

**MATHS METHODS (CAS) 3 & 4
TRIAL EXAMINATION 2
SOLUTIONS
2015**

SECTION 1 – Multiple-choice answers

1. E	9. D	17. A
2. C	10. C	18. D
3. E	11. E	19. E
4. A	12. D	20. C
5. B	13. D	21. B
6. C	14. D	22. C
7. D	15. D	
8. B	16. E	

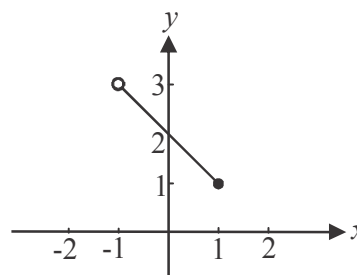
SECTION 1 – Multiple-choice solutions

Question 1

Do a quick sketch.

$$r_f = [1, 3)$$

The answer is E.



Question 2

The maximal domain occurs when $x^2 - x - 6 \geq 0$ since the square root of a negative number is undefined for $x \in \mathbb{R}$.

Solve $x^2 - x - 6 \geq 0$ for x .

$$x \leq -2 \text{ or } x \geq 3$$

In interval notation, this is expressed as $x \in (-\infty, -2] \cup [3, \infty)$.

The answer is C.

Question 3

Method 1 - intuitively

$$y = \tan\left(\frac{\pi x}{4}\right)$$

$$\text{period} = \frac{\pi}{n} \text{ where } n = \frac{\pi}{4}$$

$$= \pi \div \frac{\pi}{4}$$

$$= \pi \times \frac{4}{\pi}$$

$$= 4$$

The graph of this function is dilated by a factor of 2 from the y -axis, that is, it is stretched horizontally by a factor of 2, so its period will be 8.

The answer is E.

Method 2 – algebraically

$$y = \tan\left(\frac{\pi x}{4}\right)$$

The graph of this function is dilated by a factor of 2 from the y -axis, so replace x with $\frac{x}{2}$ in the equation.

$$y = \tan\left(\frac{\pi}{4} \times \frac{x}{2}\right)$$

$$= \tan\left(\frac{\pi x}{8}\right)$$

The period of this transformed function is

$$\frac{\pi}{n} = \pi \div \frac{\pi}{8}$$

$$= 8$$

The answer is E.

Question 4

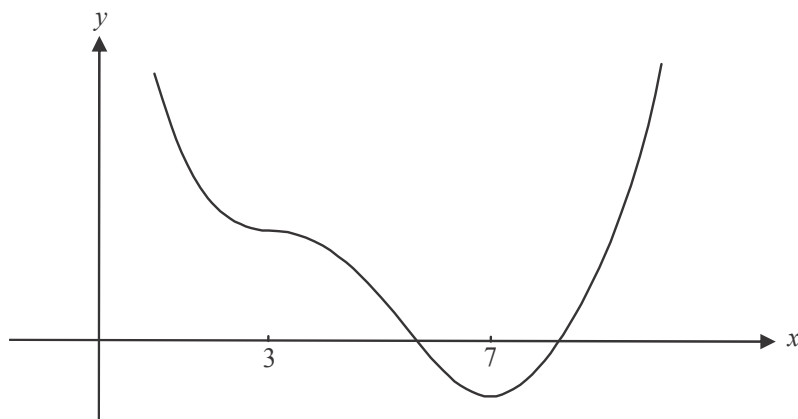
The period of the graph is π . Since the period is $\frac{2\pi}{n}$, $\frac{2\pi}{n} = \pi$ so $n = 2$.

The shape of the graph is that of an inverted cos graph that has been translated 1 unit down so the rule could be $g(x) = -\cos(2x) - 1$.

The answer is A.

Question 5

Do a quick sketch of a possible graph.



There is a stationary point of inflection at $x = 3$. Note that there could be an x -intercept at $x = 7$ if the graph shown was translated vertically up but there doesn't have to be.

The answer is B.

Question 6

$$h(x) = x^3 + 17x^2 - 24x + 6$$

$$h'(x) = 3x^2 + 34x - 24 = 0$$

$$(x+12)(3x-2) = 0$$

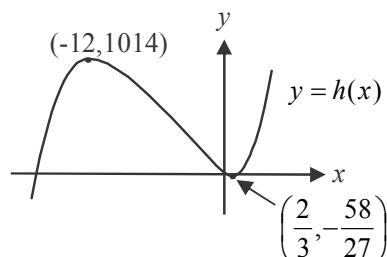
Stationary points occur at $x = -12$ and $x = \frac{2}{3}$.

