

TEST 7

Functions, graphs, algebra and calculus
Technology-free end-of-year examination
Total marks: 30
Suggested writing time: 45 minutes

Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 1

For the function $f(x) = e^x \sin(2x)$, find $f'(x)$.

$$f(x) = e^x \sin(2x)$$

Using the product rule,

$$f'(x) = e^x \sin(2x) + 2e^x \cos(2x)$$

2 marks

QUESTION 2

a For the function $f(x) = x^2 \log_e(x)$, find $f'(x)$.

$$f(x) = x^2 \log_e(x)$$

Using the product rule,

$$\begin{aligned} f'(x) &= 2x \log_e(x) + x^2 \times \frac{1}{x} \\ &= 2x \log_e(x) + x \end{aligned}$$

2 marks

b Hence find $f'(2)$.

$$f'(x) = 2x \log_e(x) + x$$

$$f'(2) = 4 \log_e(2) + 2$$

1 mark
(Total: 3 marks)

QUESTION 3

a For the function $f(x) = \frac{\log_e(x^2 + 1)}{e^{3x}}$, find $f'(x)$.

$$f(x) = \frac{\log_e(x^2 + 1)}{e^{3x}}$$

Using the quotient and chain rules,

$$\begin{aligned} f'(x) &= \frac{e^{3x} \times \frac{2x}{x^2 + 1} - \log_e(x^2 + 1) \times 3e^{3x}}{e^{6x}} \\ &= \frac{\frac{2x}{x^2 + 1} - 3 \log_e(x^2 + 1)}{e^{3x}} \end{aligned}$$

3 marks

b Hence find $f'(1)$.

$$\begin{aligned} f'(x) &= \frac{\frac{2x}{x^2 + 1} - 3 \log_e(x^2 + 1)}{e^{3x}} \\ f'(1) &= \frac{1 - 3 \log_e(2)}{e^3} \end{aligned}$$

1 mark
(Total: 4 marks)

QUESTION 4

a If $\frac{dy}{dx} = e^{3x}$, find an expression for y .

$$\frac{dy}{dx} = e^{3x}$$

$$y = \int e^{3x} dx = \frac{1}{3} e^{3x} + c$$

2 marks

b If $\frac{dy}{dx} = e^{3x}$ and $y = 1$ when $x = 0$, find y .

$$y = \frac{1}{3} e^{3x} + c$$

$$x = 0, y = 1 \text{ gives } 1 = \frac{1}{3} e^0 + c$$

$$c = \frac{2}{3}$$

$$y = \frac{1}{3} e^{3x} + \frac{2}{3}$$

2 marks
(Total: 4 marks)

QUESTION 5

Evaluate $\int_0^1 x^2 dx$.

$$\int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1$$
$$= \frac{1}{3}$$

2 marks

QUESTION 6

Evaluate $\int_0^{\frac{\pi}{6}} 2 \cos(3x) dx$.

$$\int_0^{\frac{\pi}{6}} 2 \cos(3x) dx = \left[\frac{2}{3} \sin(3x) \right]_0^{\frac{\pi}{6}}$$
$$= \frac{2}{3} \sin\left(\frac{\pi}{2}\right)$$
$$= \frac{2}{3}$$

2 marks

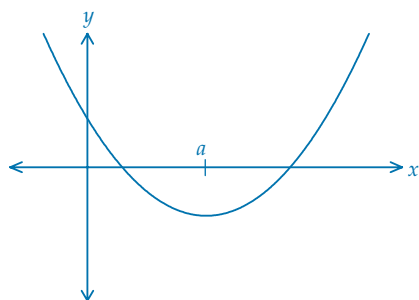
QUESTION 7

- a What type of stationary point is found at $x = a$ using the information

$$x < a, f'(x) < 0$$

$$x = a, f'(x) = 0$$

$$x > a, f'(x) > 0$$



A local minimum at $x = a$.

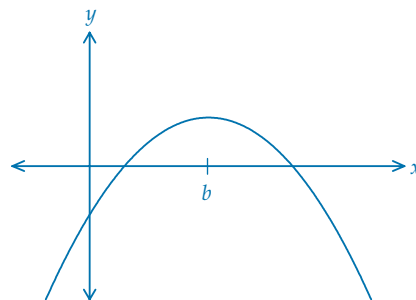
1 mark

- b What type of stationary point is found at $x = b$ using the information

$$x < b, f'(x) > 0$$

$$x = b, f'(x) = 0$$

$$x > b, f'(x) < 0$$



A local maximum at $x = b$.

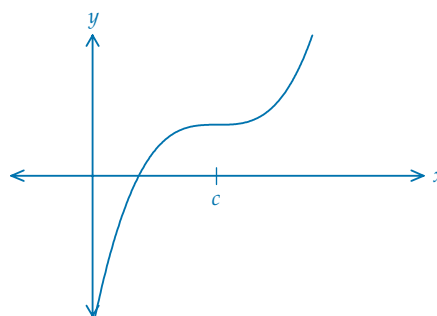
1 mark

- c What type of stationary point is found at $x = c$ using the information

$$x < c, f'(x) > 0$$

$$x = c, f'(x) = 0$$

$$x > c, f'(x) > 0$$



A stationary point of inflection at $x = c$.

1 mark

(Total: 3 marks)

QUESTION 8

- a Find the x -coordinates for the stationary points of the graph of $f(x) = \frac{1}{2}x^4 - x^2 + 3$.

$$f(x) = \frac{1}{2}x^4 - x^2 + 3$$

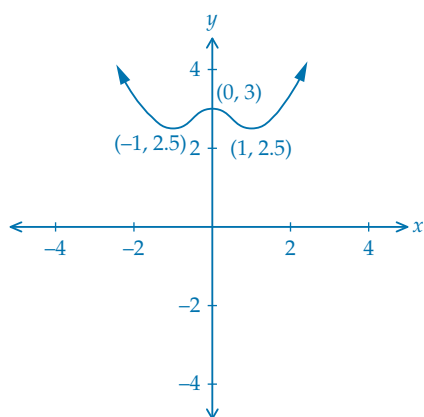
$$f'(x) = 2x^3 - 2x = 0 \text{ for stationary points.}$$

$$\Rightarrow 2x(x^2 - 1) = 0$$

$$\text{This gives } x = 0, x = 1, x = -1$$

2 marks

- b Given that there are no x -intercepts on the graph, sketch the graph of $f(x)$, labelling the coordinates of the stationary points.



2 marks

- c State the intervals over which the graph is strictly increasing.

$$x \in [-1, 0] \cup [1, \infty)$$

1 mark

- d State the intervals over which the graph is strictly decreasing.

$$x \in (-\infty, -1] \cup [0, 1]$$

1 mark

(Total: 6 marks)

QUESTION 9

It is known that $\int_1^3 f(x)dx = 1$

- a Evaluate $\int_1^3 (f(x) + 1)dx$.

$$\begin{aligned} \int_1^3 (f(x) + 1)dx &= \int_1^3 f(x)dx + \int_1^3 1dx \\ &= 1 + [x]_1^3 \\ &= 1 + 3 - 1 \\ &= 3 \end{aligned}$$

2 marks

- b Evaluate $\int_3^1 (-2f(x) + 3)dx$.

$$\begin{aligned} \int_3^1 (-2f(x) + 3)dx &= -2\int_3^1 f(x)dx + \int_3^1 3dx \\ &= (-2 \times (-1)) + [3x]_3^1 \\ &= 2 + 3 - 9 \\ &= -4 \end{aligned}$$

2 marks

(Total: 4 marks)