TEST 4

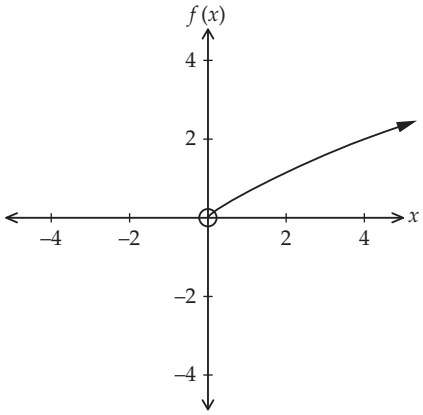
Functions, graphs and algebra
Technology-free end-of-year examination
Total marks: 30
Suggested writing time: 45 minutes

Specific instructions to students

- Answer all of the questions in the spaces provided.
- Show all workings in questions where more than one mark is available.
- An exact value must be provided in questions where a numerical answer is required, unless otherwise specified.

QUESTION 1

The graph of $f: R^+ \to R, f(x) = \sqrt{x}$ is shown below.



a The graph of $y = \sqrt{x}$ is dilated by a factor of 2 from the y-axis to become the graph of g. Write down the rule for g.

$g(x) = \sqrt{\frac{x}{2}}$

1 mark

b The graph of $y = \sqrt{x}$ is dilated by a factor of 3 from the x-axis to become the graph of h. Write down the rule for h.

$h(x) = 3\sqrt{x}$

1 mark

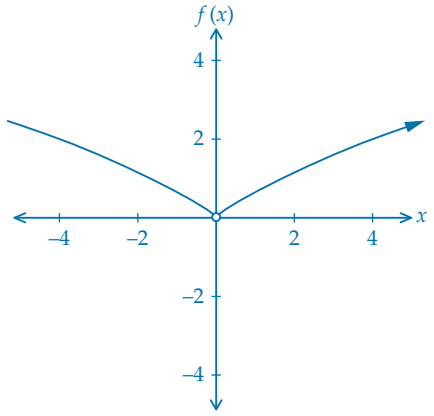
c The graph of $y = \sqrt{x}$ is translated by 4 units in the positive direction of the x-axis to become the graph of j. Write down the rule for j.

$j(x) = \sqrt{x-4}$

1 mark

d Give the equation of, and add to the diagram given, a sketch of $y = \sqrt{x}$ if it is reflected over the y-axis.

$y = \sqrt{-x}$



2 marks
(Total: 5 marks)

QUESTION 2

a For the function $f(x) = x^4 + x^3 - 7x^2 - x + 6$, $f(1) = 0$ and $f(-1) = 0$. Find the fully factorised form of $f(x)$.

Using synthetic division

	1	1	–7	–1	6
–1		–1	0	7	–6
	1	0	–7	6	0

→ remainder

and again

	1	0	–7	6
1		1	1	–6
	1	1	–6	0

→ remainder

This gives the quotient $x^2 + x - 6$.

So $x^4 + x^3 - 7x^2 - x + 6 = (x-1)(x+1)(x^2 + x - 6)$

$f(x) = (x-1)(x+1)(x+3)(x-2)$

2 marks

b If $g(x)$ is the graph of $f(x)$ translated 1 unit to the right and dilated by a factor of 3 from the x-axis, write down the equation for $g(x)$ in factorised form.

$f(x) = (x-1)(x+1)(x+3)(x-2)$

$g(x) = 3f(x-1) = 3(x-1-1)(x+1-1)(x+3-1)(x-2-1)$

$g(x) = 3(x-2)(x)(x+2)(x-3)$

$g(x) = 3x(x-2)(x+2)(x-3)$

2 marks
(Total: 4 marks)

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QUESTION 3

If $x^2 + 4x + 1 = (x - a)^2 + b$, find the values of a and b.

$x^2 + 4x + 1 = (x^2 - 2ax + a^2) + b = x^2 - 2ax + a^2 + b$

Equating coefficients,

$4 = -2a$ (coefficients of x^1)

$1 = a^2 + b$ (coefficient of x^0)

Hence $a = -2$, $b = -3$

2 marks

QUESTION 4

If $f(x) = kx$, where $k \in R$, show that $f(x) - f(y) = f(x - y)$.