Do a quick sketch.

An inverse will exist if h is a 1:1 function.

This will only occur if $D = \left[-12, \frac{2}{3}\right]$

The answer is C.

**Question 7**

$$\begin{aligned} & \int_2^4 (1 - 3h(x)) dx \\ &= \int_2^4 1 dx - 3 \int_2^4 h(x) dx \\ &= [x]_2^4 - 3 \times -1 \\ &= 4 - 2 + 3 \\ &= 5 \end{aligned}$$

The answer is D.

Question 8

$$f'(x) = e^{\sqrt{x}}, \quad x \geq 0$$

$$f(x) = \int e^{\sqrt{x}} dx$$

$$\text{So } f(x) = 2e^{\sqrt{x}}(\sqrt{x} - 1) + c \quad \text{using CAS}$$

Since $f(0) = 1$,

$$1 = 2e^0(0 - 1) + c$$

$$c = 3$$

$$\text{So } f(x) = 2e^{\sqrt{x}}(\sqrt{x} - 1) + 3$$

The answer is B.

Question 9

$$E(X) = 0 \times 0.1 + 2 \times a + 3 \times b + 5 \times 0.2 = 2.6$$

$$\text{So } 2a + 3b = 1.6 \quad \text{--- (A)}$$

$$\text{Also, } 0.1 + a + b + 0.2 = 1$$

$$a + b = 0.7 \quad \text{--- (B)}$$

$$(B) \times 2 \quad 2a + 2b = 1.4 \quad \text{--- (C)}$$

$$(A) - (C) \quad b = 0.2$$

$$\text{In (B)} \quad a = 0.5$$

The answer is D.

Question 10

Pr(no black balls)

$$= \Pr(W, W) + \Pr(W, R) + \Pr(R, W)$$

$$= \frac{5}{10} \times \frac{4}{9} + \frac{5}{10} \times \frac{1}{9} + \frac{1}{10} \times \frac{5}{9}$$

$$= \frac{30}{90}$$

$$= \frac{1}{3}$$

The answer is C.

Question 11

$$y = x^{\frac{2}{3}} + 1$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{2}{3} x^{-\frac{1}{3}} \\ &= \frac{2}{3\sqrt[3]{x}} \end{aligned}$$

When $x = -1$, $\frac{dy}{dx} = \frac{2}{3 \times -1} = -\frac{2}{3}$.

So the gradient of the tangent is $-\frac{2}{3}$ and the gradient of the normal is therefore $\frac{3}{2}$.

Equation of normal:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{3}{2}(x - -1)$$

$$y = \frac{3}{2}x + \frac{3}{2} + 2$$

$$y = \frac{3}{2}x + \frac{7}{2}$$

The y-intercept is $\frac{7}{2}$.

The answer is E.

Question 12

$$f(x) = e^{\frac{x}{2}}, \quad x \in R$$

$$\begin{aligned} (f(2x))^2 &= \left(e^{\frac{2x}{2}} \right)^2 \\ &= (e^x)^2 \\ &= e^{2x} \quad (\text{NOT } e^{x^2}) \end{aligned}$$

$$\text{Now, } f\left(\frac{x}{2}\right) = e^{\frac{\frac{x}{2}}{2}}$$

$$\begin{aligned} &= e^{\frac{x}{4}} \\ &\neq e^{2x} \end{aligned}$$

$$\begin{aligned} f(2x^2) &= e^{\frac{2x^2}{2}} \\ &= e^{x^2} \\ &\neq e^{2x} \end{aligned}$$

$$\begin{aligned} f(4x^2) &= e^{\frac{4x^2}{2}} \\ &= e^{2x^2} \\ &\neq e^{2x} \end{aligned}$$

$$\begin{aligned} f(4x) &= e^{\frac{4x}{2}} \\ &= e^{2x} \end{aligned}$$

So option D is correct.

$$\begin{aligned} \text{Note } 2(f(x))^2 &= 2 \left(e^{\frac{x}{2}} \right)^2 \\ &= 2e^x \\ &\neq e^{2x} \end{aligned}$$

The answer is D.

