 = (\sqrt{x} + 1)^2$, $2x+2 = x+2\sqrt{x}+1$, $x+1 = 2\sqrt{x}$
 $(x+1)^2 = (2\sqrt{x})^2$, $x^2 + 2x + 1 = 4x$, $x^2 - 2x + 1 = 0$, $(x-1)^2 = 0$
 $\therefore x=1$ and $y=0$, the intersection is $(1,0)$.

Q4b $y = \sqrt{2x+2} - 2$, $\frac{dy}{dx} = \frac{1}{\sqrt{2x+2}}$
 $y = \sqrt{x} - 1$, $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

The gradient of the common tangent is $-\frac{b}{a}$.

$\therefore -\frac{b}{a} = \frac{1}{\sqrt{2x+2}} = \frac{1}{2\sqrt{x}}$, $\therefore 2x+2 = 4x$, $x=1$ and $y=0$

$\therefore$ the common tangent $\frac{x}{a} + \frac{y}{b} = 1$ is at $(1,0)$ and has a gradient

of $-\frac{b}{a} = \frac{1}{2}$

$\therefore \frac{1}{a} + \frac{0}{b} = 1$, $a=1$ and $b=-\frac{1}{2}$

Q5a $2x-1 > 0$ and $x+1 > 0$, $\therefore x > \frac{1}{2}$ and $x > -1$, $\therefore x > \frac{1}{2}$

The domain is $\left(\frac{1}{2}, \infty\right)$.

Q5b As $x \rightarrow 0.5^+$, the value of $f(x) \rightarrow -\infty$, $\therefore x = \frac{1}{2}$ is an asymptote of $y = f(x)$. It is the only one.

Q5c Let $2\log_{10}(2x-1) - \log_{10}(x+1) = 0$.

$$\therefore \log_{10} \frac{(2x-1)^2}{x+1} = 0, \therefore \frac{(2x-1)^2}{x+1} = 1$$

Expand and simplify to $4x^2 - 5x = 0$, $x(4x-5) = 0$

Since $x > \frac{1}{2}$, $\therefore x = \frac{5}{4}$ and $y = 0$.

The only x -intercept is $\left(\frac{5}{4}, 0\right)$.

Q6 $\sin 46^\circ = \sin(45^\circ + 1^\circ) = \sin\left(\frac{\pi}{4} + \frac{\pi}{180}\right)$
 $\approx \sin \frac{\pi}{4} + \frac{\pi}{180} \times \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{\pi}{180} \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \left(1 + \frac{\pi}{180}\right)$

Q7a $\frac{dy}{dx} = e^x(\cos x + \sin x) + e^x(\sin x - \cos x) = 2e^x \sin x$

Q7b $\frac{dy}{dx} = 2e^x \sin x$, $\therefore \int_0^{\frac{\pi}{3}} 2e^x \sin x \, dx = \left[e^x(\sin x - \cos x)\right]_0^{\frac{\pi}{3}}$

$$\therefore \int_0^{\frac{\pi}{3}} e^x \sin x \, dx = \frac{1}{2} \left[e^x(\sin x - \cos x)\right]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{2} \left(e^{\frac{\pi}{3}} \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) - e^0(0-1)\right) = \frac{1}{4} e^{\frac{\pi}{3}} (\sqrt{3}-1) + \frac{1}{2}$$

Q8a Equation of the inverse: $(y-1)^2 + 1 = x$, $(y-1)^2 = x-1$,
 $y = 1 \pm \sqrt{x-1}$

Q8b It is the same area as the region bounded by $y = (x-1)^2 + 1$ and $y = 2$. When $y = 2$, $2 = (x-1)^2 + 1$, $x = 0, 2$

$$\text{Area} = \int_0^2 (2 - [(x-1)^2 + 1]) \, dx = \int_0^2 (1 - (x-1)^2) \, dx$$

$$= \left[x - \frac{(x-1)^3}{3}\right]_0^2 = \frac{4}{3}$$

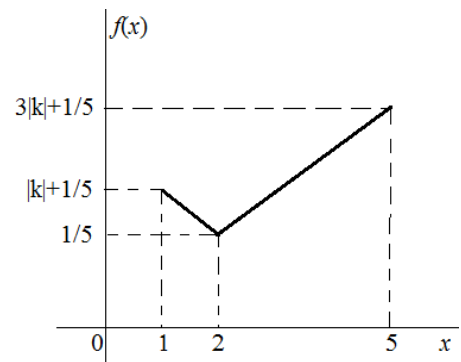
Q9 Binomial distribution, $N = 5$, $p = \frac{1}{2}$

x	0	1	2	3	4	5
$\Pr(X = x)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

$\Pr(X \geq n) = \frac{13}{16}$, $\therefore n = 2$

$\Pr(X \leq 2) = \frac{1}{32} + \frac{5}{32} + \frac{10}{32} = \frac{1}{2}$

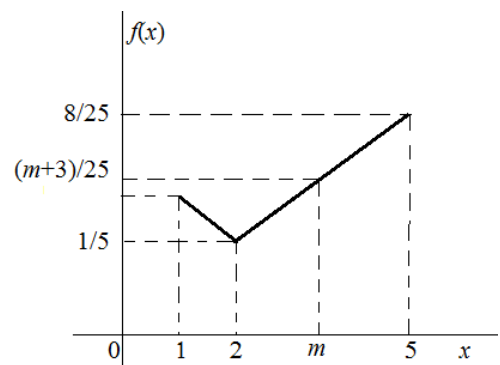
Q10a $f(1) = k + \frac{1}{5}$, $f(2) = \frac{1}{5}$, $f(5) = 3k + \frac{1}{5}$



Area under graph $= \frac{1}{2} \left(k + \frac{1}{5} + \frac{1}{5}\right) + \frac{3}{2} \left(3k + \frac{1}{5} + \frac{1}{5}\right) = 1$

$\frac{10|k|}{2} + \frac{4}{5} = 1$, $|k| = \frac{1}{25}$

Q10b By inspection of the graph, the median $m \in [2, 5]$



$f(x) = \frac{1}{25}(x-2) + \frac{1}{5} = \frac{x+3}{25}$

$\therefore f(m) = \frac{m+3}{25}$

Area under the graph from $x = m$ to $x = 5$:

$\frac{1}{2} \left(\frac{m+3}{25} + \frac{8}{25}\right) (5-m) = \frac{1}{2}$, $m^2 + 6m - 30 = 0$ and $m > 0$

$\therefore m = -3 + \sqrt{39}$

Please inform mathline@itute.com re conceptual and/or mathematical errors