LHS $= f(x) - f(y)$

$= kx - ky$

$= k(x - y)$

RHS $= k(x - y) =$ LHS

2 marks

QUESTION 5

Simplify the expression $5 \log_2(5) \times \log_5(2)$.

Using a change of base for both terms,

$\log_2(5) = \dfrac{\log_e(5)}{\log_e(2)}$ and $\log_5(2) = \dfrac{\log_e(2)}{\log_e(5)}$

$5 \log_2(5) \times \log_5(2) = \dfrac{5 \log_e(5)}{\log_e(2)} \times \dfrac{\log_e(2)}{\log_e(5)}$

$= 5$

2 marks

QUESTION 6

Evaluate

a $\sin\left(\dfrac{3\pi}{2}\right)$

$\sin\left(\dfrac{3\pi}{2}\right) = -1$

1 mark

b $\cos\left(\dfrac{7\pi}{2}\right)$

$\cos\left(\dfrac{7\pi}{2}\right) = 0$

1 mark

c $\tan\left(\dfrac{5\pi}{3}\right)$

$\tan\left(\dfrac{5\pi}{3}\right) = -\sqrt{3}$

1 mark

d $\cos(-9\pi)$

$\cos(-9\pi) = -1$

1 mark

(Total: 4 marks)

QUESTION 7

Solve for x in the equation $64^{x-2} \times 4^x = 16$

$4^{3(x-2)} \times 4^x = 4^2$

$4^{3x-6} \times 4^x = 4^2$

$4^{4x-6} = 4^2$

Equating indices,

$4x - 6 = 2$

$4x = 8$

$x = 2$

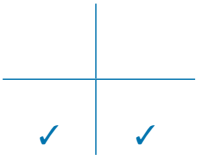
2 marks

QUESTION 8

Solve for x in the equation $2 \sin(2x) = -1$, where $x \in [0, 2\pi]$.

$\sin(2x) = -\dfrac{1}{2}$

Reference angle: $\sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}$



$2x = \pi + \dfrac{\pi}{6}, 2\pi - \dfrac{\pi}{6}, 3\pi + \dfrac{\pi}{6}, 4\pi - \dfrac{\pi}{6},$

$2x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, \dfrac{19\pi}{6}, \dfrac{23\pi}{6}$

$x = \dfrac{7\pi}{12}, \dfrac{11\pi}{12}, \dfrac{19\pi}{12}, \dfrac{23\pi}{12}$

3 marks

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QUESTION 9

a Find the general solution to the equation $\cos(3x) = \frac{\sqrt{2}}{2}$.

$3x = 2n\pi \pm \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$3x = 2n\pi \pm \frac{\pi}{4}$

$x = \frac{1}{3}\left(2n\pi \pm \frac{\pi}{4}\right)$

$x = \frac{2n\pi}{3} \pm \frac{\pi}{12}$ where $n \in \boldsymbol{Z}$

2 marks

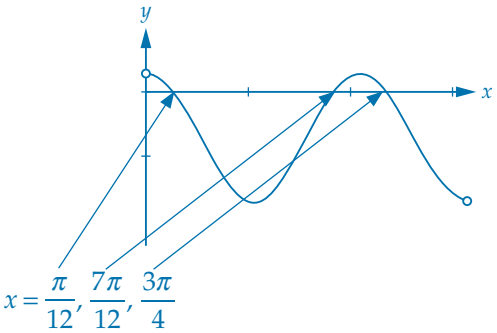
b Hence, find all the exact solutions in the interval $(0, \pi)$.

n	0	1
$\frac{2n\pi}{3} + \frac{\pi}{12}$	$\frac{\pi}{12}$	$\frac{9\pi}{12} = \frac{3\pi}{4}$
$\frac{2n\pi}{3} - \frac{\pi}{12}$	$-\frac{\pi}{12}$	$\frac{7\pi}{12}$

Answer: $x = \frac{\pi}{12}, \frac{3\pi}{4}, \frac{7\pi}{12}$

2 marks

c Hence sketch the graph of $y = \cos(3x) - \frac{\sqrt{2}}{2}$, labelling axial intercepts.



$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}$

2 marks
(Total: 6 marks)

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