Question 13

Since A and B are independent events,

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

$$0.2 = 0.5 \times \Pr(B) \quad \text{So } \Pr(B) = 0.4$$

$$\begin{aligned} \Pr(A|B) &= \frac{\Pr(A \cap B)}{\Pr(B)} \\ &= \frac{0.2}{0.4} \\ &= 0.5 \end{aligned}$$

The answer is D.

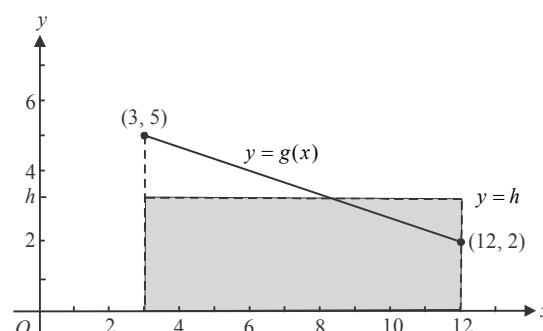
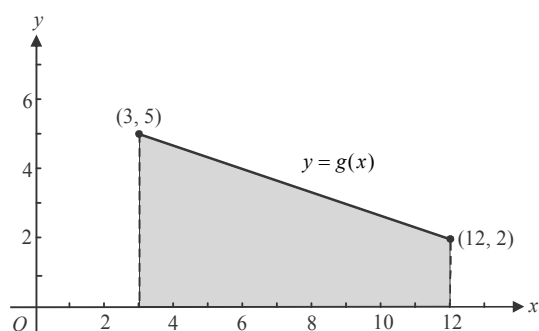
Question 14

$$\Pr(X > 4) = \int_4^6 \frac{1}{(x+2)\log_e(2)} dx$$

$$= 0.415037\dots$$

The closest answer is 0.4150.

The answer is D.

Question 15

The two shaded regions shown in the two diagrams above are equal in area. The average value of g is h , as indicated in the second diagram.

We want to find h .

shaded area in diagram 1 = shaded area in diagram 2

area of rectangle + area of Δ = base $\times$ height

$$9 \times 2 + \frac{1}{2} \times 9 \times 3 = 9 \times h$$

$$\frac{63}{2} = 9h$$

$$h = 3.5$$

The answer is D.

Question 16

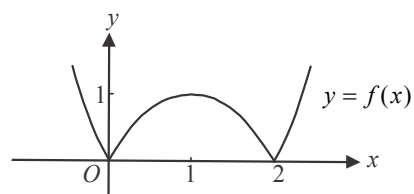
Do a quick sketch.

For f , $f(1)=1$, the graph is continuous for $x \in \mathbb{R}$,

and the gradient is positive for $0 < x < 1$ and $x > 2$.

At $x=0$ and $x=2$, the graph has a sharp point; that is, it is not smoothly continuous and therefore is not differentiable at these points. Option E is not true.

The answer is E.



Question 17

$$x + 2ay = 6$$

$$ax + 8y = a + 10$$

$$\begin{bmatrix} 1 & 2a \\ a & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ a+10 \end{bmatrix}$$

$$\Delta = 1 \times 8 - 2a^2 = 0 \text{ for no solution or infinitely many solutions}$$

$$2(4 - a^2) = 0$$

$$2(2 - a)(2 + a) = 0$$

$$a = 2 \text{ or } a = -2$$

If $a = 2$, we have

$$x + 4y = 6 \quad -(A)$$

$$2x + 8y = 12 \quad -(B)$$

$$(B) \div 2 \quad x + 4y = 6 \quad -(C)$$

Since (A) and (C) are the same line, there are infinitely many solutions for $a = 2$.

If $a = -2$, we have

$$x - 4y = 6 \quad -(A)$$

$$-2x + 8y = 8 \quad -(B)$$

$$(B) \div -2 \quad x - 4y = -4 \quad -(C)$$

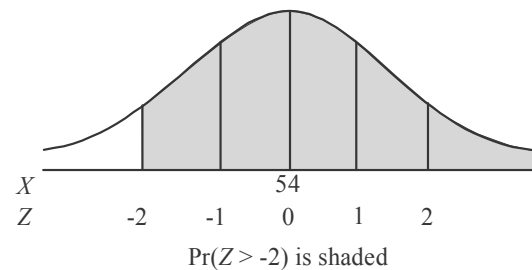
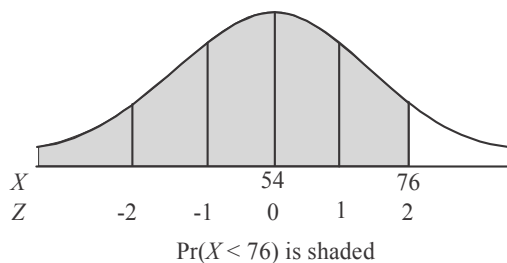
Since the lines given by (A) and (C) are parallel, there can be no solutions.

So there are infinitely many solutions only when $a = 2$.

The answer is A.

Question 18

Do a quick sketch.



Since $\Pr(X < 76) = \Pr(Z > -2)$, then 76 must lie two standard deviations above the mean.

Now $76 - 54 = 22$ so the standard deviation is 11. So the variance of X equals $11^2 = 121$.

The answer is D.

Question 19

$$y = x^2 - kx \text{ and } y = 3x - 4$$

$$x^2 - kx = 3x - 4 \quad (\text{equation of intersection})$$

$$x^2 - kx - 3x + 4 = 0$$

$$x^2 + x(-k - 3) + 4 = 0$$

The graphs intersect at two distinct points when $\Delta > 0$.

$$b^2 - 4ac > 0$$

$$(-k - 3)^2 - 4 \times 1 \times 4 > 0$$

Solve this inequation for k using CAS $k < -7$ or $k > 1$.

The answer is E.

Question 20

Since area of P = area of Q ,

$$\text{solve } \int_0^5 -x^2(x-5)dx = -1 \times \int_5^u -x^2(x-5)dx \text{ for } u.$$

Note that the x -intercepts of the graph of f occur at $x=0$ and $x=5$. Also, since area Q falls below the x -axis, we need to multiply the integral $\int_5^u -x^2(x-5)dx$ by negative one to obtain the actual area.

$$\text{Using CAS, } u=0 \text{ or } u=\frac{20}{3}.$$

$$\text{Since } u > 5, \quad u = \frac{20}{3}.$$

The answer is C.

Question 21

The relationship $f(x+0.4) \approx f(x) + 0.4 \times f'(x)$ is an application of the approximation formula $f(x+h) \approx f(x) + hf'(x)$ where $h=0.4$.

$$\text{To approximate } \frac{1}{\sqrt[3]{1.4}}, \text{ we use } f(x) = \frac{1}{\sqrt[3]{x}} \text{ and } f'(x) = \frac{-1}{3x^{\frac{4}{3}}}$$

$$\text{so } f(x+0.4) \approx f(x) + 0.4 \times f'(x)$$

$$\begin{aligned} \text{becomes } f(1+0.4) &\approx f(1) + 0.4 \times \frac{-1}{3 \times 1^{\frac{4}{3}}} \\ &= 1 - \frac{0.4}{3} \\ &= 0.86666... \end{aligned}$$

The approximation is 0.8667

The answer is B.

Question 22

A = shaded area

= area of $\triangle CDE$ + area of $\triangle ABE$

$$= \frac{1}{2} \times a \times a \times \sin(\theta) + \frac{1}{2} \times AE \times AB$$

$$= \frac{a^2}{2} \sin(\theta) + \frac{1}{2} \times a \cos(\theta) \times a \sin(\theta)$$

$$= \frac{a^2}{2} \sin(\theta) + \frac{a^2}{2} \cos(\theta) \sin(\theta)$$

Solve $\frac{dA}{d\theta} = 0$ for θ using CAS

$$\theta = \frac{\pi}{3} \text{ since } 0 < \theta < \frac{\pi}{2}$$

The answer is C.

SECTION 2

Question 1 (6 marks)

- a. At midnight on Sunday $t = 0$.

$$\begin{aligned} g(0) &= 20 - 4 \sin(0) \\ &= 20 \end{aligned}$$

The temperature is 20°C at midnight on Sunday.

(1 mark)

- b. period $= \frac{2\pi}{n}$ where $n = \frac{\pi}{12}$

$$\begin{aligned} &= 2\pi \div \frac{\pi}{12} \\ &= 24 \end{aligned}$$

(1 mark)

- c. Since the amplitude is 4, the maximum temperature is 24°C .

(1 mark)

The average temperature is 20°C .

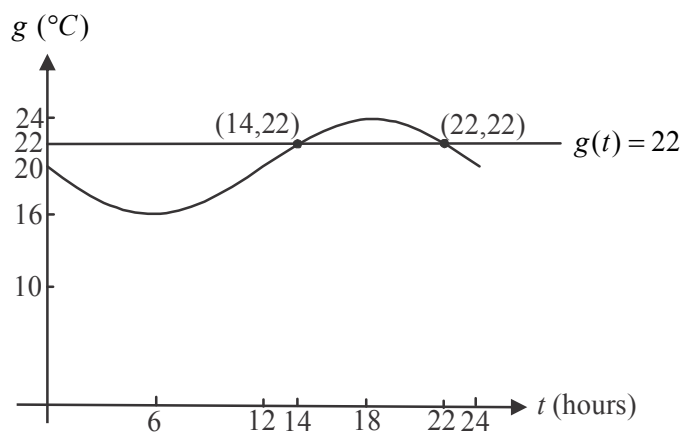
(1 mark)

- d. Solve $g(t) = 22$ for $t \in [0, 24]$

$$t = 14 \text{ or } 22$$

(1 mark)

A quick sketch of the graph of $y = g(t)$ shows that the temperature inside the greenhouse is above 22°C for 8 hours a day between $t = 14$ and $t = 22$.



$$\begin{aligned} \text{Cost} &= 7(5.20 \times 16 + 6.80 \times 8) \\ &= \$963.20 \end{aligned}$$

(1 mark)

Question 2 (15 marks)

a. $\text{mean} = E(X) = \int_2^5 \left(x \times \frac{(x-5)^2 + 1}{12} \right) dx$
 $= \frac{47}{16} \text{ hours}$ (1 mark)

b. Solve $\int_2^m \frac{(x-5)^2 + 1}{12} dx = 0.25$ for m . (1 mark)

$$m = 2.3318\dots$$

The maximum time was 2.332 hours (correct to 3 decimal places).

(1 mark)

c. $X \sim N(24, 0.3^2)$
 $\Pr(X < x) = 0.8$
 $x = 24.252486\dots$ (using the inverse normal function)
 Filters with a maximum diameter of 24.25mm (correct to 2 decimal places) can be used in a Mini car. (1 mark)

d. $\Pr(X < 23.5) = 0.047790\dots$ (using the normal cdf function)
 $0.047790\dots \times 700 = 33.4532\dots$ (1 mark)
 So 33 of the filters will fit in any car (to the nearest whole number) (1 mark)

e. i. Method 1 – using transition matrices

this car

	minor	major	
minor	q	0.6	next car
major	$1-q$	0.4	

$$T = \begin{bmatrix} q & 0.6 \\ 1-q & 0.4 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(1 mark)

$$T^3 = T^2 S_1$$

$$= \begin{bmatrix} q^2 - 0.6q + 0.6 \\ -(q-1)(q+0.4) \end{bmatrix}$$

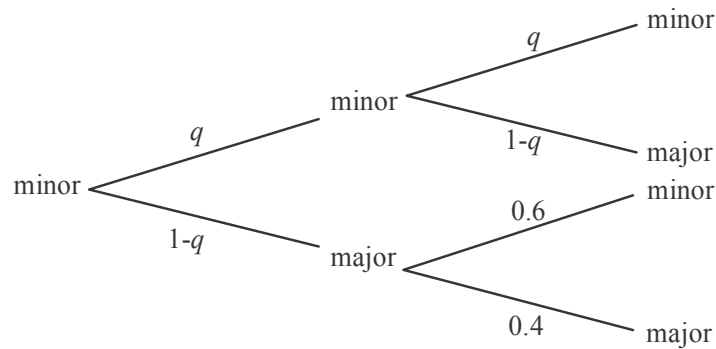
The required probability is

$$-(q-1)(q+0.4)$$

$$\text{or } -q^2 + 0.6q + 0.4$$

(1 mark)

Method 2 – using a tree diagram



(1 mark)

$$\begin{aligned}
 &\Pr(3^{\text{rd}} \text{ service is major}) \\
 &= q(1-q) + (1-q) \times 0.4 \\
 &= q - q^2 + 0.4 - 0.4q \\
 &= -q^2 + 0.6q + 0.4
 \end{aligned}$$

(1 mark)

- ii. Solve $-q^2 + 0.6q + 0.4 = 0.2$
 $q = -0.23851\dots$ or $q = 0.83851\dots$
 Since $0 < q < 1$, $q = 0.839$ (correct to 3 decimal places)

(1 mark)

(1 mark)

- iii. On this day $T = \begin{bmatrix} 0.7 & 0.6 \\ 0.3 & 0.4 \end{bmatrix}$ and $S_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
 $S_8 = T^7 S_1$
 $= \begin{bmatrix} 0.666\dots \\ 0.333\dots \end{bmatrix}$

(1 mark)

The probability that the last car had a minor service was $\frac{2}{3}$.

(1 mark)

- f. i. Solve $(0.8)^n = 0.32768$ for n
 $n = 5$

(1 mark)

- ii. This is a conditional probability question.

$$\Pr(X > 16 | X \geq 10)$$

$$= \frac{\Pr(X > 16 \cap X \geq 10)}{\Pr(X \geq 10)}$$

$$= \frac{\Pr(X > 16)}{\Pr(X \geq 10)} \quad \begin{array}{l} \text{(binom cdf(20,0.8,17,20))} \\ \text{(binom cdf(20,0.8,10,20))} \end{array}$$

(1 mark)

$$= \frac{0.4114488\dots}{0.99943\dots}$$

$$= 0.4117 \text{ (correct to 4 decimal places)}$$

(1 mark)

Question 3 (11 marks)

- a.** From the diagram, we see that the maximum height occurs at the back wall of the bay where $x = 10$.
So $h(10) = 0.82822\dots$
Height is 0.83m (correct to 2 decimal places). **(1 mark)**
- b.** Solve $h'(x) = 0$ for x . **(1 mark)**
 $x = 0.354001\dots$ or $x = 1.622882\dots$
So the interval required is $x \in [0.35, 1.62]$ where the endpoints are expressed correct to 2 decimal places. **(1 mark)**
- c.** Solve $h'(x) = h(x)$ for x . **(1 mark)**
 $x = 0.179568\dots$
So $h(0.179568\dots) = 0.142515\dots$
The height is 0.14 metres (correct to 2 decimal places). **(1 mark)**
- d.** Since the average rate of change of h between $(1, h(1))$ and $(p, h(p))$ is zero, then $h(1) = h(p)$.
Solve $h(1) = h(p)$ for p . **(1 mark)**
 $p = 0.10541\dots$ or $p = 0.999999\dots$ or $p = 2.273895\dots$
Since $p > 1$, $p = 2.27$ (correct to 2 decimal places). **(1 mark)**
- e.** volume of topsoil $= 3 \times \int_0^{10} h(x) dx$ **(1 mark)**
 $= 11.50244\dots$
The estimate for the volume of topsoil is 11.50 cubic metres (correct to 2 decimal places). **(1 mark)**
- f.** We are looking for the average value of the function h over the interval $x \in [0, 10]$.
average value $= \frac{1}{10 - 0} \int_0^{10} h(x) dx$ **(1 mark)**
 $= 0.383414\dots$
The height of the top soil would be 0.38 metres (correct to 2 decimal places). **(1 mark)**

Question 4 (12 marks)

- a. $f(x) = \log_e(x+a)$
 f is defined for $x+a > 0$

$$\begin{aligned} x &> -a \\ \text{So } d_f &= (-a, \infty) \end{aligned}$$

(1 mark)

- b. i. x -intercepts occur when $y = 0$

$$y = \log_e(x+a)$$

$$0 = \log_e(x+a)$$

$$e^0 = x+a$$

$$x+a=1$$

$$x=1-a$$

If the x -intercept is negative
 then $1-a < 0$

$$1 < a$$

$$a > 1$$

(1 mark)

- ii. From part i. the x -intercept occurs when $x = 1-a$.

If the x -intercept is positive then
 then $1-a > 0$

$$1 > a$$

$$a < 1$$

Also, we are told in the question that a is positive so $0 < a < 1$.

(1 mark)

- c. i. Given $f(g(x)) = \frac{x}{b}$

then $\log_e(g(x)+a) = \frac{x}{b}$

$$e^{\frac{x}{b}} = g(x)+a$$

$$g(x) = e^{\frac{x}{b}} - a$$

as required.

(1 mark)

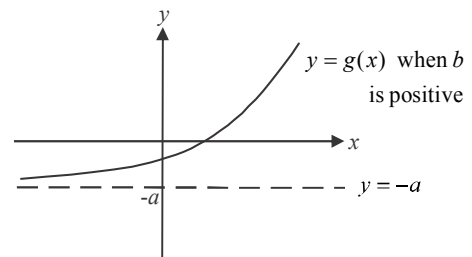
- ii. $f(g(x))$ is a composite function and $f(g(x))$ is defined iff $r_g \subseteq d_f$

Now $d_f = (-a, \infty)$ from part a.

To find the range of g , do a quick sketch.

$$r_g = (-a, \infty)$$

Since $r_g = d_f$, then $f(g(x))$ is defined.

**(1 mark)**

- d. i. $f(x) = \log_e(x+a)$
 Let $y = \log_e(x+a)$
 Swap x and y for inverse.
 $x = \log_e(y+a)$
 $e^x = y+a$
 $y = e^x - a$
 So $f^{-1}(x) = e^x - a$
 $d_{f^{-1}} = r_f$
 Since $r_f = R$
 then $d_{f^{-1}} = R$

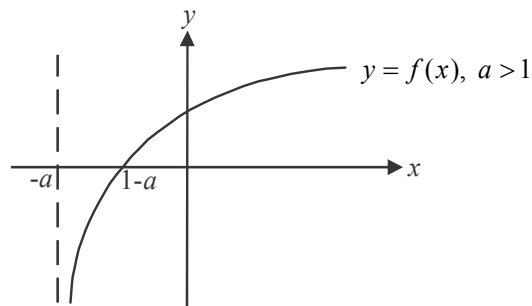
(1 mark)

(1 mark)

- ii. Since $f^{-1}(x) = e^x - a$ and $g(x) = e^{\frac{x}{b}} - a$, the transformation required to transform the graph of f^{-1} to the graph of g is a dilation from the y -axis by a factor of b units.

(1 mark)

- e. Since $a > 1$, the x -intercept of the graph of f will be negative from part b. i. Do a quick sketch.



The x -intercept occurs at $x = 1 - a$ from part b. so

$$\begin{aligned} \text{area required} &= \int_{1-a}^0 f(x) dx \\ &= a \log_e(a) - a + 1 \text{ square units} \quad (\text{using CAS}) \end{aligned}$$

(1 mark) – correct integrand

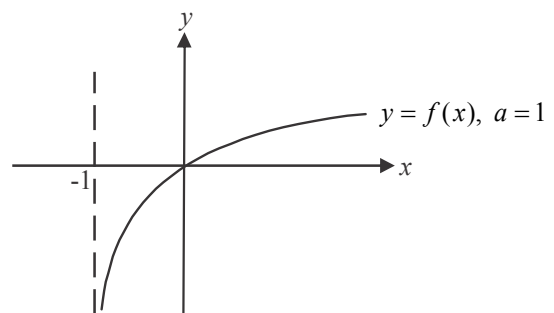
(1 mark) correct terminals

(1 mark) correct answer

- f. From part b, if $a > 1$ the graph of f has a negative x -intercept and if $0 < a < 1$, the graph of f has a positive x -intercept.

When $a = 1$, the graph of f passes through the origin hence the area enclosed by the graph of $y = f(x)$ and the x and y axes equals zero.

Alternatively, there is no area **enclosed** by the graph of $y = f(x)$ and the x and y axes.



(1 mark)

Question 5 (14 marks)

- a.** The height of the land above sea level on the island, as seen from the naval ship is given by the function l . Define this function on your CAS.
At sea level, $l(x) = 0$
Solve $l(x) = 0$ for x
 $x = 0$ or $x = 2000$
So the width of the island as observed from the naval ship is $2000 - 0 = 2000$ metres.
(1 mark)
- b.** Find $l'(x)$.
Solve $l'(x) = 0$ for x
 $x = 695.519033...$
Substitute this value into $l(x)$.
 $l(695.519033...) = 217.494939...$
The highest point on the island occurs at the point (695.5190, 217.4949) where coordinates are expressed correct to 4 decimal places.
(1 mark)
- c. i.** $d(x) = b(x) - l(x)$
$$= \frac{x^4}{10^{10}} - \frac{x^3}{2 \times 10^6} + \frac{19x^2}{2 \times 10^4} - \frac{7x}{10} + 200$$

(1 mark)
- ii.** $d_b = [0, 3000]$
 $d_l = [0, 2000]$
So $d_d = [0, 2000]$
(1 mark)
- d.** Define $d(x)$ (if you haven't already) on your CAS using your answer to part c. i.
Find $d'(x)$
Solve $d'(x) = 0$ for x
 $x = 626.586096...$
Substitute this into $d(x)$.
 $d(626.586096...) = 26.781607...$
The minimum vertical distance between the balloon and the land is 26.78m (correct to 2 decimal places).
(1 mark)
- e.** Find $l'(x)$.
Substitute $x = 1000$ into $l'(x)$.
$$l'(1000) = -\frac{1}{10}$$

(1 mark)
The equation of the tangent has a gradient of $-\frac{1}{10}$ and passes through (1000, 200).
The equation is given by
$$y - 200 = -\frac{1}{10}(x - 1000)$$

$$y = -\frac{x}{10} + 300$$

(1 mark)

- f. The missile crosses the flight path of the balloon when $b(x) = -\frac{x}{10} + 300$.

Solve this equation for x .

$$x = -1000(\sqrt{2} - 2) \text{ or } x = 1000(\sqrt{2} + 2)$$

$$= 585.7864... \quad = 3414.2135...$$

Since $d_b = [0, 3000]$, we reject this second value of x .

So $x = -1000(\sqrt{2} - 2)$.

(1 mark)

Note that the numerical values here are used only to see whether the values of x are feasible. Since we have not been asked for decimal approximations in the question, we must use the exact values of the coordinates.

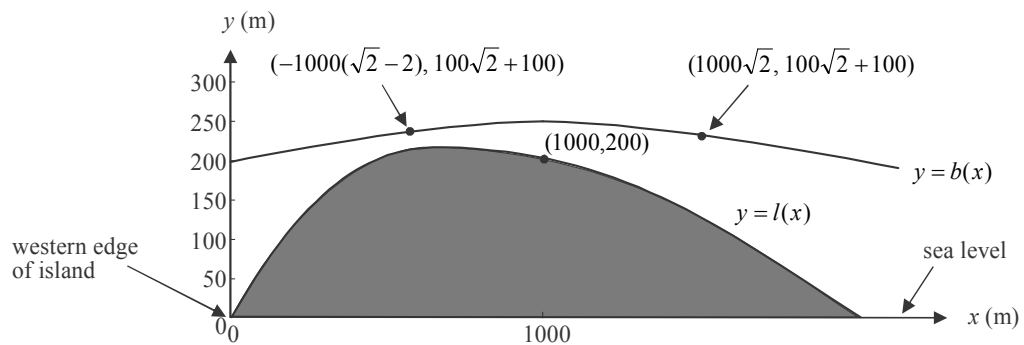
Substitute $x = -1000(\sqrt{2} - 2)$ into $b(x)$ or into $y = -\frac{x}{10} + 300$.

$$b(-1000(\sqrt{2} - 2)) = 100\sqrt{2} + 100$$

The missile crosses the flight path of the balloon at the point $(-1000(\sqrt{2} - 2), 100\sqrt{2} + 100)$

(1 mark)

g.



To find the location of the balloon when the enemy missile crosses the flight path of the balloon, solve $b(x) = 100\sqrt{2} + 100$ for x .

$$x = -1000(\sqrt{2} - 2) \text{ or } x = 1000\sqrt{2}.$$

The balloon is located at the point $(1000\sqrt{2}, 100\sqrt{2} + 100)$.

(1 mark)

At this instant, Victoria fires a missile to hit the point $(1000\sqrt{2}, 100\sqrt{2} + 100)$.

gradient of path of missile

$$= \frac{100\sqrt{2} + 100 - 200}{1000\sqrt{2} - 1000}$$

$$= \frac{1}{10} \quad \textbf{(1 mark)}$$

$$m = \tan \theta$$

So $\tan \theta = \frac{1}{10}$

$$\theta = \tan^{-1}\left(\frac{1}{10}\right)$$

So $p = 10$

(1 mark)

